Predictability and Power in Legislative Bargaining

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the power to propose

Agenda-setting rights in negotiations:

- a player is *recognized* to make a proposal.
- others vote, and votes aggregated by a voting rule.
- if proposal is rejected, fraction of pie is destroyed.

But how is the proposer determined?

Practice:

- chair nominates proposers.
- rules specify that every player / party has its turn.
- an election process selects a proposer from a group of nominees.
- players perform (political) maneuvers to become a proposer.
- seniority rules specify who makes proposals.

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Theory: *random recognition*.

Baron-Ferejohn (1989): player i is recognized with probability p_i i.i.d.

Power at time t is as unpredictable in period $t-1$ as in period 0.

But players may be able to predict bargaining power in practice:

- proposers may be pre-announced.
- constitution may exclude certain candidates.
- recognition may be history-dependent.
- chair or game with strategic maneuvering may be predictable.

our goal: understand implications of predictability for negotiations.

why study predictability?

Positive theory.

- Legislatures and organizations likely vary along this dimension.
- This dimension has not been studied in previous investigations.

Institutional design.

- Predictability is a design choice.
- Transparency about power transitions is intuitively appealing.

Theoretical curiosity.

- Compare *information structures* for a fixed recognition protocol.
- Are players better or worse off with the early resolution of uncertainty?

Outline

An example with 3 players

Baron-Ferejohn 1989

Players: $\{1, 2, 3\}$.

In each period, each is recognized with i.i.d. probability $\frac{1}{3}$.

Simple majority rule with sequential voting in a fixed order.

Each player's discount factor is $\delta < 1$.

No predictability: period-t proposer revealed in period t.

2 periods: $t \in \{0, 1\}$

$t = 1$: Final proposer captures entire dollar.

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$t = 0$: Each player's discounted continuation value is $\delta \left(\frac{1}{3}\right)$.

$\Rightarrow$ p^0 forms *minimal winning coalition* buying 1 vote.

$\Rightarrow$ first proposer captures $1 - \frac{\delta}{3}$.

all horizons, number of players = n

$$\text{Proposer Rents} = 1 - \frac{(n-1)}{2}\left(\frac{\delta}{n}\right)$$

Every finite horizon game has a sub-game perfect equilibrium in which 1^{st} proposer captures the above rents.

In infinite horizon game, this is the unique stationary sub-game perfect equilibrium outcome.

(stationary ≡ behavior is history-independent.)

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early resolution of uncertainty

Standard framework assumes all uncertainty about period-t proposer resolved in period t, and not before.

What if period-t proposer revealed before proposal in period $t-1$?

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By induction, for every finite horizon T,

> 1^{st} *proposer captures entire dollar in every SPE.*

another example

Ann, Bob, and Carol rotate in making offers.

Stationary environment in multilateral bargaining with unanimity:

$$\text{Equilibrium Shares} = \left(\frac{1}{1+\delta+\delta^2}, \frac{\delta}{1+\delta+\delta^2}, \frac{\delta^2}{1+\delta+\delta^2}\right)$$

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Instead, if voting rule is simple majority-rule we find

$$\text{Equilibrium Shares} = (1, 0, 0) \, .$$

main result

Consider non-unanimous voting rule: at least $q < n$ votes needed.

Theorem. Suppose at every history, at least q players can be ruled out with probability 1 from being tomorrow's proposer. Then, the 1^{st} proposer keeps the entire surplus in every SPE of finite horizon and every SSPE of infinite horizon.

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Strategic force:

- Information helps today's proposer target weaker coalition partners (tomorrow's non-proposers).
- Those non-proposers are weakened by tomorrow's proposer doing the same: tomorrow's proposer can't commit to being generous.

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equity and efficiency

predictability $\implies$ inequity.

predictability + risk-aversion $\implies$ inefficiency.

$u_i(x) = \sqrt{x_i}$:

$$\text{Without Predictability} = \frac{1}{\sqrt{5}}, \qquad (\approx 0.44)$$
$$\text{One-Period Predictability} = \frac{1}{3}.$$

Risk-averse players strictly prefer no early resolution of uncertainty.

Outline

environment

- A group of players: $\{1, ..., n\}$.
- Dividing a dollar: choosing x from $[0,1]^n$ such that $\sum_{i \in \mathcal{N}} x_i = 1$.
- Infinite-horizon: $t = 0, 1, 2, ...$

stages in each round

1. Proposer p^t determined by a recognition rule.
2. Information about power at $t' > t$ revealed.
3. Proposer p^t proposes a division of the dollar.
4. All vote on proposal:
   - Voting is in a fixed sequential order.
   - Proposal passes iff at least $q < n$ vote in favor.

uncertainty & recognition

Nature moves in each period, and its choices are perfectly observed.

S = a nice (complete & separable) state space.

$(\theta^t)_{t \in \mathcal{T}}$ = a time-homogeneous Markov Process that takes values in S.

$\mu(\cdot|s)$ = Markov kernel that governs transitions between states.

The recognition rule is a mapping $P : S \to \{1, \ldots, n\}$.

how to forecast next proposer

Players may infer tomorrow's proposer based on current state:

$$\Pr(i \text{ is proposer tomorrow}|s) = \mu(\{s' : P(s') = i\}|s).$$

History of proposers doesn't identify recognition probabilities:
players may learn more than observing who has been proposer thus far.

fitting examples into paradigm

Baron-Ferejohn (1989): $S = \{1, \ldots, n\}$, $P(s) = s$, and $\mu(s'|s) = \frac{1}{n}$.

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Baron-Ferejohn (1989): $S = \{1, \ldots, n\}$, $P(s) = s$, and $\mu(s'|s) = \frac{1}{n}$.

One period ahead revelation: $S = \{1, \ldots, n\}^2$, $P(s) = s_1$, and $\mu(s'|s) = 1/n$ if $s'_1 = s_2$, and 0 otherwise.

payoffs and patience

$$u_i(x, t) = \delta_i^t x_i$$

Assumption: $\delta_i < 1$ for every i.

Stationary Subgame Perfect Equilibrium

Behavior is identical across all "structurally identical" subgames.

- Payoff-relevant state is s^t: info about future power.
- Proposal strategy: $S \rightarrow \Delta X$.
- Voting strategy: $S \times X \rightarrow \Delta \{$👍, 👎$\}$.

Behavior may condition: on time ✓, history of proposers ✓, and info about future proposers ✓.
But NOT past proposals and voting decisions .

Analogue of (but slightly richer than) a MPE in this environment.

Outline

predictability

A player is a *loser* if she is known today to *not* be tomorrow's proposer:

$$L(s) = \{i : Pr(i = \text{Tomorrow's Proposer} \mid s) = 0\}$$

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Definition. A recognition rule exhibits *one-period predictability of degree* d if $|L(s)| \geqslant d$ for all $s \in S$.

main result

q-voting rule

$+$

One-period predictability of degree q

$\Downarrow$

First proposer captures entire surplus in every SSPE.

outline of proof

Construct SSPE in which proposer captures the entire surplus.

Preliminary steps for uniqueness:

- (for now, suppose $\delta_i = \delta$ for every player i.)
- every SSPE involves immediate agreement.
- proposer never offers positive shares to more than minimal winning coalition.
- individual votes 👍 if share $\geqslant$ discounted continuation value.

Proof by Contradiction

Suppose first proposer, Ann, offers > 0.

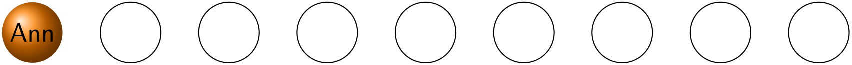
Who does Ann offer the most to?
Larry gets $x_L = \overline{x}\left(s^0\right)$.

Why Larry and neither Harry nor Mary?
they must be more expensive.

$$\min\left\{\delta V_H(s^0), \delta V_M(s^0)\right\} \geqslant \delta V_L(s^0) = \overline{x}\left(s^0\right).$$

Claim: Either Larry or someone more expensive is a loser in s^0.

Order players: Ann first, and then by discounted continuation value.



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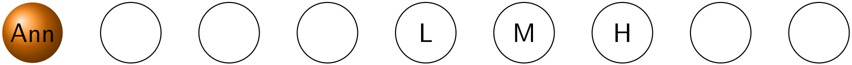

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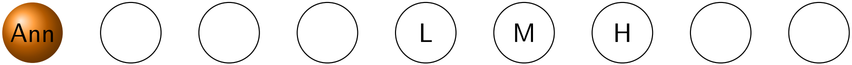

Claim: ∃ at least one Loser weakly to the right of L.

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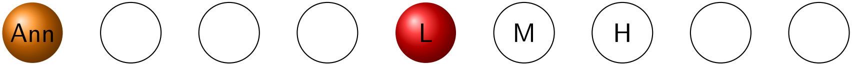

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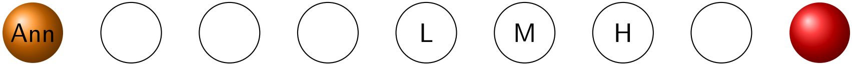

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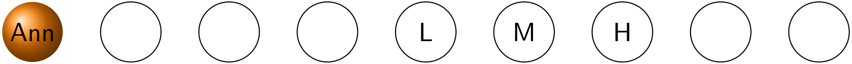

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Proof: Ann has q members in her minimal winning coalition.

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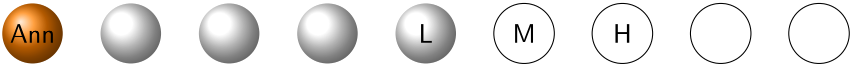

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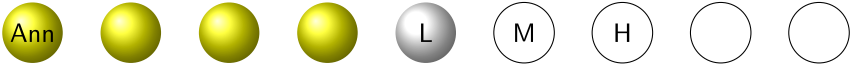

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$\implies$ Strictly to the left of L are only $q - 1$ players.

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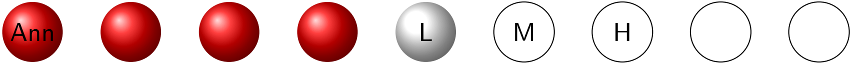

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$\implies$ Strictly to the left of L are only $q-1$ players.

But there are q losers, so at least one must be weakly to the right of L.

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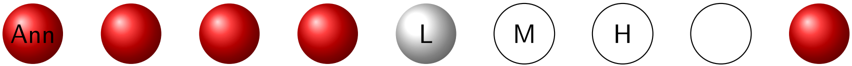

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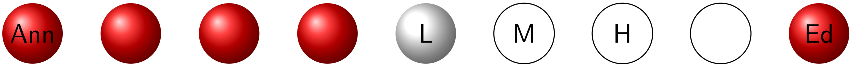

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$\Longrightarrow$ Strictly to the left of L are only $q-1$ players.

But there are q losers, so at least one must be weakly to the right of L.

(1) Ed is *at least as expensive* as Larry.

(2) Ed is a loser.

$$(1) \Longrightarrow \qquad \delta V_E(s^0) \geqslant \delta V_L(s^0) \qquad = \overline{x}(s^0)$$

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The most offered in s^1 to a non-proposer, $\overline{x}(s^1)$, satisfies

$$
\overline{x}\left(s^1\right) \geqslant \frac{\overline{x}\left(s^0\right)}{\delta}.
$$

Same pigeon-hole argument applies in period 1:

$\Rightarrow$ in some state s^2 in period 2, a proposer offers at least

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$$\overline{x}(s^2) \geqslant \frac{\overline{x}(s^1)}{\delta} \geqslant \frac{\overline{x}(s^0)}{\delta^2}.$$

$\Rightarrow$ geometrically increasing sequence of offers by a proposer:

$$\overline{x}(s^t) \geqslant \frac{\overline{x}(s^0)}{\delta^t}$$

$\delta < 1 \Rightarrow$ eventually, a proposer offers more than 1. Contradiction.

Comments

If $\delta_i \neq \delta_j$: replace δ in proofs with highest δ_i.

Can consider general stage payoff function $u_i(x_i)$ where $u_i(0) = 0$, $u_i(\cdot)$ is strictly increasing, continuous, and concave.

Neither relative patience nor risk-aversion influence bargaining shares.

Outline

robustness to perturbations

Two perturbations appear especially important:

a) Players may only be *virtually* ruled out at each time.

b) Players may not expect predictability to *perfectly persist* over time.

Investigate robustness to *almost-persistent virtual predictability*.

A player is an ϵ-loser if she isn't tomorrow's proposer with probability exceeding $\epsilon > 0$.

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States where there is virtual predictability of degree d:

$$\mathcal{P}_{d,\epsilon} = \{s : |L_\epsilon(s)| \geqslant d\}$$

Definition. The bargaining process exhibits $(1-\rho)$-persistent one-period ϵ-predictability of degree d if $s^0 \in \mathcal{P}_{d,\epsilon}$ and

$$\Pr\left(s^t \in P_{d,\epsilon} \mid s^{t-1}\right) \geqslant 1-\rho \text{ for all } s^{t-1} \in P_{d,\epsilon}.$$

Theorem

$(1-\rho)$-*persistent one-period* ϵ-*predictability of degree* q

$$\Downarrow$$

First proposer never offers more than $\frac{\delta(\epsilon+\rho)}{1-\delta(1-\epsilon-\rho)}$ *in any MPE.*

Proof

comments on robustness result

First proposer never offers more than $\frac{\delta(\epsilon+\rho)}{1-\delta(1-\epsilon-\rho)}$ in any MPE.

a) Existence? Countable state space $\rightarrow$ Duggan (2014).

b) Taking period length to 0, bound on offer is $\frac{\epsilon+\rho}{\epsilon+\rho+r}$.

(patience = $e^{-r\Delta}$, virtualness = $e^{-\epsilon\Delta}$, almost-persistence = $e^{-\rho\Delta}$.)

varying degree of predictability

Baron-Ferejohn $\leftrightarrow$ Perfect one-period predictability.

Ex ante recognition probability at time t is $\frac{1}{n}$.

At t, players learn about proposer power at $t+1$:

$$p_1 \leqslant p_2 \leqslant \ldots \leqslant p_n \text{ such that } \sum_{i=1}^{n} p_i = 1.$$

- Baron-Ferejohn: $p_1 = \ldots = p_n = 1/n$.
- Predictability of degree d: $p_1 = \ldots = p_d = 0$.

Result 1: If $q < n$, the share obtained by the first proposer is increasing in the degree of one-period predictability.

Result 2: In a large legislature,

$$\alpha_v = \lim_{n \to \infty} \frac{q_n}{n}, \alpha_p = \lim_{n \to \infty} \frac{d_n}{n}$$

Proposer's share is $1 -$ constant $\times$ $(\alpha_v - \alpha_p)$

- Gap between proportional predictability and voting rule matters.
- Proposer's share is convex in α_p.

other default options

Concern: Analysis exploits the $(0, \ldots, 0)$ disagreement outcome.

Reasons disagreement may not be so costly:

- policy reverts to previous agreement. (Baron 96, Kalandrakis 04)
- chair or caretaker govt choosing policy. (Austen-Smith & Banks 88)

Disagreement $\Rightarrow$ player i receives $\bar{x}_i > 0$.

Perhaps $\sum_{i=1}^{n} \bar{x}_i \geqslant 1$.

No cost in waiting for the default option. $(\delta = 1)$

Assumptions:

a) Generic: $\bar{x}_i \neq \bar{x}_j$ for distinct i, j.
b) Majority Improvements: $\sum_{i \in C} \bar{x}_i < 1$ for all C with $\frac{n+1}{2}$ members.
c) Perfect One-Period Predictability.
d) Simple Majority Rule.

Theorem

In a finite horizon game with at least 3 periods and at least 7 players, the first proposer captures the entire surplus in every SPE.

Every other player receives 0, despite a strictly positive default option.

Proof

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Proof

Results apply in the infinite horizon with a stochastic deadline if the deadline is one-period predictable.

A broader point: with an endogenous status quo, the number of bargaining periods in each session influences the outcome.

other extensions

a) Coalitional Bargaining

b) Political Maneuvers

c) Private Learning

d) Inequity Aversion (Fehr-Schmidt)

e) Optimism / overconfidence

Outline

one-period predictability + *excludability* $\Rightarrow$ *extreme power*.

Neither feature so powerful in isolation.

Rubinstein (1982): alternating offer $\Rightarrow$ perfect predictability.

but unanimity rule $\Rightarrow$ equal split as $\delta \to 1$.

Baron-Ferejohn (1989): excludability $\Rightarrow$ minimal winning coalition.

but no predictability $\Rightarrow$ proposer keeps approximately half surplus.

Predictability of power robustly generates inequality.

Early resolution of uncertainty disadvantages those with future bargaining power, and can be *ex ante* inefficient.

May matter more than traditional determinants of power.

Key strategic force:

- future proposers cannot commit.
- information helps each proposer target weaker coalition partners.

Thank you!

Consider a virtual loser .

$$V_i(s^0) \leqslant \epsilon + \rho + (1 - \epsilon - \rho)\overline{x}\left(s^1\right)$$

Return to Theorem

Consider a virtual loser whose vote is expensive.

$$\frac{\overline{x}\left(s^0\right)}{\delta} \leqslant V_i(s^0) \leqslant \epsilon + \rho + (1 - \epsilon - \rho)\overline{x}\left(s^1\right)$$

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$$\Rightarrow \overline{x}\left(s^1\right) \geqslant \frac{\overline{x}\left(s^0\right) - \delta(\epsilon + \rho)}{\delta(1 - \epsilon - \rho)} = f\left(\overline{x}\left(s^0\right)\right)$$

Return to Theorem

Logic of Result

1 2 3 4 5 6 7

Return to Statement

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Proceed by backward induction from final period T.

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$T - 1$: Players $5, 6$, and 7 expect 0 if disagreement today.

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If $(T - 1)$-proposer$= 7$: she needs a single vote from $\{1, 2, 3\}$.

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$T - 2$: Suppose 4 is proposer. Only players $1, 2, 7$ have positive continuation value.

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$\Rightarrow$ if $(T - 1)$-proposer $\in \{1, 2, 3, 4\}$, she has 3 free votes.

If $(T - 1)$-proposer$= 7$: she needs a single vote from $\{1, 2, 3\}$.

$T - 2$: Suppose 4 is proposer. Only players $1, 2, 7$ have positive continuation value.

$\Rightarrow$ player 4 captures entire surplus, all others 0.

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Logic of Result

Toughest case: final proposer $\in \{1, 2, 3, 4\}$.

$T - 1$: Players $5, 6$, and 7 expect 0 if disagreement today.

$\Rightarrow$ if $(T - 1)$-proposer $\in \{1, 2, 3, 4\}$, she has 3 free votes.

If $(T - 1)$-proposer$= 7$: she needs a single vote from $\{1, 2, 3\}$.

$T - 2$: Suppose 4 is proposer. Only players $1, 2, 7$ have positive continuation value.

$\Rightarrow$ player 4 captures entire surplus, all others 0.

By induction, first proposer captures surplus.

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