Reselling Information

S. Nageeb Ali, Ayal Chen-Zion, & Erik Lillethun

Penn State, Amazon, & Colgate

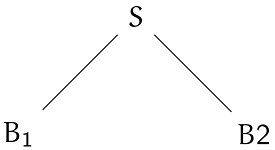
What is the impact of resale on the price of information?

Outline

1. Example: The Problem
2. Example: A Solution
3. General Model
4. Discussion

This talk is mostly set in *Exampleland*.
I am happy to remain there for as long as audiences like.

benchmark: vanilla world without resale



Seller has information (e.g., knowledge of ω).

Buyer's value for information $= 1$; payoff of 0 until then.

Each link meets with probability $\lambda\,dt$ in period of length dt.

Each player discounts future at rate $r > 0$.

Frequency of interaction per unit of *effective time* is λ/r.

Each buyer obtains info only from the seller.

Equilibrium = Nash Bargaining + Rational Expectations.

So an equilibrium price p solves

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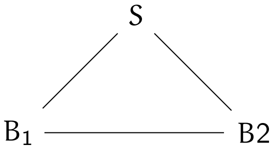
$\implies p = \frac{1}{2}$.

Without resale, buyers and seller split the surplus.

Seller's payoff $\to \frac{1}{2}\times$ social surplus as $\lambda/r \to \infty$



Once a buyer obtains info, he can sell it to the other buyer at the next trading opportunity.

Key idea: information is replicable $\Rightarrow$ buyer can both consume and sell it.

Sale of information is publicly observed.

Payoff-relevant state is the set of informed players:

$$s \in \Big\{\{S\}, \{S, B_1\}, \{S, B_2\}, \{S, B_1, B_2\}\Big\}.$$

Equilibrium $\equiv$ value functions $V_i(s)$ and prices $p_{ij}(s)$ where

1. Value functions satisfy *rational expectations* given prices,
2. Prices satisfy *symmetric Nash bargaining* given value functions:
   - Trade today iff trading today increases bilateral surplus.
   - prices split the gains from trade equally.

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Study both *immediate agreement* and *seller's optimal* equilibria.

$s = (S, B_1)$

Proceed by backward induction: suppose S and B_1 have information.

B_2 can buy information from either S or B_1: 2 trading partners.

Prices $p_{S2}(s) = p_{12}(s)$ and solve

$$\underbrace{p - p\int_0^\infty e^{-rt}e^{-2\lambda t}\lambda\, dt}_{\text{Seller's Gain from Trading Today}} = \underbrace{(1-p) - (1-p)\int_0^\infty e^{-rt}e^{-2\lambda t}2\lambda\, dt}_{\text{Buyer's Gain from Trading Today}}.$$

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$$\implies p(2) = \frac{r}{2r+\lambda},$$

which converges to 0 in a frictionless market ($\lambda/r \to \infty$).

key idea

For buyer, gain from trading today is cost of delay ≈ 0.

For a seller, gain from trading today $>> 0$ because she may lose buyer to other seller.

Equating these two gains implies prices must vanish.

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Equating these two gains implies prices must vanish.

Is this intuitive?

- Yes: Bertrand outcome expected if B_2 met S and B_1 simultaneously.
- No: B_2 meets only one at a time, faces costs from delay, and so *Diamond Paradox* may apply.

Slight bargaining power to the buyer averts the Diamond Paradox.

two uninformed buyers remain

Let $\gamma \equiv \int_0^\infty e^{-rt} e^{-2\lambda t} \lambda \, dt$, which converges to $\frac{1}{2}$ as $\lambda / r \to \infty$.

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Suppose S meets a buyer.

Buyer's payoff:

- Trading today: $1 - p(1) + \gamma p(2)$.
- Waiting: $\gamma(1 - p(1) + \gamma p(2)) + \gamma(2\gamma)(1 - p(2))$.

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- Waiting: $\gamma(1 - p(1) + \gamma p(2)) + \gamma(2\gamma)(1 - p(2)) \to 1 - \frac{p(1)}{2}$.

The payoff from waiting is higher if $p(1) > 0$.

Therefore, $p(1) \to 0$ as $\lambda/r \to \infty$.

The seller is a *monopolist* on information.

But neither he nor the first buyer cannot commit to selling information to the second buyer.

$\implies$ the second buyer gets information for virtually free.

Little incentive for the first buyer to pay a lot for info:

- Resale price is low.
- Waiting to be the second buyer involves minimal delay.

seller-optimal equilibrium

The seller-optimal equilibrium may involve delayed agreements.

Structure of equilibrium:

- Seller never sells info to B_2 before she sells info to B_1.
- Once seller sells info to B_1, then both compete to sell it to B_2.

In this equilibrium, every meeting between S and B_2 has no trade before B_1 is informed.

$\Rightarrow$ B_1 knows that he is always first buyer and so he pays $\frac{1}{2}$.

not-trading must be credible

Is it credible for S and B_2 to not trade?

$$\underbrace{\frac{\lambda}{r+\lambda}\left(\frac{1}{2}+\gamma p(2)\right)}_{\text{Seller's cont value}}+\underbrace{\frac{\lambda}{r+\lambda}\left(2\gamma(1-p(2))\right)}_{\text{Buyer's cont value}}>\underbrace{1+2\gamma p(2)}_{\text{Joint Surplus with Trade}}.$$

whenever $\lambda/r > 5$.

summary

Seller-optimal equilibrium features delay.

Seller obtains bilateral bargaining price from at most 1 buyer in any eqm.

- Even if $n > 2$, the seller does not obtain a non-trivial price from any buyer other than the first to whom she sells.
- Key idea: Once the seller sells information to any buyer, then in every equilibrium, prices $\to 0$ in the frequent-offer limit.

Clearly, seller can do better if she can prohibit resale. But are there non-contractual solutions?

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preview

One way to view previous results is that it identifies what happens in Markov Perfect Equilibria in which all trading decisions condition only on the payoff-relevant aspects of history.

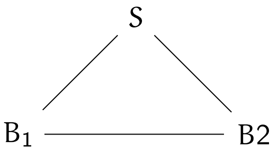
Here, we are going to augment the game with “tokens” to encode a slight degree of history dependence.

To understand this, let us begin with an interlude.

Led Zeppelin, Past, Present and Future

what if information weren't replicable?

an interlude



Suppose the good were non-replicable:

- There is only a single copy of the good, of value 1 to each buyer.
- A buyer who possesses it can consume or re-sell it.

Once a buyer obtains the good, there is no reason to re-trade.

Equilibrium prices solve

$$\underbrace{p(1-2\gamma)}_{\text{Seller's Gain from Trading Today}} = \underbrace{(1-p)\gamma}_{\text{Buyer's Gain from Trading Today}}.$$

Recall that $\gamma = \int_0^\infty e^{-rt}e^{-2\lambda t}\lambda \, dt \to \frac{1}{2}$.

$\Rightarrow$ seller obtains the entire social surplus.

By being on the short side of the market, seller captures all of the buyer's gains from trade.

We will exploit this idea.

Seller will sell a single token that is intrinsically worthless but will affect continuation play.

We will show that by doing so, she can capture the full intellectual property value of her information.

a prepay scheme

Seller first sells a single token to either buyer.

If buyer B_i buys token, then seller always sells info first to B_j.

Token = Right to be the 2nd buyer of info, who buys info at ≈ 0.

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- Value of token $= p(1) - p(2) \approx 1/2$.
- Fewer tokens than buyers $\Rightarrow$ Seller captures full value of token.
- Seller obtains $\approx 1/2$ for the token and $\approx 1/2$ for info!

Problem Solved! ●

maybe that was a bit fast. ●

let's do some algebra to convince ourselves.

Buyer's value from purchasing token is V_t and price of token is p_t.

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In symmetric Nash Bargaining,

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As $\lambda/r \rightarrow \infty$: $\quad 0 = \frac{1-p_t}{2} - \frac{1-p(1)}{2} \implies p_t \rightarrow p(1) = 1/2$

Tokens play the role of encoding a minimal degree of history dependence:

- Tokens need not be "physical."
- Info Resale: Value of token = Value of buying info for ≈ 0 later.
- With n buyers, seller sells $n - 1$ tokens.
- Competition: Scarce tokens $\rightarrow$ S captures buyer's value for token.
- Scheme exploits competition + resale.

Could also implement solution by slicing / encrypting information into different bits, and selling each bit separately.

Solution solves the commitment problem by exploiting it.

Outline

environment without tokens

- A set of buyers $\mathcal{B} \equiv \{b_1, \ldots, b_n\}$; value of info for each player is v.
- A set of sellers $\mathcal{S} \equiv \{s_1, \ldots, s_m\}$, all with identical info.
- The set of agents is $\mathcal{A} \equiv \mathcal{B} \cup \mathcal{S}$.
- Each pair of agents have a “trading relationship.”
- All players have discount rate r.

discrete-time

Trading opportunities occur at time periods $0, \Delta, 2\Delta, \ldots$.

In each period, a "link" is recognized.

Each link is recognized with uniform probability.

Results consider limiting behavior as $\Delta \to 0$.

solution concept

Set of informed players is payoff-relevant state space.

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Value function satisfies rational expectations: $V_i : \mathcal{M} \to \mathfrak{R}$.

Trading functions satisfy Nash Bargaining:

- $\alpha_{ij} : \mathcal{M} \to \{$👎, 👍$\}$,
- $p_{ij} : \mathcal{M} \to \mathfrak{R}$.

nash bargaining

Trading functions satisfies **Nash Bargaining** if for all $M \in \mathcal{M}$, $i \in M$, and $j \in \mathcal{A} \backslash M$,

$$\alpha_{ij}(M) = \text{👍} \Leftrightarrow \underbrace{V_i(M \cup \{j\}) + v + V_j(M \cup \{j\})}_{\text{Joint Surplus with Trade}} \geqslant \underbrace{V_i(M) + V_j(M)}_{\text{Joint Surplus with No Trade}}$$

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and $p_{ij}(M)$ is set to divide the change in surplus equally:

$$(1-w) \times \left(\underbrace{p_{ij}(M) + V_i(M \cup \{j\}) - V_i(M)}_{\text{Change in Seller's Surplus}} \right)$$

$$= w \times \left(\underbrace{v - p_{ij}(M) + V_j(M \cup \{j\}) - V_j(M)}_{\text{Change in Buyer's Surplus}} \right).$$

An equilibrium is a triple (V, α, p) such that V satisfies rational expectations given (α, p), and (α, p) satisfy Nash Bargaining given V.

Proposition

For every $\Delta > 0$, an equilibrium exists.

a statement about all equilibria

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The proof proceeds by induction on $|M|$.

Base Step: Suppose $|M| = |\mathcal{A}| - 1$.

Then we know that every equilibrium involves immediate agreement and prices converging to 0.

induction hypothesis

Suppose result is true when $|M| = k + 1$. We consider $|M| = k$.

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- If there is only one active buyer-seller pair, and limiting price is $p > 0$, then trade cannot happen when buyer i meets any other seller.
  - But joint surplus from disagreement is $v - p$.
  - Joint surplus from agreement is v.
  - $\Rightarrow$ contradiction.

seller-optimal equilibria

Proposition. In the seller-optimal equilibrium, with a single seller, she obtains a price of $\approx wv$ from first buyer, and ≈ 0 from any other buyer.

Prior result establishes that prices converge to 0 once there are two sellers, so only opportunity to gain is by holding up the first buyer.

Suppose there is a single seller initially and n buyers.

We allow the seller to sell $n-1$ tokens as "prepayment".

Proposition. $\exists\overline{\Delta}$ such that if $\Delta < \overline{\Delta}$, a prepay equilibrium exists. As $\Delta \to 0$, the price paid for each token and the first sale of the information good all converge to wv.

Outline

Potentially relevant for thinking about trading for information, incentives to acquire expertise, etc.

Classical narrative attributes underinvestment in information b/c info is a public good (non-rivalrous + non-excludable).

But in our setting, a market for info can exclude buyers.

Our analysis focuses on different channel:

Commitment problems $\rightarrow$ Low seller payoffs.

But commitment problem can be exploited to solve the resale problem.

related literature

Hinting at problem: Arrow (1962).

Appropriability Problem: Anton & Yao (1994); Beccara & Razin (2007); Horner & Skrzypacz (2014).

Resale problem: Polanski (2007, 2019), Manea (2020).

Intermediation / bargaining: Condorelli, Galeotti, & Renou (2016), Manea (2018), Elliot & Talamas (2019).

Thank you.