The Perverse Politics of Polarization*

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November 13, 2023

Abstract

Voters often do not know who gains or loses from a policy. This paper studies how asymmetric information about a policy's distributional effects influences voting behavior. We show that asymmetric information and distributional uncertainty together forge a powerful adverse selection effect in which voters are wary of policies supported by other voters. This force impels a majority of voters to support policies contrary to their preferences and information even if they ascribe low probability to others being better informed. Formally, we establish that collective choice is fragile to asymmetric information and fails to aggregate information. We identify and interpret a form of "adverse correlation" that is necessary and sufficient for these electoral failures.

*This is a major revision, with new results, of a working paper circulated as "Adverse Selection in Distributive Politics." We thank Peter Buisseret, Wioletta Dziuda, Ben Golub, Yingni Guo, Jacopo Perego, Philipp Strack, Sevgi Yuksel, numerous seminar and conference audiences, and owe special thanks to Navin Kartik and Richard Van Weelden for their insightful discussions.

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1 Introduction

Many policies benefit some voters at a cost borne by others. At the outset, voters may not know who gains and loses; indeed, voters may not only be uncertain but also asymmetrically informed. We study how this asymmetric information influences voting behavior. Our main finding is that the mere prospect of asymmetric information can induce voters to support policies contrary to their own interests. This electoral effect is reminiscent of Akerlof's Lemons Problem: a voter may vote against policies that command the support of others out of fear of what their support means for her. Thus, even if the policy choice is not zero sum, zero-sum thinking may prevail in equilibrium.

To see how this happens, consider the following example. Ann, Bella, and Carol vote between policies a and b, and the policy that obtains more votes wins. Each voter's payoff from policy a is 0. By contrast, policy b results in two *winners*, each of whom accrue a payoff of 2, and one *loser*, who obtains a payoff of -3. Ex ante, voters are symmetric: there are three equally likely states of the world, $\{\omega_A, \omega_B, \omega_C\}$, where ω_i is the state in which voter i is the loser.

Given these payoffs, policy b would win the election were voters *fully informed*, i.e., to know the state of the world. That policy would also prevail if all voters were known to be uninformed, i.e., the *no-information* benchmark.[1] But a different possibility emerges once voters may be asymmetrically informed.

Suppose that each voter privately learns her payoff from policy b with (independent) probability $\lambda > 0$ and otherwise remains uninformed. We take λ to be relatively small, embodying the idea that private information is scarce. As we show, even the slight chance that others may be better informed impacts voting behavior. Consider the following strategy profile:

- An informed loser votes a and an informed winner votes b;
- An uninformed voter votes a.

We claim that this is a strict equilibrium. As an informed voter votes for her strictly preferred policy, it suffices to consider the incentives of an uninformed voter, say Ann. Her vote affects her payoff only if it breaks a tie: of Bella and Carol, one votes a and the other votes b. Given the stipulated voting behavior, Ann knows that only an informed winner would vote for b. By contrast, a vote for a may be cast by an uninformed voter (just like Ann) or an informed loser. For $\lambda \approx 0$, Ann ascribes much

[1] Throughout our analysis, we focus on weakly undominated equilibria.

higher probability to the former event. In this case, she and the other uninformed voter have an equal chance of being the remaining winner. Hence, whenever her vote is decisive, Ann believes that she is a winner with probability one-half, lower than the ex ante odds of two-thirds. At these interim odds, Ann finds it strictly optimal to vote a, ratifying that the strategy profile is a strict equilibrium.[2] That is, when uniformed voters fear that support for policy b comes from others who are better informed, they find it optimal to vote a, thereby reinforcing the initial suspicion.

Moreover, if each voter obtains private information with a small probability ($\lambda \approx 0$), then all voters are likely to be uninformed. As they all vote a, that policy wins with near certainty. This outcome contrasts with what prevails in the no-information benchmark; it's also informationally inefficient because if signals were realized publicly, with high probability, voters would all see that they are uninformed and choose policy b.

In this paper, we formalize this strategic logic to see generally how asymmetric information impacts distributive politics. We model an electoral choice that has distributional consequences, like a trade or tax policy. Voters are uncertain both about the number of people who benefit from a policy as well as their identities, and some voters obtain private information about these variables. To isolate the role of asymmetric information, we begin with a benchmark in which all voters share the same information and ex ante preferences. In that case, the unique equilibrium (in weakly undominated strategies) selects the ex ante optimal policy. We compare that benchmark with an "informationally-scarce" setting in which each voter obtains additional private information with a small probability. Our analysis evaluates whether the prospect of asymmetric information leads the electorate astray.[3]

Addressing this question requires us to assess what a voter makes of a policy when she conditions on others learning privately that the policy will benefit them. Should an uninformed voter, say Ann, see the good news for others as good or bad news for herself? On one hand, others may receive good news about a policy because the policy results in many winners, which improves Ann's odds of being a winner as well. This force of *advantageous selection* is potent when uncertainty about the number of

[2]The probability that Ann is a winner, conditional on being pivotal, is $\frac{1}{2-\lambda}$, and she has a strict incentive to vote a if $\lambda < \frac{1}{3}$.

[3]Our emphasis on informationally-scarce settings may be reasonable from an applied standpoint as experts themselves disagree about the impact of many policies. For example, some predictions on how trade treaties affect wages and employment focus on factor abundance and heterogeneity across industries while others highlight within-industry heterogeneity, emphasizing the differential impact on low and high skill workers.

winners trumps distributional uncertainty, e.g., in a pure common-value election where either all voters gain or all lose from policy b. The second force is *adverse selection*: for any given number of winners, Ann's odds of being a winner reduce when others receive good news. This crowding-out effect features in our example above as it holds the number of winners fixed.

More generally, which force dominates depends on the correlation between a voter's own payoff and the signals of other voters. We define the relevant correlation measure in term of primitives. We say that the collective choice problem is *adversely correlated* if the adverse selection effect dominates; otherwise, we deem it *advantageously correlated*. Our main result (Theorem 1) characterizes how correlation shapes equilibrium behavior in informationally-scarce settings; stated informally, it finds:

Main Result. *If the collective choice problem is adversely correlated, there is a strict equilibrium that selects the ex ante inferior policy with near certainty. Otherwise, every equilibrium selects the ex ante optimal policy with near certainty.*

Adverse correlation is thus a source of political fragility as elections may then select a policy distinct from that which would prevail were all voters known to be uninformed. The mere prospect that voters are asymmetrically informed, however unlikely, causes the inferior policy to win in an equilibrium of the voting game. Not all equilibria select this policy: there also exists an equilibrium in which the optimal policy prevails.[4] But nevertheless, elections may fail to pick the right outcome and collective choice hinges on voters' ability to coordinate on a "good" equilibrium. By contrast, if the collective choice problem is advantageously correlated, such coordination is obviated; all equilibria—pure or mixed, symmetric or asymmetric—result in the optimal policy.[5]

For expositional clarity, we consider a model with a known number of voters, all of whom are ex ante symmetric, aggregating votes by simple-majority rule. But the logic of adverse selection goes beyond this. Similar results hold for other voting rules, imposing a restriction to symmetric equilibria, and an electorate of random size. Moreover, ex ante asymmetries can amplify these forces because the presence of elite voters makes non-elites fear adverse selection even more.

The primitive condition of adverse correlation lends itself to concrete interpretation. We show that policy choices that are more polarizing, in terms of the proportional loss

[4] In the example, such an equilibrium is that in which all uninformed voters cast their votes for b.

[5] From this standpoint, our work connects to the growing interest in institutional design under adversarial equilibrium selection, as in Mathevet, Perego, and Taneva (2020), Halac, Lipnowski, and Rappoport (2021), Ali, Haghpanah, Lin, and Siegel (2022), Inostroza and Pavan (2023), and others.

suffered by losers relative to the gains that accrue to winners, have a greater potential to induce adverse correlation. We also formulate the "crowding-out effect" from learning that others are winners in terms of the standard likelihood-ratio dominance order. These comparative statics reveal features that make elections more prone to suspicion.

Adverse correlation also relates to the information conveyed by private signals. If voters learn only about aggregate outcomes—in our model, the number of winners, but more broadly, say, aggregate GDP or economic growth—without any distributional consequences, then the collective choice problem cannot be adversely correlated. By contrast, purely distributional information that identifies who is first in line to obtain gains induces adverse correlation. This contrast dovetails with analyses of how media outlets profit from selling information that polarizes rather than unifies voters (Martin and Yurukoglu, 2017; Perego and Yuksel, 2022). Our results call attention to a pernicious "downstream" effect of this market competition on voting behavior.

We view our result as capturing how distrust may prevail when voters vote on policies that have a strong distributional component. Recent evidence shows that voters appear to hold polarized perceptions of reality (Alesina, Miano, and Stantcheva, 2020) and see their interests as conflicting with those of other voters (Levendusky and Malhotra, 2016). Zero-sum thinking, where voters perceive gains that accrue to others as being correlated with losses they would suffer, appears to be pervasive (Davidai and Ongis, 2019; Chinoy, Nunn, Sequeira, and Stantcheva, 2022). Our analysis suggests a strategic mechanism for zero-sum thinking and how it may prevail even if the policy choice is not zero sum: the fear of what other voters may have learned can push many voters to support a policy that they view to be inferior. Our results show that this fear and distrust can emerge independently of identity politics or partisan interests, and even with voters who are ex ante symmetric.

Our study complements prior work on distributive politics. Fernandez and Rodrik (1991) show that a reform that benefits a majority of voters ex post may not pass ex ante if the majority do not find the lottery worthwhile; their setting involves uncertainty without private information. We identify instead the potency of private information: relative to a complete information game devoid of any private information, the electorate may be worse off when some voters are better but privately informed.[6]

[6] That better informed voters may, in equilibrium, make worse choices offers a new channel to evaluate the importance of "voter competence." In a different context, Ashworth and Bueno De Mesquita (2014) argue that more competent voters may be worse off; their analysis focuses on the effect of voter competence on politicians' behavior.

Incomplete information amplifies the scope for political failures as a policy may be ex ante optimal and yet fail to pass because each voter fears what others have learned. That adverse selection leads to inferior choices and makes collective choice fragile to private information are novel to our framework.

This interest in the sensitivity of collective choice to private information connects our work to that on global games and higher-order beliefs (Carlsson and Van Damme, 1993; Kajii and Morris, 1997). Closer to our work, Morris and Shin (2012) find that small amounts of private information can disrupt asset markets through adverse selection. This strand of research shows that higher-order uncertainty renders some equilibrium outcomes of complete information games untenable in nearby incomplete information games; i.e., the equilibrium outcome correspondence fails lower hemicontinuity. While related, our analysis emphasizes a different kind of fragility: incomplete information games with scarce private information generate equilibrium outcomes that do not obtain in the complete information game. Formally, the outcome correspondence of weakly undominated equilibria fails upper hemicontinuity. As we show, second-order uncertainty alone triggers this discontinuity.

Adverse selection is a source of informational inefficiency in our model as the electorate selects a policy that would be rejected if all information were public. In this regard, our work also connects to that on information aggregation; see, for instance, Feddersen and Pesendorfer (1997), Razin (2003), Kim and Fey (2007), Bhattacharya (2013), and Barelli, Bhattacharya, and Siga (2022). Our focus on the fragility to asymmetric information leads us to adopt an approach that differs from this prior work. Specifically, we fix a population size and study the limit equilibria as private information becomes increasingly scarce; by contrast, prior approaches to information aggregation fix an information structure with conditionally independent signals and consider the limit equilibria for an increasingly large electorate, where collective information is abundant but privately distributed. Both limits are conceptually meaningful but address different questions. Our specific interest in distributive politics also necessitates a model in which voter types (both payoffs and signals) are correlated (i.e., not conditionally independent) so that a voter may perceive gains that accrue to others as being correlated with losses that she suffers.

2 Model

The Collective Choice Problem. Each of $\mathcal{N} := \{1, \ldots, n\}$ voters cast a vote for one of two policies $p \in \{a, b\}$. Our baseline analysis assumes that $n \geq 3$ is odd and votes are aggregated by simple-majority rule: policy p wins if and only if it receives *strictly* more than $\tau := \frac{n-1}{2}$ votes.

Voters are uncertain about their payoffs from each policy. A payoff for voter i is a tuple $v_i = (v_i^a, v_i^b) \in \mathcal{V} := \mathcal{V}^a \times \mathcal{V}^b$, where $\mathcal{V}^p$ is a finite set of potential payoffs from policy p. Before casting her vote, voter i receives a private signal s_i drawn from a finite set of signals $\mathcal{S} := \{s^0, \ldots, s^K\}$. Thus, a state of the world ω consists of a payoff profile $v \in \mathcal{V}^n$ and a signal profile $s \in \mathcal{S}^n$. Uncertainty is described by a probability distribution P on the state space $\Omega := \mathcal{V}^n \times \mathcal{S}^n$. We interpret the signal profile as describing the *private* information that voters receive above and beyond any public information that all have access to. The random variable V^p denotes the payoff profile from choosing policy p and S denotes the signal profile.[7] The population of voters and the probability space together define a *collective choice problem*.

Our baseline framework makes three assumptions. For the first assumption, we say that state (v', s') *permutes* state (v, s) if there is a one-to-one mapping $\psi : \mathcal{N} \to \mathcal{N}$ such that $(v_i', s_i') = (v_{\psi(i)}, s_{\psi(i)})$ for all $i \in \mathcal{N}$.

Assumption 1. *Voters are exchangeable: if ω' permutes ω, then $P(\omega') = P(\omega)$.*

Exchangeability focuses attention on the conflict generated by asymmetric information rather than heterogeneous ex ante preferences; we relax this assumption in Section 5.3. Given Assumption 1, all voters agree on the (ex ante) optimal policy, which we denote p^*, and the (ex ante) inferior policy, which we denote p_*. We define the random variable $V^d := V^{p^*} - V^{p_*}$ to denote the payoff difference between these two policies. We denote the payoff difference conditioning on any non-null event $E \subseteq \Omega$ by

$$V_i^d(E) := \sum_{\omega \in \Omega} \Big(V_i^{p^*}(\omega) - V_i^{p_*}(\omega) \Big) P(\omega | E).$$

If E is null, we define $V_i^d(E) = 0$ to simplify exposition.

Our second assumption distinguishes signal s^0, which is *uninformative*, from the remaining signals $\mathcal{M} := \{s^1, \ldots, s^K\}$, which are *informative*.

[7] In general, capital letters denote random variables and lower-case letters denote realizations. For random variable X, $\{x\} := \{\omega : X(\omega) = x\}$ is the event where x is realized, omitting the brackets when clear from the context.

Assumption 2. *Signal s^0 is uninformative and every other signal is sufficient to deduce the ex post ordinal preference: for all $(v, s) \in \Omega$,*

(a) *if $s_i = s^0$, then $P(v, s) = P(v, s_{-i})P(s_i)$; and*

(b) *if $s_i \neq s^0$, then $V_i^d(s) > 0$ if and only if $V_i^d(s_i) > 0$.*

Assumption 2(a) asserts that signal s^0 is realized independently of the payoff profile and the other signals, and hence conveys no private information about these variables. Assumption 2(b) speaks to the informativeness of the remaining signals: for a voter who obtains an informative signal, learning the signals of others conveys no additional information about her *ordinal* preference over policies. Hence, relative to the entire signal profile s, the private signal s_i is sufficient to deduce voter i's preferred policy. A special case is where the informed signal reveals directly one's cardinal payoffs from each policy, as in our introductory example. Assumption 2(b) goes beyond this special case by encompassing settings in which informed voters do not know all that much about their cardinal payoffs but are just well-informed *relative* to the electorate about their ordinal preferences. We exploit this generality in Section 4.

Among the informative signals, we say that signal s^k conveys *good news* if $V_i^d(S_i = s^k) > 0$ and *bad news* if $V_i^d(S_i = s^k) < 0$. Being ordinal, these notions convey whether an informed voter favors the ex ante optimal or inferior policy. For a signal profile $s \in \mathcal{S}^n$, $M(s)$ is the number of voters who receive informative signals, $G(s)$ is the number with good news, and $B(s)$ is the number with bad news.

Our final assumption simplifies exposition without playing a substantive role.

Assumption 3. *Voters have strict preferences and both bad news and good news are possible:*

(a) *For every voter $i \in \mathcal{N}$ and non-null event $E \subseteq \Omega$, $V_i^d(E) \neq 0$.*

(b) *$P(B \geq 1) > 0$ and $P(G \geq \tau) > 0$.*

Assumption 3(a) obviates the need to describe how voters behave when indifferent and, with finitely many states, strict preferences are generic in the space of possible payoff profiles. Assumption 3(b) rules out some trivial cases: when it is impossible to receive bad signals, voting for the inferior policy is a weakly dominated strategy; if $P(G \geq \tau) = 0$, there is always an uninteresting equilibrium where all uninformed voters select the inferior policy only because their votes are then never decisive.

This completes the description of the baseline collective choice problem. We contrast behavior in this collective choice problem with private information to that in a *public information benchmark*, where every voter observes the entire signal profile s.

Solution Concept. A strategy for a voter is a signal-contingent probability of voting for p^*. We study Bayes-Nash equilibria in weakly undominated strategies, which implies that those with good news vote for p^* and those with bad news vote for p_*. Henceforth, we refer to these as equilibria. The following summarizes equilibrium existence in the baseline collective choice problem and the public information benchmark.

Proposition 1. *An equilibrium exists under both private and public information; in the latter case, the equilibrium is unique.*

3 The Fragility of Collective Choice

3.1 Scarce Information

In this section, we formalize how we take information to be scarce holding all other variables fixed. The general idea is that the primitive probability distribution P decomposes into a two-stage process: first, a probability distribution chooses a state in which every voter obtains an informative signal; and second, for each voter i, her signal is deleted with some probability $\lambda > 0$, independently of other voters, which results in her obtaining the uninformative signal s^0.

Formally, let $\Omega_{\mathcal{I}} := \{\omega \in \Omega : \text{ for every voter } i, S_i(\omega) \in \mathcal{M}\}$ denote the states in which every voter obtains an informative signal, and $P_{\mathcal{I}}(.) := P(.|\Omega_{\mathcal{I}})$ be the corresponding conditional probability distribution. Given a signal profile $s \in \mathcal{S}^n$, we say that signal profile $s' \in \mathcal{M}^n$ is a *clarification* if, for every voter i, $s_i \neq s^0$ implies $s'_i = s_i$. In other words, s' might have been the "true" signal profile had information not been deleted. We denote the set of all clarifications of profile s by $C(s)$. Finally, let $\lambda := 1 - P(S_i = s^0)$ denote the probability that any voter receives an informative signal under our primitive distribution P.

Lemma 1 in the Appendix shows that, under our assumptions, the two-stage process envisioned above is a valid interpretation for the primitive probability distribution P:

$$\text{For every } (v, s) \in \Omega, \qquad P(v, s) = \lambda^{M(s)}(1-\lambda)^{n-M(s)} \sum_{\tilde{s} \in C(s)} P_{\mathcal{I}}(v, \tilde{s}).$$

Probabilities are therefore separable in the probability of who is informed, which depends on λ, and a probability distribution on $\Omega_{\mathcal{I}}$ (states where every player is informed), which is independent of λ. This separability implies that the collective choice problem $\mathcal{C} := (\mathcal{N}, \Omega, P)$ decomposes uniquely into an *informed* collective choice problem $\mathcal{C}_{\mathcal{I}} := (\mathcal{N}, \Omega_{\mathcal{I}}, P_{\mathcal{I}})$, in which every voter is informed, and a *dilution* λ that describes the probability that a voter obtains an informative signal.[8] By representing the collective choice problem as the tuple $(\mathcal{C}_{\mathcal{I}}, \lambda)$, we can then vary λ while holding $\mathcal{C}_{\mathcal{I}}$ fixed. This is how we take information to be scarce independently of other parameters.

The following defines what it means for a policy to win with scarce information.

Definition 1. *Policy p* ***wins in an equilibrium with scarce information*** *if, for every ε in $(0,1)$, there exists λ_ε in $(0,1)$ such that, for all λ in $(0, \lambda_\varepsilon)$, policy p wins with probability at least $1-\varepsilon$ in an equilibrium of the collective choice problem $(\mathcal{C}_{\mathcal{I}}, \lambda)$.*

It immediately follows that the ex ante optimal policy wins with scarce *public* information.

Proposition 2. *In the public information benchmark, policy p^* wins in the unique equilibrium with scarce information.*

The logic is that when information is scarce and public, all voters are likely uninformed and this fact is commonly known. Hence, each voter casts her vote for policy p^* thereby securing its victory. Collective choice therefore is robust to scarce *public* information. By contrast, when signals are private, it is still probable that all voters are uninformed but this is *not* commonly known. We turn to the condition that determines whether collective choice is robust to scarce *private* information.

3.2 Adverse Correlation

Is good news for others also good news for me? This question is at the core of our analysis. The answer depends on how a voter's payoff is correlated with others' signals.

Consider a voter, Ann, who receives the uninformative signal. Based only on her own information, the ex ante optimal remains optimal. Now suppose that she conditions on κ voters obtaining good news and no voter obtaining bad news. Her conditional expected payoff is then

$$V^G(\kappa) := V_i^d(G = \kappa, B = 0, S_i = s^0),$$

[8] Our terminology here borrows from Pomatto, Strack, and Tamuz (2023).

i.e., the expected payoff difference between the optimal and inferior policies conditioning on Ann receiving the uninformative signal, κ others receiving good news, and no one receiving bad news.[9]

We elaborate on the interpretation of this term. First, this posterior expected payoff considers an event in which no voter obtains bad news; our analysis in Section 3.3 confirms that such events play an important role in equilibrium behavior. Second, in principle, it's unclear if Ann favors policy p^* more or less once she conditions on κ other voters obtaining good news. On one hand, the likelihood of drawing κ good signals from κ random draws is higher if policy p^* is likely to benefit many voters. On the other hand, for any given number of beneficiaries, Anne's chances of benefiting from p^* herself is lower once κ "slots" are already taken by others. These effects push in opposing directions, one bearing on aggregate considerations and the other on distributional considerations.

The sign of $V^G(\kappa)$ determines which effect is dominant at a specific value of κ; we note how this sign can also vary with κ. For example, if no voter obtains good news ($\kappa = 0$), the posterior expected payoff difference coincides with the prior expected payoff difference, which is strictly positive. Once some voters obtain good news, the sign depends on the relative importance of the aggregate and distributional considerations. Our introductory example features only distributional uncertainty and $V^G(\kappa) < 0$ for every $\kappa > 0$; knowing that anyone else obtained good news is bad news for Ann. By contrast, when voter preferences are perfectly aligned—as in a common-value problem—then there is only aggregate uncertainty in that all voters gain or all lose from policy p^*. In that case, $V^G(\kappa) > 0$ for every $\kappa > 0$.

Our correlation condition assesses if V^G is negative for a relevant domain.

Definition 2. *The collective choice problem is* ***adversely correlated*** *if there is some* κ *in* $\{1, \ldots, \tau\}$ *such that* $V^G(\kappa) < 0$. *Otherwise, the collective choice problem is* ***advantageously correlated***.[10]

Recall that $\tau := (n-1)/2$ is the number of votes for policy p^* that makes a voter pivotal. The criterion above evaluates if there is any coalition of weakly smaller size whose good news, on net, makes an uninformed voter favor the inferior policy p_*. This criterion jointly evaluates the payoff and information structure; fixing a distribution

[9]Under our assumptions, V^G does not depend on λ, as we show in Lemma 1 in the Appendix. As such, it is a property only of $\mathcal{C}_{\mathcal{I}}$, which we hold fixed as we consider the limit with scarce information.

[10]By Assumption 3(a), advantageous correlation equates to $V^G(\kappa) > 0$ for every κ in $\{1, \ldots, \tau\}$.

on payoffs, some information structures render the collective choice problem adversely correlated whereas others make it advantageously correlated.

3.3 Main Result

We show that the fragility of collective choice hinges on its correlation.

Theorem 1. *The inferior policy p_* wins in an equilibrium with scarce information if the collective choice problem is adversely correlated. Otherwise the optimal policy p^* wins in every equilibrium with scarce information.*

Recall that were each voter known to be uninformed, or if all information were public and scarce, every equilibrium would select policy p^*. Theorem 1 shows, by contrast, that the slightest prospect of private information could result in a different equilibrium outcome. Voting behavior is fragile to private information whenever the collective choice problem is adversely correlated: formally, the outcome correspondence of weakly undominated equilibria violates upper hemicontinuity as $\lambda \to 0$. Conversely, advantageous correlation assures robustness. Not only does the optimal policy p^* win with non-trivial odds, it does so with near certainty in any equilibrium, pure or mixed, asymmetric or symmetric.

To see how adverse correlation results in policy p_* winning in an equilibrium, we start with the simplest case, namely $V^G(\tau) < 0$. Then, a strict equilibrium akin to that of our introductory example implements the inferior policy: all uninformed voters vote for policy p_*. An uninformed Ann infers, conditional on being pivotal, that τ voters have voted for policy p^*. By construction, each of these voters must have heard good news. As information is scarce, Ann believes all those who voted for p_* are likely uninformed, i.e., the number of voters who received bad news is likely 0. Her expected payoff difference conditional on being pivotal then approximates $V^G(\tau) < 0$, which implies that she strictly prefers to vote for policy p_*. As most voters are uninformed, it wins with high probability.

Other cases require more elaborate constructions. Let $V^G(\tau) > 0$ and κ^* be the highest value of κ in $\{1, \ldots, \tau\}$ such that $V^G(\kappa) < 0$. We construct a (strict) pure-strategy equilibrium that is asymmetric. In this equilibrium, we label some voters as "suspicious" and all others are "sanguine." A suspicious (resp., sanguine) voter casts her vote for policy p_* (resp., p^*) when she is uninformed. We specify that $(\tau - \kappa^*)$

voters are sanguine and all others are suspicious. Our argument establishing that this is an equilibrium hews to the following logic:

- A suspicious uninformed voter, conditional on being pivotal, places high odds on the event where (i) of the τ votes for policy p^*, $(\tau - \kappa^*)$ are cast by uninformed sanguine voters and the remaining κ^* votes are cast by suspicious voters who obtained good news; (ii) all the $(n - \tau - 1)$ votes for policy p_* are cast by uninformed suspicious voters. Hence, her conditional expected payoff difference is close to $V^G(\kappa^*) < 0$. Thus, she strictly prefers to vote for policy p_*.

- A sanguine uninformed voter makes a different calculation that stem from her being sanguine: conditional on being pivotal, she puts high odds that of the τ votes for policy p^*, $(\tau - \kappa^* - 1)$ are cast by uninformed sanguine voters and $(\kappa^* + 1)$ votes are cast by suspicious voters who obtained good news. Hence, her conditional expected payoff difference is close to $V^G(\kappa^* + 1)$, which is strictly positive as κ^* is the highest value of κ such that $V^G(\kappa) < 0$. Thus, she strictly prefers to vote for policy p^*.

Having argued that the strategy profile is an equilibrium, we note that as $\lambda \to 0$, the election is likely decided by the uninformed voters. Given that sanguine voters are in a minority, the inferior policy p_* wins with near certainty.

Theorem 1 also asserts that under advantageous correlation, the optimal policy prevails in *every* equilibrium with near certainty. Hence adverse correlation is also necessary for electoral failures. This direction is considerably more challenging to prove given the large number of candidate equilibria, including those that are asymmetric and in mixed strategies. The key idea, we show, is that as private information becomes increasingly scarce ($\lambda \to 0$), the relevant posterior payoff of an uninformed voter approximates a weighted sum of the $V^G(\kappa)$ for different values of $\kappa \in \{0, .., \tau\}$ for *any* weakly undominated strategy profile. The specific weights depend on the strategy profile but, regardless of the weights, advantageous correlation implies that the sum is strictly positive because $V^G(\kappa) > 0$ for *every* κ. Hence, the decisions of others in any equilibrium never generates enough bad news to sway uninformed voters to vote for policy p_* with non-trivial probability.

In light of our main result, the reader may wonder if adverse correlation nevertheless accommodates a "good" equilibrium that selects the optimal policy? Yes.

Theorem 2. *Regardless of correlation, policy p^* wins in at least some equilibrium with scarce information.*

The logic of Theorem 2 is that although the information structure is a primitive of the collective choice problem, how an uninformed voter interprets others' votes is determined in equilibrium. Our construction in Theorem 1 sways an uninformed Ann towards policy p_* by having most votes in favor of policy p^* to be cast by those who have good news so that, under adverse correlation, policy p^* is selected adversely to her interests. By the same logic, a strategy profile in which most votes in favor of policy p_* are cast by those who have bad news could make policy p_* selected adversely to Ann's interests, which would push Ann to vote for policy p^*. Our proof of Theorem 2 shows that the ex ante inferiority of policy p_* makes it always feasible to construct such an equilibrium. As uninformed voters overwhelmingly vote for policy p^* in that equilibrium, it wins when information is scarce.

Theorem 2 suggests that fragility is not a concern *if* voters can coordinate on equilibria that maximize ex ante welfare. We take the perspective that presuming voter coordination is not modest, particularly in large electorates or in the absence of an ex ante stage at which all voters are known to be symmetrically informed. Theorem 1 implies that, under adverse correlation, voter coordination is *necessary* to avoid an inferior outcome that is rationalized by neither voters' information nor their preferences; the prospect of asymmetric information, however slight, can result in this perverse outcome. By contrast, advantageous correlation obviates voter coordination as the optimal policy is assured across all equilibria.

Our analysis applies beyond our baseline setting. We assume simple-majority rule for simplicity but our analysis invokes it only for simplicity. Suppose that policy p^* passes if it obtains at least $\tau+1$ votes where τ is now any number in $\{1,\ldots,n-1\}$. We would continue to define adverse and advantageous correlation as per Definition 2, and Theorem 1 still holds. Therefore, increasing the number of votes required for policy p^* to pass only expands the scope for adverse correlation and electoral failures.

Section 5 describes elaborations that require further analysis. A similar characterization obtains for symmetric equilibria under a slightly different correlation condition; that analysis also accommodates population uncertainty. We also relax the condition that voters are ex ante symmetric, allowing for some voters to be "elites" who are more likely to benefit from the optimal policy. We show that this ex ante conflict only amplifies the forces described here. Before describing these extensions, we focus on a tractable class of the baseline model to identify various sources of adverse correlation.

4 Sources of Adverse Correlation

In this section, we study what features of a collective choice problem—its payoffs, the probability distribution on the number of winners, and its information structure—render it adversely correlated. For this comparison, we focus on a class of collective choice problem with *binary payoffs*: $V_i^d \in \{v_w, -v_\ell\}$ for some v_w and v_ℓ strictly positive.[11] The event $W_i := \{\omega : V_i^d(\omega) > 0\}$ are all states where voter i gains from policy p^*, and is referred to as a *winner*. The number of winners in state ω is $W(\omega) := |\{i \in \mathcal{N} : \omega \in W_i\}|$. As policy p^* is ex ante optimal, we are assuming throughout that $P(W_i)v_w - (1 - P(W_i))v_\ell > 0$.

In this class of collective choice problems, we study how varying the payoffs, the probability distribution on the number of winners, and the nature of the information structure affects the collective choice problem.

4.1 Polarization Ratios and the Crowding-Out Effect

We first fix a simple information structure to focus on payoffs: suppose that informative signals convey *perfect news* in that an informed voter learns if she is a winner or loser, i.e., $P(W_i|S_i = s^k) \in \{0, 1\}$ for each informative signal s^k. The collective choice problem can then be represented through the tuple (P_W, v) where P_W is the (marginal) distribution on the number of winners, $P_W : \{0, \ldots, n\} \to [0, 1]$ and $v := (v_w, v_\ell)$ specifies the ex post payoff differences. We write $V^G(\kappa; P_W, v)$ as the value of $V^G(\kappa)$ for this collective choice problem. We order these collective choice problems by their propensity for adverse correlation.

Definition 3. *A collective choice problem (P_W, v) is* ***more adversely correlated*** *than the collective choice problem (P'_W, v') if for every $\kappa \in \{1, \ldots, \tau\}$,*

$$V^G(\kappa|P'_W, v') < 0 \implies V^G(\kappa|P_W, v) < 0.$$

We denote this binary relation by $\succcurlyeq_{AC}$.[12]

[11] For instance, $\Omega = \{v_w, -v_\ell\}^n \times \{0\}^n \times \mathcal{S}^n$ for some $v_w > 0$ and $v_\ell > 0$, and $p^* = a$.

[12] We note that $\succcurlyeq_{AC}$ is a preorder in that it is reflexive and transitive but not complete.

We identify two sources of adverse correlation. The first views payoffs through their *polarization ratio*: given binary payoffs (v_w, v_ℓ), the *polarization ratio* v_ℓ/v_w measures the proportional costs incurred by losers relative to the gains that accrue to winners. The second views the crowding-out effect through likelihood ratios: the probability distribution P'_W likelihood-ratio dominates P_W, denoted $P'_W \succsim_{LR} P_W$, if $P'_W(w')P_W(w) \geq P'_W(w)P_W(w')$ whenever $w' > w$.

Proposition 3. *The following comparative statics results hold:*

(a) *A higher polarization ratio induces more adverse correlation:* $(P_W, v) \succsim_{AC} (P_W, v')$ *if* $v_\ell/v_w \geq v'_\ell/v'_w$.

(b) *Likelihood-ratio dominance induces less adverse correlation:* $(P'_W, v) \succsim_{AC} (P_W, v)$ *if* $P_W \succsim_{LR} P'_W$.

A higher polarization ratio increases adverse correlation because, relative to the inferior policy p_*, a higher polarization ratio worsens the gamble from the optimal policy p^* both ex ante and ex interim. By contrast, a likelihood-ratio dominant increase in the marginal distribution over the number of winners has the opposite effect because it reduces crowding-out from learning that others received good news. In combination with Theorem 1, Proposition 3 implies that increasing the polarization ratio or reducing the likelihood ratio makes collective choice more fragile to asymmetric information.

4.2 Aggregate and Distributional Information

Electoral choices are also shaped by the kind of information voters obtain. Contrast the following messages by Republican Presidents about the economic consequences of trade liberalization:

> *"Free trade serves the cause of economic progress..."* – Reagan, 1982

> *"Members of the club—the consultants, the pollsters, the politicians, the pundits, and the special interests—grow rich and powerful while the American people grow poorer and more isolated..."* – Trump, 2016

The former emphasizes aggregate consequences without referring to who benefits and loses whereas the latter focuses on those distributional consequences without speaking about aggregate gains. Politicians and media outlets can choose between focusing their messaging on aggregate or distributional consequences. Here, we identify how this choice affects voting behavior.

To this end, we depart from the perfect news considered in Section 4.1. Instead, we say that signals *convey only aggregate news* if $P(V = v|S = s, W = w) = P(V = v|W = w)$ for any payoff profile v, signal profile s, and number of winners w such that $P(W = w) > 0$. That is, conditioning on the number of winners, voters learn nothing further about the payoffs from their signals.

Proposition 4. *Suppose signals convey only aggregate news. Then the collective choice problem is advantageously correlated for all* (P_W, v).

We contrast this case with one where signals *convey only distributional news*: $P(W = w|S = s) = P(W = w)$ for any signal profile s and number of winners w, i.e., signals are uninformative about the number of winners.

Proposition 5. *Suppose signals convey only distributional news. Then holding the signal structure fixed, there is a polarization ratio* ρ *such that the collective choice problem is adversely correlated whenever* $v_\ell/v_w \geq \rho$.[13]

The contrast between Propositions 4 and 5 reveals how the information structure may preclude or induce adverse correlation by focusing on aggregate or distributional news. This finding dovetails with Martin and Yurukoglu (2017) and Perego and Yuksel (2022) who show that media providers may profit from delivering information that polarizes voters. In conjunction with their analysis, Proposition 5 suggests that this profit-seeking behavior may have detrimental effects on elections.[14]

5 Extensions

5.1 A Characterization for Symmetric Equilibria

The analysis of Theorem 1 considers all equilibria, including those that are asymmetric.[15] Restricting to symmetric equilibria, a stronger form of adverse correlation is necessary for the inferior policy to win with scarce information, while a weaker form of advantageous correlation is sufficient to guarantee that the optimal policy wins across all symmetric equilibria.

[13] In the proof, we show that this critical ρ is low enough that policy p^* remains optimal.

[14] Focusing on a different force, Yuksel (2022) shows that segregation in news consumption has a further polarizing effect on the electoral platforms chosen by parties.

[15] We note that the equilibria described in Proposition 1-2 and Theorem 2 are symmetric.

In a symmetric (weakly undominated) equilibrium, informed voters choose their preferred policy and all uniformed voters choose p^* with the same probability $\alpha \in [0,1]$. When $\alpha \in (0,1)$, a voter can be pivotal when κ others receive good news for any $\kappa \in \{0, ..., \tau\}$, because it is always possible for exactly $\tau - \kappa$ uninformed voters to choose p^* along with the κ voters who received good news. However, these events are not equally likely, depending both on the primitive probability of events and the behavior of uninformed voters. The relevant correlation measure for symmetric equilibria therefore involves a weighted sum of the conditional payoffs $V^G(\cdot)$ evaluated at different values of $\kappa \in \{0, ..., \tau\}$:

$$\mathcal{K}(\theta) = \sum_{\kappa=0}^{\tau} \theta^{\kappa} \binom{\tau}{\kappa} P(G = \kappa | B = 0) V^G(\kappa).$$

This sum again focuses on the perspective of an uninformed voter, whose expected payoff difference conditional on κ others receiving good news and no one receiving bad news is $V^G(\kappa)$. In each summand, $P(G = \kappa | B = 0)$ is the primitive probability that κ voters receive good news, conditional on no one receiving bad news. The binomial coefficient is an adjustment factor to account for the number of ways that the uninformed voter can be pivotal when no one receives bad news.[16] As we elaborate below, the variable θ encodes the relative likelihood that, in a symmetric mixed strategy profile, a vote for policy p^* is cast by an informed rather than an uninformed voter, when κ voters received good news and no one received bad news. In particular, θ^{κ} offers additional flexibility in weighting the summands: taking $\theta \to 0$ concentrates weight on the term that involves $V^G(0)$ whereas $\theta \to \infty$ focuses the sum on the term that involves $V^G(\tau)$.

The relevant *correlation measure* here is the infimum, $\mathcal{K}_* := \inf_{\theta \in \mathbb{R}_{++}} \mathcal{K}(\theta)$.

Definition 4. *The collective choice problem is* ***strongly adversely correlated*** *if* $\mathcal{K}_* < 0$ *and* ***weakly advantageously correlated*** *if* $\mathcal{K}_* > 0$.

On one hand, $V^G(\kappa) > 0$ for all $\kappa \in \{0, ..., \tau\}$ is sufficient but not necessary for $\mathcal{K}_* > 0$, and so advantageous correlation implies weak advantageous correlation but not vice versa. On the other hand, $V^G(\kappa) < 0$ for some $\kappa \in \{0, ..., \tau\}$ is necessary but

[16] Note that $\binom{\tau}{\kappa} = \binom{n}{\tau}^{-1} \binom{n}{\kappa} \binom{n-\kappa}{\tau-\kappa}$, where $\binom{n}{\kappa}$ is the number of ways of selecting the κ voters that receive good news, $\binom{n-\kappa}{\tau-\kappa}$ is the number of ways of selecting $\tau - \kappa$ uninformed voters to vote for the optimal policy along with voters who received good news, and their product is normalized by the total number of ways of selecting τ votes for the optimal policy $\binom{n}{\tau}$.

not sufficient for $\mathcal{K}_* < 0$, and so strong adverse correlation implies adverse correlation but not vice versa. However, $V^G(\tau) < 0$ does imply $\mathcal{K}_* < 0$ because, as $\theta \to \infty$, the τ-th summand in $\mathcal{K}(\theta)$ is dominant.

Every collective choice problem is either strongly adversely or weakly advantageously correlated except in the knife-edge case where $\mathcal{K}_* = 0$, which is possible only on a measure-0 set of parameters. Parallel to Theorem 1, the following theorem therefore essentially characterizes when a collective choice problem is fragile to asymmetric information with a restriction to symmetric equilibria.

Theorem 3. *If the collective choice problem is strongly adversely correlated, the ex ante inferior policy p_* wins in a symmetric equilibrium with scarce information. By contrast, if the collective choice problem is weakly advantageously correlated, then the ex ante optimal policy p^* wins in every symmetric equilibrium with scarce information.*

Hence, fragility also emerges with symmetric equilibria. However, establishing this fragility requires a different argument. Suppose that the collective choice problem is strongly adversely correlated; moreover, assume that $V^G(\tau) > 0$.[17] We show that a sequence of symmetric mixed strategy equilibria selects policy p_* with probability approaching 1 as $\lambda \to 0$. The idea is that $\mathcal{K}_* < 0$ implies the existence of $\tilde{\theta}$ such that $\mathcal{K}(\tilde{\theta}) = 0$. For each $\lambda \in (0, 1)$, we then choose α such that $\tilde{\theta} = \frac{\lambda}{(1-\lambda)\alpha}$; in other words, $\tilde{\theta}$ is the relative likelihood that a vote for p^* is cast by an informed voter. Choosing α at this rate guarantees that an uninformed voter is indifferent between policies p^* and p_* conditional on being pivotal, which rationalizes her mixing. Moreover, as $\lambda \to 0$, the probability that an uninformed voter votes for p^* converges to 0. As most voters are uninformed, policy p_* then wins with near certainty.

The converse also requires a different argument because weak advantageous correlation does not imply advantageous correlation. However, weak advantageous correlation is sufficient to ensure that, regardless of how one pools votes for p^* from uninformed voters and those who hear good news, uninformed voters are never swayed to vote for policy p_* with substantial probability in any symmetric equilibrium.

5.2 Population Uncertainty

The characterization for symmetric equilibria can also accommodate population uncertainty. Suppose that, as in Myerson (1998), the population size is random. For

[17] If $V^G(\tau) < 0$, the equilibrium constructed in the proof of Theorem 1 is already symmetric.

simplicity, we assume that the population size is always odd and a policy $p \in \{a, b\}$ is then chosen by simple majority rule: for a realized population of n voters, policy p wins if it receives at least $\tau(n) + 1$ votes, where $\tau(n) := \frac{n-1}{2}$. We make the following assumption about the random population size.

Assumption 4. *The population size N is drawn from a probability measure Q with a finite expectation and support $\mathcal{Q}$, which is a subset of the odd positive integers strictly greater than one.*

Let $\Omega^n = \mathcal{V}^n \times \mathcal{S}^n$ be the state space for a realized population size n. Uncertainty is described by the random population size $(\mathcal{Q}, Q)$ and a stochastic process $\{(\Omega^n, P_n) : n \in \mathcal{Q}\}$, where P_n is a probability distribution over payoff and signals profiles, $\omega \in \Omega^n$, for the voters in an election for population size n. We adapt our main assumptions in Section 2 to apply conditional on each population size n (see online appendix for a formal description). In addition, we assume that the ex-ante optimal policy p^* is also optimal conditioning on only the population size for every population size.

We generalize the correlation measure considered in Section 5.1, considering inferences that a voter draws were she to think that the population size is n_0, which is the smallest population size in $\mathcal{Q}$. Adapting our previous notation, let $V^G(\kappa, n_0) := V_i^d(S_i = s^0, G = \kappa, B = 0, N = n_0)$ denote the expected payoff difference for a voter who receives the uninformative signal, conditioning on κ voters obtaining good news, no voter obtaining bad news, and there being n_0 voters (where $n_0 \geq \kappa$). This term feeds into the relevant correlation measure:

$$\mathcal{K}_*(n_0) := \inf_{\theta \in \mathbb{R}_{++}} \sum_{\kappa=0}^{\tau(n_0)} \theta^\kappa \binom{\tau(n_0)}{\kappa} P(G = \kappa | B = 0) V^G(\kappa, n_0).$$

Theorem 4. *If $\mathcal{K}_*(n_0) < 0$, then policy p_* wins in a symmetric equilibrium with scarce information. By contrast, if $\mathcal{K}_*(n_0) > 0$, then policy p^* wins in every symmetric equilibrium with scarce information.*

The key idea is that an uninformed voter believes she is most likely to be pivotal in a small election, and increasingly so as information becomes scarce. Hence, her beliefs about adverse correlation at population size n_0 drive behavior.

5.3 The Role of Elites

Our baseline model isolates the role of asymmetric information by assuming away all other differences between voters. Here, we allow for voters to be ex ante heterogeneous and show that such differences only exacerbate the scope for electoral failures.

Formally, we continue to assume that all voters agree on the ex ante optimal policy p^* to give us unambiguous no-information and public information benchmarks, but weaken Assumption 1.[18] Specifically, we decompose voters into "elites" and "non-elites." Voters are exchangeable within each group, but not necessarily across these groups. The collective choice problem therefore consists of the set of elite voters $\mathcal{E}$, non-elite voters $\mathcal{NE}$, and a probability space $(\Omega, P) = (\mathcal{V}^n \times \mathcal{S}^n, P)$ such that Assumptions 2 and 3 are satisfied, and for any permutations, $\psi_E : \mathcal{E} \to \mathcal{E}$ and $\psi_N : \mathcal{NE} \to \mathcal{NE}$, and state $\omega, P(\omega_{\mathcal{E}}, \omega_{\mathcal{NE}}) = P(\omega_{\psi_E(\mathcal{E})}, \omega_{\psi_N(\mathcal{NE})})$.

Given a non-empty set of voters $\mathcal{H}$, let $G_{\mathcal{H}}$ denote the random variable describing the number of voters in $\mathcal{H}$ who received good news.

Definition 5. *The collective choice problem is **elite-adversely correlated** if there exists a binary partition of the electorate, $\{\mathcal{E}, \mathcal{NE}\}$ such that the following hold:*

(a) *Elites are a minority: $|\mathcal{E}| < \tau$.*

(b) *Elites do not fear the support of others:*

$$V_i^d(S_i = s^0, B = 0, G = G_{\mathcal{NE}} = \tau - |\mathcal{E}| + 1) > 0 \text{ for every } i \in \mathcal{E}.$$

(c) *Non-elites fear the support of others:*

$$V_i^d(S_i = s^0, B = 0, G = G_{\mathcal{NE}} = \tau - |\mathcal{E}|) < 0 \text{ for every } i \in \mathcal{NE}.$$

Definition 5(a) asserts that the elites are a minority of the electorate. Part (b) states that these voters continue to support the optimal policy even after conditioning on the support of others. In other words, they are not concerned about adverse selection. Part (c), by contrast, states that non-elites *are* concerned by adverse selection: knowing that all elites vote for the optimal policy, each views her odds of gaining from p^* to be low when sufficiently many non-elite voters obtain good news. Parts (b) and (c) together imply that elites are more likely to benefit from policy p^*.

[18] By agreeing on the ex ante optimal policy, we mean that for every pair of voters, i and j, $\mathbb{E}[V_i^a - V_i^b]\mathbb{E}[V_j^a - V_j^b] > 0$.

A special case of Definition 5 is where each elite voter has a higher "rank" than every non-elite voter in that an elite voter is guaranteed to gain from the optimal policy p^* whenever a non-elite voter does. An elite voter then is elated to learn that any non-elite voter has good news for it assures that she too gains from p^*. By contrast, non-elites are crowded out from being winners both by elites and other non-elites. In this vein, Definition 5 views the optimal policy as a gamble that is simply more likely to benefit elites before its rewards trickle down to non-elite voters.

As voters agree that p^* is ex-ante optimal, it remains the winner with scarce *public* information. But collective choice may still be fragile to private information.

Proposition 6. *Suppose payoffs are elite-adversely correlated. Then, the inferior policy p_* wins in a strict equilibrium with scarce information.*

The idea is that elite voters, unconcerned by adverse selection, vote for p^* even when they are uninformed. As elites are a minority, p^* can only be in the race if it has sufficient support from non-elites. An uninformed non-elite voter then worries about being crowded out and hence has a strict incentive to vote for p^*.

Moreover, the presence of elite voters can exacerbate adverse selection for non-elites. To see how, we specialize to the binary payoff setup described in Section 4.1, in which informed voters obtain perfect news, and assume that $W_i \subseteq W_j$ for all $i \in \mathcal{NE}$ and $j \in \mathcal{E}$. The collective choice problem is then fully described by the probability that a voter receives an informative signal λ, the ex-ante distribution over the number of winners P_W, the payoffs $v = (v_W, v_L)$, and the number of elite voters $|\mathcal{E}| := e$. Let $\tilde{E}(e) = \{\omega \in \Omega : S_i = s^0, B = 0, G = G_{\mathcal{NE}} = \tau - e\}$.

Proposition 7. *For a non-elite voter i, the conditional expected payoff $V_i^d(\tilde{E}(e)|P_W, v, e)$ is strictly decreasing in the number of elites e.*

Hence, increasing the size of the elite group expands the range of polarization ratios for which the collective choice problem is majority-adversely correlated, leading to a greater scope for political failures.

6 Conclusion

We find that asymmetric information leads to adverse selection in distributive politics. Voters may choose policies that do not match their collective preferences and information, and equilibria itself may be fragile to asymmetric information. Broadly, a form of zero-sum thinking may prevail even if the policy choice is not zero sum.

We obtain this electoral failure in a strict equilibrium. There may be other equilibria in which voters fare better. As to how voters behave in practice is an empirical question. Motivated by this question, Ali, Mihm, Siga, and Tergiman (2021) study the extent to which people account for adverse and advantageous selection. People appear to be significantly better at accounting for adverse selection: across a range of experimental treatments, subjects distrust better-informed partners who have conflicting interests but fail to trust those with aligned interests. This finding dovetails with the evidence that zero-sum thinking is pervasive, noted in social psychology (Meegan, 2010; Różycka-Tran, Boski, and Wojciszke, 2015; Davidai and Ongis, 2019) and economics (Chinoy, Nunn, Sequeira, and Stantcheva, 2022). That people see the world as zero-sum and account for adverse selection lends support to the strategic underpinnings of our results.

The strategic calculus of adverse selection operates through voters considering what it means for a policy to be *viable*, i.e., command enough support that it might actually pass. This thought process involves, much like the lemon's problem, identifying who the supporters of a policy are, why they might favor that policy, and whether their support for it is bad news. We find this line of thinking to resonate with political rhetoric. For instance, populist movements often suggest that those who oppose them are elites who do not share the interests of the common voter. Although we model this logic as applying at an individual level, one could also do so at a group-level, in the spirit of ethical voter models (Coate and Conlin, 2004; Feddersen and Sandroni, 2006).[19] We also see this logic as potentially manifesting in other political contexts, such as legislative action and lobbying, where the interested parties do not all expect to gain but have to act jointly to select a policy. Each player may then worry about being crowded out from gains when she conditions on others supporting a policy.

Our study shows that voting behavior in distributive politics may be highly sensitive to private information. An equivalent way to frame our results is through the lens of public and private information. Suppose that all voters obtain noisy public information that indicates that policy p^* is superior. If it were commonly known that no voter obtains any additional private information, the electorate would use this public

[19]Ethical voter models typically address costly voting; we consider the same calculus here but with costless voting. Say each voter of our model is a stand-in for workers in a different sector, and members of each group vote in the group's interest. Identical results then ensue if the collective choice problem is adversely correlated across these sectors.

information to select policy p^* in every weakly undominated equilibrium. However, if some voters obtain additional private information with even the slightest chance, under adverse correlation, there is an equilibrium in which the electorate fails to use public information to select the better policy. Thus, a better informed electorate might make worse choices when some of that better information is delivered privately to voters.

While our analysis tackles several important questions, it leaves others unanswered. We abstract from costly turnout decisions and it may be useful to see how endogenous participation affects collective choice. Equally, it would be interesting to see how parties strategically choose policies to exploit polarization.[20] One may also envision the design of information structures to capitalize on zero-sum thinking. We hope to address these questions in future work.

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[20]This has been a theme of recent work, for example, Buisseret and Van Weelden (2022), Bueno De Mesquita and Dziuda (2022), and Levy and Razin (2022).

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A Appendix

The appendix has the following structure:

1. Appendix A.1 includes preliminary results and notation, proves Proposition 1, and describes the decomposition detailed in Section 3.1.
2. Appendix A.2 contains all proofs for Section 3.
3. Appendix A.3 contains all proofs for Section 4.

The Online Appendix contains all proofs for our extensions described in Section 5.

A.1 Preliminaries

We first provide a formal description of the voting game and establish equilibrium existence, and then describe how we analyze equilibria with scarce information.

Throughout, $\mathcal{G} = \{s^k \in \mathcal{M} : V_i^d(S_i = s^k) > 0\}$ is the set of signals that convey good news, and $\mathcal{B} = \mathcal{M} \setminus \mathcal{G}$ the set of signals that convey bad news. For $g \in \{0, ..., n-1\}$ and $m \in \{g, ..., n-1\}$,

$$Z(g, m) = P(G = g|M = m)V_i^d(G = g, M = m, S_i = s^0)$$

is the expected payoff difference for an uninformed voter who learns that m other voters received news g of whom received good news, weighted by the probability that g voters receive good news when m receive news. Finally s_0 is the signal profile where all voters receive the uninformative signal.

A.1.1 Strategies and Equilibrium

Private information: A strategy-profile is a mapping $\sigma : \mathcal{N} \times \mathcal{S} \to [0, 1]$, where $\sigma(i, s_i) := \sigma_i(s_i)$ represents the probability that voter $i \in \mathcal{N}$ votes for p^* when she receives the signal $s_i \in \mathcal{S}$. Let Σ be the set of all strategy-profiles, and let

$$\Sigma_i^* = \{\sigma \in \Sigma : \forall\, i \in \mathcal{N}, \sigma_i(s^k) = 1\ \forall s^k \in \mathcal{G} \text{ and } \sigma_i(s^k) = 0\ \forall s^k \in \mathcal{B}\}.$$

The set Σ^* differs from Σ only by excluding strategy-profiles where informed voters vote against their own signals, which are weakly dominated strategy-profiles by Assumption 2.[21] For a strategy-profile $\sigma \in \Sigma^*$, we simplify notation by letting $\sigma_i := \sigma_i(s^0)$.

An action-profile is a mapping $a : \mathcal{N} \to \{0, 1\}$, where $a(i) = 1$ represents a vote for p^* and $a(i) = 0$ represents a vote for p_*. Let $\mathcal{A}$ be the set of all action-profiles. For $a \in \mathcal{A}$, $a^{-1}(1) = \{i \in \mathcal{N} : a(i) = 1\}$ is the set of voters who vote for p^*. For any voter $i \in \mathcal{N}$ and $a \in \mathcal{A}$, let $a_{-i}^{-1}(1) = a^{-1}(1) - \{i\}$.

Given a collective choice problem $\mathcal{C} = (\mathcal{N}, \Omega, P)$, we denote by P_σ the probability distribution on $\mathcal{A} \times \Omega$ induced by strategy-profile $\sigma \in \Sigma$ and the primitive distribution

[21] A strategy-profile $\sigma \in \Sigma$ is weakly undominated if, for each voter $i \in \mathcal{N}$, there does not exists an alternative strategy σ_i' such that voter i's expected payoff from $(\sigma_i', \sigma_{-i}')$ is greater than equal to her expected payoff from (σ_i, σ_{-i}') for all $\sigma_{-i}' \in \Sigma_{-i}$ and strictly greater for some $\sigma_{-i}' \in \Sigma_{-i}$.

P on Ω, defined by

$$P_\sigma(a,\omega) := P(\omega) \prod_{i\in a^{-1}(1)} \sigma_i\big(S_i(\omega)\big) \prod_{j\in a^{-1}(0)} \Big(1-\sigma_j\big(S_j(\omega)\big)\Big).$$

For a voter i, we then denote by $\Pi_i(\mathcal{C},\sigma)$ the difference between the expected payoff when voter i votes for p^* and the expected payoff when she votes for p_* conditional on her receiving the uninformative signal s^0, which equates to

$$\Pi_i(\mathcal{C},\sigma) := \sum_{\{\omega\in\Omega: P(\omega)>0\}} V_i^d(\omega) P_\sigma\left(\left|a_{-i}^{-1}(1)\right| = \tau \middle| \omega\right) P(\omega|S_i = s^0)$$

since voter i impacts the election outcome only for action-profiles with $|a_{-i}^{-1}(1)| = \tau$.

Let $\mathcal{N}_i(g,m)$ be the collection of all (N_0,N_1) such that $N_0, N_1 \subseteq \mathcal{N} - \{i\}$ with $N_0 \cap N_1 = \emptyset$, $|N_0| = \tau - (m-g)$, and $|N_1| = \tau - g$. Then, when $\sigma \in \Sigma^*$,

$$\Pi_i(\mathcal{C},\sigma) = \sum_{g=0}^{\tau} \sum_{m=g}^{\tau+g} p_i(\sigma|g,m) Z(g,m)$$

where[22]

$$\begin{aligned} p_i(\sigma|g,m) &:= P_\sigma\left(|a_{-i}^{-1}(1)| = \tau \,\middle|\, G = g, M = m, S_i = s^0\right) \\ &= \binom{n-1}{m}^{-1} \sum_{(N_0,N_1)\in\mathcal{N}_i(g,m)} \prod_{j\in N_1} \sigma_j(s^0) \prod_{k\in N_0} (1-\sigma_k(s^0)). \end{aligned} \tag{1}$$

We observe that a strategy-profile $\sigma \in \Sigma$ is an equilibrium (in weakly undominated strategies) if and only if $\sigma \in \Sigma^*$ and, for all $i \in \mathcal{N}$, $\Pi_i(\mathcal{C},\sigma) > 0$ implies $\sigma_i(s^0) = 1$ and $\Pi_i(\mathcal{C},\sigma) < 0$ implies $\sigma_i(s^0) = 0$.

Public information: In the public information benchmark, a strategy-profile is a mapping $\phi : \mathcal{N} \times \mathcal{S}^n \to [0,1]$ where $\phi(i,s) := \phi_i(s)$ is the probability that voter i votes for p^* when she observes the signal-profile $s \in \mathcal{S}^n$.

[22]We follow the standard convention that the product over terms in the empty set is 1.

A.1.2 Proof of Proposition 1

In the private information baseline, let $\sigma^\alpha \in \Sigma^*$ denote the symmetric strategy-profile where $\sigma_i^\alpha = \alpha \in [0,1]$ for all $i \in \mathcal{N}$. Then, for $g \in \{0, ..., \tau\}$ and $m \in \{g, ..., g+\tau\}$,

$$p_i(\sigma^\alpha | g, m) = \begin{cases} \mathbb{1}[g = \tau] & \text{if } \alpha = 0 \\ \mathbb{1}[m - g = \tau] & \text{if } \alpha = 1 \\ \binom{n-1-m}{\tau-g} \alpha^{\tau-g}(1-\alpha)^{\tau-(m-g)} & \text{if } \alpha \in (0,1) \end{cases},$$

Hence, $p_i(\sigma^\alpha|g,m)$ is continuous in α, and so $\Pi_i(\mathcal{C}, \sigma^\alpha)$ is continuous in α. If $\Pi_i(\mathcal{C}, \sigma^1) \geq 0$, then σ^1 is an equilibrium; if $\Pi_i(\mathcal{C}, \sigma^0) \leq 0$, then σ^0 is an equilibrium; otherwise, there exists $\alpha^* \in (0,1)$ such that $\Pi_i(\mathcal{C}, \sigma^{\alpha^*}) = 0$ and so σ^{α^*} is an equilibrium.

In the public information benchmark, the only weakly undominated strategy-profile is ϕ^*, where $\phi_i^*(s) = \mathbb{1}[V_i^d(s) > 0]$ for all $i \in \mathcal{N}$, which is the unique equilibrium. ■

A.1.3 Scarce information

For our equilibrium analysis with scarce information, the following Lemma shows how a collective choice problem $\mathcal{C}$ can be decomposed into an informed collective choice problem $\mathcal{C}_\mathcal{I}$ and the probability λ that any one voter is informed.

Lemma 1. *Let $\mathcal{C} = (\mathcal{N}, \Omega, P)$ satisfy Assumptions 1 and 2. Then, for any state $\omega = (v, s) \in \Omega$,*

$$P(\omega) = \lambda^{M(s)}(1-\lambda)^{n-M(s)} \sum_{s' \in C(s)} P_\mathcal{I}(v, s').$$

Moreover, if $\sigma \in \Sigma^*$, then in each term of $\Pi_i(\mathcal{C}, \sigma)$, $P(M = m | S_i = s^0)$ depends only on λ, $p_i(\sigma|g,m)$ depends only on σ, and $Z(g,m)$ depend only on $\mathcal{C}_\mathcal{I}$.

Proof. Let $\omega = (v, s) \in \Omega$ with $I = \{i \in \mathcal{N} : S_i(\omega) \in \mathcal{I}\}$ and $m = |I|$. Then, by Assumptions 1 and 2,

$$\begin{aligned} P(v,s) = P(v, s_I)(1-\lambda)^{n-m} &= \frac{P(v, s_I, S_{\mathcal{N}-I} \in \mathcal{M}^{n-m})}{\lambda^{n-m}}(1-\lambda)^{n-m} \\ &= \frac{P(v, s_I | S \in \mathcal{M}^n)\lambda^n}{\lambda^{n-m}}(1-\lambda)^{n-m} = \lambda^m(1-\lambda)^{n-m} \sum_{s' \in C(s)} P_\mathcal{I}(v, s'). \end{aligned}$$

Moreover, by Assumptions 1 and 2,

$$P(M = m|S_i = s^0) = \binom{n-1}{m}\lambda^m(1-\lambda)^{n-1-m},$$

which depends only on λ. From Equation (1) it is immediate that $p_i(\sigma|g,m)$ depends only on σ when $g \leq \tau$ and $m \leq \tau + g$, and $p_i(\sigma|g,m) = 0$ otherwise. Finally, we show that $Z(g,m)$ depends only on $\mathcal{C}_{\mathcal{I}}$. For $g \in \{0,...,n-1\}$ and $m \in \{g,...,n-1\}$, let $\Omega(g,m) = \{(v,s) \in \Omega : G(s) = g, M(s) = m\}$ and $\Omega_i(g,m) = \{\omega \in \Omega(g,m) : S_i = s_0\}$. Then,

$$P(G = g|M = m, S_i = s^0) = \frac{\sum_{(v,s)\in\Omega(g,m)} \lambda^m(1-\lambda)^{n-m} \sum_{s'\in C(s)} P_{\mathcal{I}}(v,s')}{\binom{n-1}{m}\lambda^m(1-\lambda)^{n-m}}$$

which depends only on $\mathcal{C}_{\mathcal{I}}$, and so

$$\begin{aligned}
V_i^d(G = g, M = m, S_i = s^0) &= \sum_{(v,s)\in\Omega_i(g,m)} P(v,s|G = g, M = m)v_i^d \\
&= \sum_{(v,s)\in\Omega_i(g,m)} \frac{P(v,s)}{P(G = g, M = m, S_i = s^0)} v_i^d \\
&= \sum_{(v,s)\in\Omega_i(g,m)} \frac{\lambda^m(1-\lambda)^{n-m}\sum_{s'\in C(s)} P_{\mathcal{I}}(v,s')}{P(G = g|M = m, S_i = s^0)\binom{n-1}{m}\lambda^m(1-\lambda)^{n-m}} v_i^d,
\end{aligned}$$

depends only on $\mathcal{C}_{\mathcal{I}}$. ■

A.2 Proofs for Section 3

In Section 3, we fix $\mathcal{C}_{\mathcal{I}} := (\mathcal{N}, \Omega_{\mathcal{I}}, P_{\mathcal{I}})$, and look at the equilibrium outcomes of the collective choice problems $\{(\mathcal{C}_{\mathcal{I}}, \lambda) : \lambda \in (0,1)\}$ as λ gets small. By Lemma 1, the correlation structure is a property of $\mathcal{C}_{\mathcal{I}}$: if any collective choice problem in $\{(\mathcal{C}_{\mathcal{I}}, \lambda) : \lambda \in (0,1)\}$ is adversely/advantageously correlated, then all collective choice problems in the class have the same correlation.

Since $\mathcal{C}_{\mathcal{I}}$ is fixed, we write the expected payoff difference $\Pi_i(\mathcal{C}, \sigma)$ simply as a function of λ and σ. For $\kappa \in \{0,...,\tau\}$, assumption 3 implies that $P(G = \kappa|M = \kappa, S_i = s^0) > 0$ and $V^G(\kappa) \neq 0$; hence, $Z(\kappa,\kappa) \neq 0$ and $Z(\kappa,\kappa) > 0 \iff V^G(\kappa) > 0$.

A.2.1 Proof of Proposition 2

Fix $\varepsilon \in (0,1)$ and let $\lambda_\varepsilon = 1-(1-\varepsilon)^{\frac{1}{n}}$. Now consider the equilibrium strategy profile ϕ^* from the proof of Proposition 1. Since $V_i^d(s_0) > 0$, $\sigma_i^*(s_0) = 1$ for all $i \in \mathcal{N}$, and so p^* wins in this event. Hence, for all $\lambda \in (0, \lambda_\varepsilon)$, the probability that p^* wins is greater than $P(S = s_0) = (1-\lambda)^n > (1-\lambda_\varepsilon)^n = (1-\varepsilon)$. ■

A.2.2 Proof of Theorem 1

Suppose the informed collective choice problem is adversely correlated: $V^G(\kappa) < 0$ for some $\kappa \in \{1, ..., \tau\}$. For any $t \in \{0, ..., n\}$, let σ^t be the strategy-profile in Σ^* where $\sigma_i^t = \mathbb{1}[i \leq t]$ for all $i \in \mathcal{N}$ (i.e., the first t voters vote for p^* when uninformed and the remaining voters vote for p_* when uninformed). Let $\mathcal{N}_t = \{1, ..., t\}$ and $\mathcal{N}_t^c = \{t+1, ..., n\}$.

Now fix $\varepsilon \in (0,1)$ and let $\lambda_\varepsilon = 1-(1-\varepsilon)^{\frac{1}{n}}$. If $t \leq \tau$, p_* wins in the event $\{S = s_0\}$ and so, for all $\lambda \in (0, \lambda_\varepsilon)$, the probability that p_* wins is greater than $(1-\varepsilon)$. We can therefore complete the proof by showing that there exists $\bar{\lambda}$ such that, for all $\lambda < \bar{\lambda}$, σ^t is an equilibrium for some $t \leq \tau$. We do this by considering the two cases where $V^G(\tau) < 0$ and $V^G(\tau) > 0$.

Case 1: When $V^G(\tau) < 0$, σ^0 is an equilibrium for λ sufficiently small.

For any voter $i \in \mathcal{N}$, $p_i(\sigma^0 | g, m) = \mathbb{1}[g = \tau]$ because only voters with good signals vote for p^*. Hence,

$$\Pi_i(\lambda, \sigma^0) = \sum_{m=\tau}^{\tau+g} \binom{n-1}{m} \lambda^m (1-\lambda)^{n-1-m} Z(g, m).$$

Since $\lambda^\tau (1-\lambda)^\tau > 0$, it follows that $\Pi_i(\lambda, \sigma^0) < 0$ if and only if

$$\sum_{m=\tau}^{\tau+g} \binom{n-1}{m} \left(\frac{\lambda}{1-\lambda}\right)^{m-\tau} Z(g, m) < 0,$$

where the lhs converges to $\binom{n-1}{\tau} Z(\tau, \tau) < 0$ as $\lambda \to 0$.

Case 2: When $V^G(\tau) > 0$, there exists $\kappa \in \{1, ..., \tau-1\}$ such that $V^G(\kappa) < 0$ and $V^G(\kappa') > 0$ for $\kappa' \in \{\kappa+1, ..., \tau\}$, and $\sigma^{\tau-\kappa}$ is an equilibrium for λ sufficiently small.

For any voter $i \in \mathcal{N}_{\tau-\kappa}$, $p_i(\sigma^{\tau-\kappa} | g, m) = 0$ if $g < \tau - (\tau - \kappa - 1) = \kappa + 1$ because there only $\tau - \kappa - 1$ other voters who vote for p^* when uninformed. Hence,

$$\Pi_i(\lambda, \sigma^{\tau-\kappa}) = \sum_{g=\kappa+1}^{\tau} \sum_{m=g}^{\tau+g} \binom{n-1}{m} \lambda^m (1-\lambda)^{n-1-m} p_i(\sigma^{\tau-\kappa}|g,m) Z(g,m)$$

Since $\lambda^{\kappa+1}(1-\lambda)^{n-1-(\kappa+1)} > 0$, it follows that $\Pi_i(\lambda, \sigma^{\tau-\kappa}) > 0$ if and only if

$$\sum_{g=\kappa+1}^{\tau} \sum_{m=g}^{\tau+g} \binom{n-1}{m} \left(\frac{\lambda}{1-\lambda}\right)^{m-(\kappa+1)} p_i(\sigma^{\tau-\kappa}|g,m) Z(g,m) > 0,$$

where the lhs converges to $\binom{n-1}{\kappa+1} p_i(\sigma^{\tau-\kappa}|\kappa+1, \kappa+1) Z(\kappa+1, \kappa+1)$ as $\lambda \to 0$. Since i is pivotal in the non-null event where all $\kappa+1$ of the voters in $\mathcal{N}^c_{\tau-\kappa}$ receive good news, $p_i(\sigma^{\tau-\kappa}|\kappa+1, \kappa+1) > 0$, and $Z(\kappa+1, \kappa+1) > 0$ because $V^G(\kappa+1) > 0$. Hence, $\Pi_i(\lambda, \sigma^{\tau-\kappa}) > 0$ for λ sufficiently small.

For any voter $i \in \mathcal{N}^c_{\tau-\kappa}$, $p_i(\sigma^{\tau-\kappa}|g,m) = 0$ if $g < \tau - (\tau - \kappa) = \kappa$ because there $\tau - \kappa$ other voters who vote for p^* when uninformed. Hence,

$$\Pi_i(\lambda, \sigma^{\tau-\kappa}) = \sum_{g=\kappa}^{\tau} \sum_{m=g}^{\tau+g} \binom{n-1}{m} \lambda^m (1-\lambda)^{n-1-m} p_i(\sigma^{\tau-\kappa}|g,m) Z(g,m)$$

Since $\lambda^{\kappa}(1-\lambda)^{n-1-(\kappa)} > 0$, it follows that $\Pi_i(\lambda, \sigma^{\tau-\kappa}) < 0$ if and only if

$$\sum_{g=\kappa}^{\tau} \sum_{m=g}^{\tau+g} \binom{n-1}{m} \left(\frac{\lambda}{1-\lambda}\right)^{m-\kappa} p_i(\sigma^{\tau-\kappa}|g,m) Z(g,m) < 0,$$

where the lhs converges to $p_i(\sigma^{\tau-\kappa}|\kappa, \kappa) Z(\kappa, \kappa)$ as $\lambda \to 0$. Since i is pivotal in the non-null event where κ voters in $\mathcal{N}^c_{\tau-\kappa}$ receive good news, $p_i(\sigma^{\tau-\kappa}|\kappa, \kappa) > 0$, and $Z(\kappa, \kappa) < 0$ because $V^G(\kappa) < 0$. Hence, $\Pi_i(\lambda, \sigma^{\tau-\kappa}) < 0$ for λ sufficiently small.

Together, the two cases show that, with adverse correlation, there exists some $\bar{\lambda} \in (0,1)$ such that, for all $\lambda \in (0, \bar{\lambda})$ there is an equilibrium in which a majority of voters vote for p_* when they are uninformed. Hence, p_* wins when all voters are uninformed and so, for $\lambda \leq \min\{\bar{\lambda}, \lambda_\varepsilon\}$, there is an equilibrium in which p_* wins with probability exceeding $1 - \varepsilon$.

For the converse, suppose the informed collective choice problem is advantageously correlated: $V^G(\kappa) > 0$ for all $\kappa \in \{1, ..., \tau\}$. Fix some $\varepsilon \in (0,1)$, let $\sigma \in \Sigma^*$ and, without loss of generality, let $\sigma_i \geq \sigma_{i+1}$ for $i = 1, ..., n-1$. We show that, for the strategy profile σ and λ sufficiently small, either p^* wins with probability exceeding $1-\varepsilon$ or σ is not an equilibrium. Hence, for λ sufficiently small, if σ is an equilibrium, then p^* wins with probability exceeding $1-\varepsilon$.

Let $\delta := (1-\varepsilon)^{\frac{1}{2(\tau+1)}} \in (0,1)$ and $\lambda'_\varepsilon = 1-(1-\varepsilon)^{\frac{1}{2n}}$. If $\sigma_{\tau+1} \geq \delta$, then, in the event $\{S = s_0\}$ where all voters are uninformed, voters $i = 1, ..., \tau+1$ vote for p^* with probability at least δ. Therefore, for all $\lambda \in (0, \lambda'_\varepsilon)$,

$$P_\sigma(|a^{-1}(1)| \geq \tau+1) \geq \delta^{\tau+1}(1-\lambda)^n > (1-\varepsilon)^{\frac{\tau+1}{2(\tau+1)}}(1-\varepsilon)^{\frac{n}{2n}} = 1-\varepsilon.$$

To complete the proof, it sufficies to show that, if $\sigma_{\tau+1} < \delta$, then there exists λ_ε such that, for all $\lambda \in (0, \lambda_\varepsilon)$, σ is not an equilibrium.

Therefore, suppose $\sigma_{\tau+1} < \delta$. We consider voter n, who when uninformed votes p^* with probability less than 1, and show this is not a best-response. For $g \in \{0, ..., \tau\}$ and $m \in \{g, ..., \tau+g\}$, it follows from Equation (1) that

$$\binom{n-1}{m}^{-1}(1-\delta)^{\tau-(m-g)}\prod_{j=1}^{\tau-g}\sigma_j \leq p_n(\sigma|g,m) \leq \binom{n-1-m}{\tau-g}\prod_{j=1}^{\tau-g}\sigma_j,$$

and, therefore, for $g \in \{0, ..., \tau\}$,

$$\sum_{m=g}^{\tau+g} p_n(\sigma|g,m)P(M=m|S_n=s^0)Z(g,m) \geq$$
$$\lambda^g(1-\lambda)^{n-1-g}\prod_{j=1}^{\tau-g}\sigma_j\left((1-\delta)^\tau Z(g,g) - \sum_{m=g+1}^{\tau+g}\left(\frac{\lambda}{1-\lambda}\right)^{m-g}\mathcal{M}(m,g)\big|Z(g,m)\big|\right)$$

where $\mathcal{M}(m,g)$ is shorthand for the multinomial coefficient

$$\mathcal{M}(m,g) := \binom{n-1}{m, \tau-g, \tau-(m-g)}.$$

Since $\delta \in (0,1)$ and $Z(g,g) > 0$, there exists $\lambda_g \in (0,1)$ such that

$$(1-\delta)^\tau Z(g,g) - \sum_{m=g+1}^{\tau+g}\left(\frac{\lambda}{1-\lambda}\right)^{m-g}\mathcal{M}(m,g)\big|Z(g,m)\big| > 0$$

for all $\lambda \in (0, \lambda_g)$. In particular, λ_g depends only on $\mathcal{C}_\mathcal{I}$. Let $\lambda_\varepsilon = \min\{\lambda_g : g \in \{0, ..., \tau\}\}$. Since $\prod_{j=1}^{\tau-g}\sigma_j > 0$ when $g = \tau$, it follows that $\Pi_n(\lambda, \sigma) > 0$ for all $\lambda \in$

$(0, \lambda_\varepsilon)$. But then σ is not an equilibrium for $\lambda \in (0, \lambda_\varepsilon)$ because $\sigma_n < 1$. ■

A.2.3 Proof of Theorem 2

We first observe that, for $\sigma \in \Sigma^*$, $\Pi_i(\lambda, \sigma)$ can equivalently be written in terms of number of other voters who receive bad news:

$$\Pi_i(\lambda, \sigma) = \sum_{b=0}^{\tau} \sum_{m=b}^{\tau+b} q_i(\sigma|b, m) \binom{n-1}{m} \lambda^m (1-\lambda)^{n-1-m} Z_B(b, m)$$

where, for $b \in \{0, ..., \tau\}$ and $m \in \{b, ..., \tau + b\}$, $q_i(\sigma|b, m) = p_i(\sigma|m - b, m)$ and $Z_B(b, m) = Z(m - b, m)$.

Now fix $\varepsilon \in (0, 1)$ and let $\sigma^\alpha \in \Sigma^*$ be the symmetric strategy-profile from the proof of Proposition 1, where $\sigma_i := \alpha \in [0, 1]$ for all $i \in \mathcal{N}$. We consider three cases.

Case 1: If $Z_B(\tau, \tau) > 0$, then there exists λ_ε such that, for all $\lambda \in (0, \lambda_\varepsilon)$, σ^1 is an equilibrium in which p^* wins with probability exceeding $1 - \varepsilon$.

Let $\lambda_\varepsilon = 1 - (1 - \varepsilon)^{\frac{1}{n}}$. For the strategy-profile σ^1, p^* wins in the event $\{S = s_0\}$ and so, for all $\lambda \in (0, \lambda_\varepsilon)$, the probability that p^* wins is greater than $(1 - \varepsilon)$. We can therefore complete the proof by showing that $\Pi_i(\lambda, \sigma^1) > 0$ for λ sufficiently small.

From Equation (1), $q_i(\sigma^1|b, m) = \mathbb{1}[\tau = b]$ and so

$$\Pi_i(\lambda, \sigma^1) = \lambda^\tau (1-\lambda)^\tau \sum_{m=\tau}^{n-1} \binom{n-1}{m} \left(\frac{\lambda}{1-\lambda}\right)^{m-\tau} Z_B(b, m)$$

Since $\lambda^\tau (1 - \lambda)^\tau > 0$, it follows that $\Pi_i(\lambda, \sigma^1) > 0$ if and only if

$$\sum_{m=\tau}^{n-1} \binom{n-1}{m} \left(\frac{\lambda}{1-\lambda}\right)^{m-\tau} Z_B(b, m) > 0$$

where the lhs converges to $\binom{n-1}{\tau} Z_B(\tau, \tau) > 0$ as $\lambda \to 0$.

Case 2: If $Z_B(\tau, \tau) < 0$, there exists λ_ε such that, for all $\lambda \in (0, \lambda_\varepsilon)$, there is $\alpha \in (0, 1)$ such that σ^α is an equilibrium in which p^* wins with probability exceeding $1 - \varepsilon$.

Let $\lambda_\varepsilon = 1 - (1 - \varepsilon)^{\frac{1}{2n}} = \bar{\alpha}$. Then for all $\lambda \in (0, \lambda_\varepsilon)$ and $\alpha \in (\bar{\alpha}, 1]$, the probability that p^* wins for the strategy-profile σ^α in the event $\{S = s_0\}$ exceeds α^n, and so p^*

wins with probability exceeding

$$\alpha^n(1-\lambda)^n > \bar{\alpha}^n(1-\lambda_\varepsilon)^n = 1-\varepsilon.$$

Hence, it suffices to show that there exists $\bar{\lambda} \in (0, \lambda_\varepsilon)$ such that, for all $\lambda \in (0, \bar{\lambda})$, there exists $\alpha_\lambda \in (\bar{\alpha}, 1)$ such that σ^{α_λ} is an equilibrium.

Analogous to the argument in case 1, $Z_B(\tau, \tau) < 0$ implies that there exists $\lambda_1 \in (0, \lambda_\varepsilon)$ such that, for all $\lambda \in (0, \lambda_1)$, $\Pi_i(\lambda, \sigma^1) < 0$.

Since $\bar{\alpha} \in (0, 1)$, for $b \in \{0, ..., \tau\}$ and $m \in \{b, ..., \tau + b\}$,

$$q_i(\sigma^{\bar{\alpha}}|b, m) = \binom{n-1-m}{\tau-b}(1-\bar{\alpha})^{\tau-b}\bar{\alpha}^{\tau-(m-b)} > 0$$

and, hence,

$$\Pi_i(\lambda, \sigma^{\bar{\alpha}}) = (1-\lambda)^{n-1}\sum_{b=0}^{\tau}\sum_{m=b}^{\tau+b}\left(\frac{\lambda}{1-\lambda}\right)^m\binom{n-1}{m}q_i(\sigma^{\bar{\alpha}}|b, m)Z_B(b, m),$$

which converges to $\binom{n-1}{\tau}(1-\bar{\alpha})^\tau\bar{\alpha}^\tau Z_B(0, 0)$ as $\lambda \to 0$. Since $Z_B(0, 0) > 0$, there exists $\lambda_2 \in (0, \lambda_\varepsilon)$ such that, for all $\lambda \in (0, \lambda_2)$, $\Pi_i(\lambda, \sigma^{\bar{\alpha}}) > 0$.

Now let $\bar{\lambda} = \min\{\lambda_1, \lambda_2, \lambda_\varepsilon\}$ and let $\lambda \in (0, \bar{\lambda})$. Then, $\Pi_i(\lambda, \sigma^1) < 0 < \Pi_i(\lambda, \sigma^{\bar{\alpha}})$ and, since $\Pi_i(\lambda, \sigma^\alpha)$ is continuous in α, there exists $\alpha_\lambda \in (\bar{\alpha}, 1)$ such that $\Pi_i(\lambda, \sigma^{\alpha_\lambda}) = 0$, and so σ^{α_λ} is an equilibrium in which p^* wins with probability exceeding $1-\varepsilon$.

Case 3: If $Z_B(\tau, \tau) = 0$ then $P(B = \tau|M = m) = 0$ for all $m \in \{\tau, ..., n-1\}$. Since $p_i(\sigma^1|m-b, m) = 0$ for all $b \in \{0, ..., \tau-1\}$ and $m \in \{b, ..., \tau\}$, it follows that $\Pi_i(\lambda, \sigma^1) = 0$ for all λ, and so σ^1 is an equilibrium in which p^* wins in the event $\{S = s_0\}$. If $\lambda \leq 1-(1-\varepsilon)^{\frac{1}{n}}$, then $P(S = s_0) \geq 1-\varepsilon$, and so p^* wins in an equilibrium with probability exceeding $1-\varepsilon$.

Together, the three cases show that, with advantageous correlation, there exists $\lambda_\varepsilon \in (0, 1)$ such that, for all $\lambda \in (0, \lambda_\varepsilon)$, there exists $\alpha \in (0, 1]$ such that σ^α is a symmetric equilibrium in which p^* wins with probability exceeding $1-\varepsilon$. ■

A.3 Proofs for Section 4

For a binary collective choice problem $\mathcal{C} = (\mathcal{N}, \Omega, P)$, we let $W_i = \{\omega \in \Omega : V_i^d(\omega) > 0\}$ be the set of states in which voter i is a winner, and $\mathcal{W}(\omega) = \{i \in \mathcal{N} : \omega \in W_i\}$ denote the set of winners in state ω.

A.3.1 Proof of Proposition 3

Suppose $\mathcal{C}$ is binary collective choice problem in which signals are fully-informative. We first show that, for any $\kappa \in \{0, ..., \tau\}$,

$$V^G(\kappa|P_W, v) = \sum_{w=\kappa}^{n} \left(\left(\frac{w-\kappa}{n-\kappa} \right) v_W - \left(\frac{n-w}{n-\kappa} \right) v_L \right) \frac{\binom{w}{\kappa} P_W(w)}{\sum_{w'=\kappa}^{n} \binom{w'}{\kappa} P_W(w')}. \qquad (2)$$

For $\kappa \in \{0, ..., \tau\}$, $w \in \{\kappa + 1, ..., n\}$, and $i \in \mathcal{N}$,

$$P(G = \kappa|S_i = s^0, M = \kappa, W = w) = \left(\frac{w}{n} \right) \frac{\binom{w-1}{\kappa}\binom{n-1-(w-1)}{0}}{\binom{n-1}{\kappa}} + \left(\frac{n-w}{n} \right) \frac{\binom{w}{\kappa}\binom{n-1-w}{0}}{\binom{n-1}{\kappa}} = \frac{\binom{w}{\kappa}}{\binom{n}{\kappa}}$$

and

$$P(G = \kappa|S_i = s^0, M = \kappa, W = \kappa) = \frac{1}{\binom{n}{\kappa}}.$$

Therefore, for any $\kappa \in \{0, ..., \tau\}$ and $w \in \{\kappa, ..., n\}$, and $i \in \mathcal{N}$,

$$P(W = w|S_i = s^0, G = M = \kappa) = \frac{\binom{w}{\kappa} P_W(w)}{\sum_{w'=\kappa}^{n} \binom{w'}{\kappa} P_W(w')}.$$

Equation (2) then follows by observing that,

$$P(W_i|S_i = s^0, G = M = \kappa, W = w) = \frac{w-\kappa}{n-\kappa}.$$

Parts (a) and (b) follow because $V^G(\kappa)$ is increasing in v_W for a fixed v_L and P_W, and decreasing in v_L for a fixed v_W and P_W. Part (c) follows because, for $w' > w$, $P'_W(w')P_W(w) \geq P'_W(w)P_W(w')$ implies that

$$\frac{\binom{w'}{\kappa} P'_W(w')}{\sum_{w''=\kappa}^{n} \binom{w''}{\kappa} P'_W(w'')} \frac{\binom{w}{\kappa} P_W(w)}{\sum_{w''=\kappa}^{n} \binom{w''}{\kappa} P_W(w'')} \geq \frac{\binom{w}{\kappa} P'_W(w)}{\sum_{w''=\kappa}^{n} \binom{w''}{\kappa} P'_W(w'')} \frac{\binom{w'}{\kappa} P_W(w')}{\sum_{w''=\kappa}^{n} \binom{w''}{\kappa} P_W(w'')},$$

and so $P_W \succcurlyeq_{LR} P'_W$ implies $P_W(.|S_i = s^0, G = M = \kappa) \succcurlyeq_{LR} P'_W(.|S_i = s^0, G = M = \kappa)$. Since $P(W_i|S_i = s^0, G = M = \kappa, W = w)$ is increasing in w, $P_W(.|S_i = s^0, G = M = \kappa) \succcurlyeq_{LR} P'_W(.|S_i = s^0, G = M = \kappa)$ then implies that $V^G(\kappa|P_W, v_W, v_L) \geq$

$V^G(\kappa|P'_W, v_W, v_L)$. ■

A.3.2 Proof of Proposition 4

Suppose $\mathcal{C}$ is a binary collective choice problem in which signals convey only aggregate news. For any signal profile $s \in \mathcal{S}^n$, it follows that

$$V_i^d(s) = \sum_{w=0}^{n} V_i^d(s, W = w)P(W = w|S = s) = \sum_{w=0}^{n} V_i^d(W = w)P(W = w|S = s),$$

and therefore, $V_i^d(s) = V_j^d(s)$ for all $i, j \in \mathcal{N}$. Now consider a signal profile s such that $s_i \in \mathcal{G}$ and $s_j \in \mathcal{M}$ for some voters $i \neq j$. Then $V_i^d(s_i) > 0$ and so $V_i^d(s) > 0$ by Assumption 2(b). It follows that $V_j^d(s) > 0$, and so $V_j^d(s_j) > 0$ by Assumption 2(b). Hence, $s_j \in \mathcal{G}$. Analogously, if $s_i \in \mathcal{B}$ and $s_j \in \mathcal{M}$, then $s_j \in \mathcal{B}$.

Let $G(\kappa) = \{s \in \mathcal{M}^n : G(s) \geq \kappa\}$. By Lemma 1 and the preceding argument, for a voter who is uninformed, and learns that $\kappa \in \{1, ..., \tau\}$ other voters received good news,

$$\begin{aligned} P(W_i|S_i = s^0, M = G = \kappa) &= \lambda^\kappa(1-\lambda)^{n-\kappa} \sum_{s \in G(\kappa)} P(W_i|s)P(s|s \in G(\kappa)) \\ &= \lambda^\kappa(1-\lambda)^{n-\kappa} \sum_{s \in \mathcal{G}^n} P(W_i|s)P(s|s \in \mathcal{G}^n) \\ &= \lambda^\kappa(1-\lambda)^{n-\kappa} P(W_i|S \in \mathcal{G}^n) \end{aligned}$$

and, therefore, $V^G(\kappa) > 0$. ■

A.3.3 Proof of Proposition 5

We say that binary collective choice problems $\mathcal{C} = (\mathcal{N}, \Omega, P)$ and $\mathcal{C}' = (\mathcal{N}, \Omega', P')$ are informationally equivalent if, for any $\mathcal{N}' \subseteq \mathcal{N}$ and $s \in \mathcal{S}^n$, $P(\mathcal{W} = \mathcal{N}', S = s) = P'(\mathcal{W} = \mathcal{N}', S = s)$; that is, the problems differ in terms of the payoff but not the information structure. We show that, when all news is distributional, for any binary collective choice problem, there is an informationally equivalent problem that has adverse correlation.

Suppose $\mathcal{C} = (\mathcal{N}, \Omega, P)$ is a binary collective choice problem in which signals convey only distributional news. Without loss of generality, let $P(W_i|S_i = s^k) \geq P(W_i|S_i = s^{k+1})$ for $k = 1, ..., K-1$ (where it does not matter in the following how ties are broken). Let $\mathcal{G}' = \{k = 1, ..., K : P(W_i|S_i = s^K) \geq P(W_i)\}$ and $\mathcal{B}' = \mathcal{M} - \mathcal{G}'$. Since $P(B \geq 1) > 0$, both $\mathcal{G}'$ and $\mathcal{B}'$ are non-empty, and $P(W_i|S_i = s^1) > P(W_i)$. For a signal profile $s \in \mathcal{S}^n$, $G'(s)$ is the number of voters with a signal in $\mathcal{G}'$.

We first show that, for any voter $h \in \mathcal{N}$,

$$P(W_h) > P(W_h|S_h = s^0, G' = M = 1). \tag{3}$$

Let $w \in \{0, ..., n\}$ and E be any event such that $E \cap W^{-1}(w)$ is non-null. Then,

$$\begin{aligned}
\sum_{i=1}^{n} P(W_i|E, w) &= \sum_{i=1}^{n} \sum_{\omega \in \Omega(w) \cap E} P(W_i|E, w, \omega) P(\omega|E, w) \\
&= \sum_{\omega \in \Omega(w) \cap E} P(\omega|E, w) \sum_{i=1}^{n} P(W_i|E, w, \omega) \\
&= w \sum_{\omega \in \Omega(w) \cap E} P(\omega|E, w) = w
\end{aligned}$$

Therefore, for any voter $i \neq h$ and $w \in \{0, ..., n\}$ with $P(W = w) > 0$, Assumption 1 implies that

$$\begin{aligned}
\sum_{j=1}^{n} P(W_j|S_i \in \mathcal{G}', M = 1, w) &= P(W_i|S_i \in \mathcal{G}', M = 1, w) + \sum_{j \neq i} P(W_j|S_i \in \mathcal{G}', M = 1, w) \\
&= P(W_i|S_i \in \mathcal{G}', M = 1, w) + (n-1) P(W_h|S_i \in \mathcal{G}', M = 1, w) \\
&= \sum_{j=1}^{n} P(W_j|w) = P(W_i|w) + \sum_{j \neq i} P(W_j|w) \\
&= P(W_i|w) + (n-1) P(W_h|w).
\end{aligned}$$

Since $P(W_i|w) < P(W_i|S_i \in \mathcal{G}', M = 1, w)$, it follows that $P(W_h|w) > P(W_h|S_i \in \mathcal{G}', M = 1, w)$. Moreover, by Assumption 1,

$$\begin{aligned}
P(W_h|S_k = s^0, G' = M = 1, w) &= \sum_{j \neq h} P(W_h|S_j \in \mathcal{G}', M = 1, w) P(S_j \in \mathcal{G}'|G' = M = 1, w) \\
&= \frac{1}{n-1} \sum_{j \neq h} P(W_h|S_j \in \mathcal{G}', M = 1, w) \\
&= P(W_h|S_i \in \mathcal{G}', M = 1, w),
\end{aligned}$$

and, therefore, $P(W_h|w) < P(W_h|S_k = s^0, G' = M = 1, w)$. Since signals convey only distributional information,

$$\begin{aligned} P(W_h) &= \sum_{w=0}^{n} P(W_h|w)P(w) \\ &< \sum_{w=0}^{n} P(W_h|S_k = s^0, G' = M = 1, w)P(w) \\ &= \sum_{w=0}^{n} P(W_h|S_k = s^0, G' = M = 1, w)P(w|S_h = s^0, G' = M = 1) \\ &= P(W_h|S_h = s^0, G' = M = 1), \end{aligned}$$

and, therefore, $P(W_h) > P(W_h|S_k = s^0, G' = M = 1)$.

Now let $k^* = \min\{k = 1, ..., K : s^k \in \mathcal{B}'\}$ and let $P^* = \max\{P(W_i|S_i = s^{k^*}), P(W_h|S_k = s^0, G' = M = 1)\}$. From the preceding argument, $P(W_h) > P^*$ and so there exists $(v'_W, v'_L) >> 0$ such that

$$P(W_h)v'_W - (1 - P(W_h))v'_L > 0 > P^* v'_W - (1 - P^*)v'_L.$$

The binary collective choice problem $\mathcal{C}' = (\mathcal{N}, \Omega, P')$ uniquely defined by letting $P'(\mathcal{W} = \mathcal{N}', S = s) = P(\mathcal{W} = \mathcal{N}', S = s)$ for any $\mathcal{N}' \subseteq \mathcal{N}$ and $s \in \mathcal{S}$ is informationally equivalent to $\mathcal{C}$, and $\mathcal{C}'$ is adversely correlated because the set of good news signals for $\mathcal{C}'$ is exactly $\mathcal{G}'$. As a result, for any binary collective choice problem $\mathcal{C}$ in which signals convey only distributional information there exists an informationally equivalent collective choice problem with adverse correlation. ■

B Online Appendix

The Online Appendix contains all preliminary arguments, notation, and proofs for the extensions described in Section 5.

B.1 Symmetric Equilibria (Section 5.1)

Preliminaries: Let σ^α be the symmetric strategy-profile defined in the proof of Proposition 1. Recall that, for $g \in \{0, ..., \tau\}$ and $m \in \{g, ..., g+\tau\}$,

$$p_i(\sigma^\alpha | g, m) = \begin{cases} \mathbb{1}[g = \tau] & \text{if } \alpha = 0 \\ \mathbb{1}[m - g = \tau] & \text{if } \alpha = 1 \\ \binom{n-1-m}{\tau-g} \alpha^{\tau-g}(1-\alpha)^{\tau-(m-g)} & \text{if } \alpha \in (0,1) \end{cases},$$

and so

$$\Pi_0(\sigma^1, \lambda) = \sum_{g=0}^{\tau} \binom{n-1}{\tau+g} \lambda^{\tau+g}(1-\lambda)^{\tau-g} Z(g, \tau+g),$$
$$\Pi_0(\sigma^0, \lambda) = \sum_{m=\tau}^{n-1} \binom{n-1}{m} \lambda^m (1-\lambda)^{n-1-m} Z(\tau, m),$$

and, for $\alpha \in (0,1)$,

$$\Pi_0(\sigma^\alpha, \lambda) = \sum_{g=0}^{\tau} \sum_{m=g}^{\tau+g} \mathcal{M}(g,m) \lambda^m (1-\lambda)^{n-1-m} \alpha^{\tau-g}(1-\alpha)^{\tau+g-m} Z(g,m),$$
$$= \alpha^\tau (1-\alpha)^\tau (1-\lambda)^{n-1} \sum_{g=0}^{\tau} \left(\frac{\lambda(1-\alpha)}{\alpha(1-\lambda)}\right)^g \sum_{m=g}^{\tau+g} \left(\frac{\lambda}{(1-\alpha)(1-\lambda)}\right)^{m-g} \mathcal{M}(g,m) Z(g,m)$$

where $\mathcal{M}(g,m)$ is shorthand for the multinomial coefficient $\binom{n-1}{\tau-g, m, \tau+g-m}$.

Proof of Theorem 3

(1) Suppose $\mathcal{C}$ is strongly adversely correlated: $\sum_{\kappa=0}^{\tau} \theta^\kappa \binom{\tau}{\kappa} Z(\kappa,\kappa) < 0$ for some $\theta \in \mathbb{R}_{++}$, and fix some $\varepsilon \in (0,1)$.

By Case 1 in the proof of Theorem 1, if $Z(\tau,\tau) < 0$, then there exists $\bar{\lambda} \in (0, 1 - (1-\varepsilon)^{\frac{1}{n}})$ such that σ^0 is a symmetric equilibrium in which p_* wins with probability exceeding $1-\varepsilon$ for all $\lambda \in (0,\bar{\lambda})$. Therefore, we can focus on the case $Z(\tau,\tau) > 0$. In that case, since

$$\lim_{\lambda\to 0}\sum_{m=\tau}^{n-1}\binom{n-1}{m}\lambda^{m-\tau}(1-\lambda)^{n-1-m}Z(\tau,m) = \binom{n-1}{\tau}Z(\tau,\tau)$$

it follows that $\Pi_0(\sigma^0,\lambda) > 0$ for $\lambda > 0$ sufficiently small.

Let $\bar{\alpha} := 1-(1-\varepsilon)^{\frac{1}{2(\tau+1)}}$. If $\alpha < \bar{\alpha}$ and $\lambda < 1-(1-\varepsilon)^{\frac{1}{2n}}$, then p_* wins in strategy profile σ^α with probability exceeding

$$(1-\bar{\alpha})^{\tau+1}(1-\lambda)^n > (1-\varepsilon)^{\frac{\tau+1}{2(\tau+1)}}(1-\varepsilon)^{\frac{n}{2n}} = 1-\varepsilon$$

Since $\Pi_0(\sigma^0,\lambda) > 0$ for λ sufficiently small, it therefore sufficies to show that there exists $\bar{\lambda} \in (0,1)$ such that, for all $\lambda \in (0,\bar{\lambda})$, there is a $\alpha_\lambda \in (0,\bar{\alpha})$ such that $\Pi_0(\sigma^{\alpha_\lambda},\lambda) < 0$.

For any $\lambda \in (0,1)$, let $\alpha_\lambda := \frac{\lambda}{(1-\lambda)\theta+\lambda}$; hence, $\alpha_\lambda \in (0,1)$, is increasing in λ, and converges to 0 as $\lambda \to 0$. Therefore,

$$\begin{aligned}\lim_{\lambda\to 0}\sum_{g=0}^{\tau}\left(\frac{\lambda(1-\alpha_\lambda)}{\alpha_\lambda(1-\lambda)}\right)^g\sum_{m=g}^{\tau+g}\left(\frac{\lambda}{(1-\alpha_\lambda)(1-\lambda)}\right)^{m-g}\mathcal{M}(g,m)Z(g,m)\\ = \sum_{g=0}^{\tau}\theta^g\mathcal{M}(g,g)Z(g,g) = \binom{n-1}{\tau}\mathcal{K}(\theta) < 0.\end{aligned}$$

Hence, there is $\bar{\lambda} \in (0,1)$ such that $\Pi_0(\sigma^{\alpha_\lambda},\lambda) < 0$ for all $\lambda \in (0,\bar{\lambda})$.

(2) Suppose $\mathcal{C}$ is weakly advantageously correlated: $\sum_{\kappa=0}^{\tau}\theta^\kappa\binom{\tau}{\kappa}Z^G(\kappa) > 0$ for all $\theta \in \mathbb{R}_{++}$.

The correlation structure implies that $Z(\tau,\tau) > 0$, and so there exists $\bar{\theta}$ such that $\sum_{\kappa=0}^{\tau}\theta^\kappa\binom{\tau}{\kappa}Z(\kappa,\kappa) > Z(0,0)$ for all $\theta > \bar{\theta}$. Since $\lim_{\theta\to 0}\sum_{\kappa=0}^{\tau}\theta^\kappa\binom{\tau}{\kappa}Z(\kappa,\kappa) = Z(0,0)$, it follows that $\mathcal{K}$ attains a minimum on $[0,\bar{\theta}]$, which is strictly positive. As a result, there exists $\delta > 0$ such that $\sum_{\kappa=0}^{\tau}\theta^\kappa\binom{\tau}{\kappa}\left(Z^G(\kappa)-\delta\right) > 0$ for all $\theta \in \mathbb{R}_{++}$.

Now fix some $\varepsilon \in (0,1)$, and let $\bar{\alpha} := (1-\varepsilon)^{\frac{1}{2(\tau+1)}} \in (0,1)$. If $\alpha \in [\bar{\alpha},1]$ and $\lambda < 1-(1-\varepsilon)^{\frac{1}{2n}}$, then p^* wins with probability exceeding

$$\alpha^{\tau+1}(1-\lambda)^n > (1-\varepsilon)^{\frac{\tau+1}{2(\tau+1)}}(1-\varepsilon)^{\frac{n}{2n}} = 1-\varepsilon$$

in the strategy profile σ^α.

For $g \in \{0, ..., \tau\}$, let $\phi(g) := \mathcal{M}(g,g)Z(g,g)$ and, for $\alpha, \lambda \in (0,1)$, let

$$\phi(g, \alpha, \lambda) := \sum_{m=g+1}^{\tau+g} \left(\frac{\lambda}{(1-\alpha)(1-\lambda)} \right)^{m-g} \mathcal{M}(g,m)Z(g,m)$$

so that

$$\begin{aligned} \Pi_0(\sigma^\alpha, \lambda) &= \alpha^\tau (1-\alpha)^\tau (1-\lambda)^n \sum_{g=0}^{\tau} \left(\frac{\lambda(1-\alpha)}{\alpha(1-\lambda)} \right)^g \Big(\phi(g) + \phi(g, \alpha, \lambda) \Big) \\ &\geq \alpha^\tau (1-\alpha)^\tau (1-\lambda)^n \sum_{g=0}^{\tau} \left(\frac{\lambda(1-\alpha)}{\alpha(1-\lambda)} \right)^g \Big(\phi(g) - |\phi(g, \alpha, \lambda)| \Big) \end{aligned}$$

For $g \in \{0, ..., \tau\}$ and $\alpha \in (0, \bar{\alpha})$, $|\phi(g, \alpha, \lambda)| \leq |\phi(g, \bar{\alpha}, \lambda)|$, and so there exists $\bar{\lambda} \in (0,1)$ such that $|\phi(g, \bar{\alpha}, \lambda)| \leq \binom{n-1}{\tau}\delta$ for all $\lambda \in (0, \bar{\lambda})$ and $g \in \{0, ..., \tau\}$. Hence, for all $\lambda \in (0, \bar{\lambda})$,

$$\begin{aligned} \Pi_0(\sigma^\alpha, \lambda) &\geq \alpha^\tau (1-\alpha)^\tau (1-\lambda)^n \sum_{g=0}^{\tau} \left(\frac{\lambda(1-\alpha)}{\alpha(1-\lambda)} \right)^g \left(\phi(g) - \binom{n-1}{\tau} \delta \right) \\ &= \alpha^\tau (1-\alpha)^\tau (1-\lambda)^n \binom{n-1}{\tau} \sum_{\kappa=0}^{\tau} \left(\frac{\lambda(1-\alpha)}{\alpha(1-\lambda)} \right)^\kappa \binom{\tau}{\kappa} (Z(\kappa,\kappa) - \delta) > 0, \end{aligned}$$

since $\frac{\lambda(1-\alpha)}{\alpha(1-\lambda)} \in \mathbb{R}_{++}$. Therefore, for all $\lambda \in (0, \bar{\lambda})$, σ^α is not an equilibrium for any $\alpha \in (0, \bar{\alpha})$. Moreover, if $Z(\tau, \tau) > 0$, there exists $\bar{\lambda}_0 \in (0,1)$ such that $\Pi_0(\sigma^0, \lambda) > 0$ for all $\lambda \in (0, \bar{\lambda}_0)$.

Hence, for $\lambda_\varepsilon = \min\{1 - (1-\varepsilon)^{\frac{1}{2n}}, \bar{\lambda}, \bar{\lambda}_0\}$, the preceding arguments show that, for all $\lambda \in (0, \lambda_\varepsilon)$ and any $\alpha \in [0,1]$, either p^* wins with probability exceeding $(1-\varepsilon)$ in the strategy profile σ^α or the strategy profile σ^α is not an equilibrium. ■

B.2 Population Uncertainty (Section 5.2)

Preliminaries: We first describe how we adapt our main assumptions from Section 2 to the setting with population uncertainty. As previously, let $\mathcal{M} = \{s^1, ..., s^K\}$ be the set of informative signals for any population size. For $\omega \in \Omega^n$, $V(\omega)$ and $S(\omega)$ are the payoff and signal profiles in state ω, and $V_i^d(E, n)$ is the expected payoff difference between the ex-ante optimal and inferior policies for a voter i who conditions on the population size n and the event $E \subseteq \Omega^n$. We continue to define $V_i^d(E, n) := 0$ when E

is a null-event and assume that $V_i^d(E,n) \neq 0$ otherwise.

Assumption 5. *Voters are exchangeable for any population size $n \in \mathcal{Q}$: if $\omega, \omega' \in \Omega^n$ and ω permutes ω', then $P_n(\omega) = P_n(\omega')$.*

Let p_n^* be the optimal policy when voters learn only that the population size is n. By assumption 5, voters agree on p_n^*. We assume that p_n^* does not depend on n.

Assumption 6. *For all $n, n' \in \mathcal{N}$, $p_n^* = p_{n'}^*$.*

Assumption 7. *There is an uninformative signal, and other signals are sufficient:*

(a) **Uninformative signal:** *For $n \in \mathcal{Q}$, $\omega \in \Omega^n$ with $S_i(\omega) = s^0$,*

$$P_n(\omega) = P_n(V(\omega), S_{-i}(\omega))(1-\lambda).$$

for some $\lambda \in (0,1)$.

(b) **Informative signals:** *For $n \in \mathcal{Q}$ and $s_i \in \mathcal{M}$, $V_i^d(s_i, n) > 0$ if and only if $V_i^d(s', n') > 0$ for all $n' \in \mathcal{Q}$, $s' \in \mathcal{S}^{n'}$ such that $s'_i = s_i$.*

By Assumption 7, we can again classify informative signals as good or bad news.We let $\tau_0 := \tau(n_0)$ and $P_0 := P_{n_0}$.

Assumption 8. *$P_0(B \geq 1) > 0$ and $P_0(G \geq \tau_0) > 0$.*

We denote the mean population size by μ and the CDF of Q by F. As observed by Myerson (1998), being selected to participate in an election, leads a voter to update their beliefs about the size of the electorate. To perform this updating when $\mathcal{Q}$ may be countably infinite, we follow Myerson (1998) by first assuming $\bar{N} \in \mathcal{Q}$ players are pre-selected, each of whom is equally likely to be recruited as a voter. We then calculate voter i's beliefs about the size of the electorate, conditional on the event R_i that i is a voter, and take the limit as $\bar{N} \to \infty$. Hence,

$$\begin{aligned}
Q(N=n|R_i) &:= \lim_{\bar{N}\to\infty} Q(N=n|R_i, N \leq \bar{N}) \\
&= \lim_{\bar{N}\to\infty} \frac{Q(R_i|N=n, N\leq \bar{N})Q(N=n|N\leq\bar{N})}{\sum_{n'=n_0}^{\bar{N}} Q(R_i|N=n', N\leq\bar{N})Q(N=n|N\leq\bar{N})} \\
&= \lim_{\bar{N}\to\infty} \frac{\frac{n}{\bar{N}}\frac{\mathbb{1}[n\leq\bar{N}]Q(N=n)}{F(\bar{N})}}{\sum_{n'=n_0}^{\bar{N}} \frac{n'}{\bar{N}}\frac{Q(N=n')}{F(\bar{N})}} = \frac{nQ(N=n)}{\mu}
\end{aligned}$$

By Assumption 7(b), a voter who receives an informative signal has a unique undominated action. Given the population uncertainty, we focus on the set of symmetric undominated strategy profiles σ^α, where voters with good signals vote for p^*, voters with bad signals vote for p_*, and voters with the signal s^0 independently vote for p^* with probability α for some $\alpha \in [0,1]$. Adapting our previous notation, let $\Pi(\alpha, \lambda)$ be the expected payoff difference between a vote for p^* and vote for p_* for a voter who receives signal s^0 when $P(S_i \in \mathcal{M}) = \lambda$ and other voters follow the strategy-profile σ^α. A subscript n means "conditional on population size n," with a subscript 0 for the case when $n = n_0$. By Assumption 7(a), signal s^0 is not informative about the population size, payoff-profile or signal-profile of the other voters. Hence, for a voter i and $m \in \{0, ..., n-1\}$,

$$P_n(M = m | S_i = s^0, N = n) = \binom{n-1}{m} \lambda^m (1-\lambda)^{n-1-m}.$$

We formulate the following elementary property of absolutely convergent series for later reference.

Lemma 2. *Let $a : \mathbb{N}^2 \to \mathbb{R}$ such that, for all t, $\sum_{n=0}^{\infty} a(n,t)$ is absolutely convergent and, for all n, $a(n,t)$ converges monotonically to 0. Then, $\lim_{t\to 0} \sum_{n=0}^{\infty} a(n,t) = 0$.*

Proof. For $(n,t) \in \mathbb{N}^2$, let $a^+(n,t) = \mathbb{1}[a(n,t) \geq 0]a(n,t)$ and $a^-(n,t) = \mathbb{1}[a(n,t) < 0]|a(n,t)|$. Since $\sum_{n=0}^{\infty} a(n,t)$ is absolutely convergent for any t,

$$\sum_{n=0}^{\infty} a(n,t) = \sum_{n=0}^{\infty} a^+(n,t) - \sum_{n=0}^{\infty} a^-(n,t)$$

(where both series on the right-hand side converge, hence converge absolutely). We show that $\lim_{t\to 0} \sum_{n=0}^{\infty} a^+(n,t) = 0$, and analogous argument then applies for the series of negative terms.

Let $\varepsilon > 0$. First fix some some t^*. Since $\sum_{n=0}^{\infty} a^+(n,t^*)$ converges absolutely, there exists $\bar{n}$ such that $\sum_{n=\bar{n}+1}^{\infty} a^+(n,t^*) \leq \frac{\varepsilon}{2}$. Now fix $\bar{n}$, since $\lim_{t\to 0} a^+(n,t) = 0$ for all $n \in \{0, ..., \bar{n}\}$, there exists $\bar{t} \geq t^*$ such that $\sum_{n=0}^{\bar{n}} a^+(n,t) < \frac{\varepsilon}{2}$ for all $t \geq \bar{t}$. Moreover, since $a(n,t)$ is decreasing in t, $\sum_{n=\bar{n}+1}^{\infty} a^+(n,t) \leq \sum_{n=\bar{n}+1}^{\infty} a^+(n,t^*) = \frac{\varepsilon}{2}$ for all $t \geq t^*$. Hence, $\sum_{n=0}^{\infty} a^+(n,t) \leq \varepsilon$ for all $t \geq \bar{t}$. ∎

Proof of Theorem 4

For notational convenience, let $\mathcal{R}_n(g,m) := \mathcal{M}_n(g,m)Z_n(g,m)\frac{nQ(n)}{\mu}$, where $\mathcal{M}_n(g,m) := \binom{n-1}{m,\tau(n)-g,\tau(n)+g-m}$, and

$$Z_n(g,m) = P(G=g|M=m,S_i=s^0,N=n)V_i^d(S_i=s^0,G=g,M=m,N=n),$$

and let $v^* = \max\{|v^a - v^b| : (v^a,v^b) \in \mathcal{V}^a \times \mathcal{V}^b\}$.

Proof. First, suppose $\mathcal{K}_*(n_0) < 0$ and fix $\varepsilon \in (0,1)$. We consider two cases.

Case 1: Suppose $V_0^G(\tau_0) < 0$, which implies $\binom{n_0-1}{\tau_0}Z_0(\tau_0,\tau_0)\frac{n_0Q(n_0)}{\mu} < 0$ by Assumption 8. By Assumption 4, $\lim_{n\to\infty} nQ(n) = 0$, and so

$$\sum_{n=n_0+1}^{\infty}\sum_{m=\tau(n)}^{n-1}\binom{n-1}{m}\lambda^{m-\tau_0}(1-\lambda)^{n-1-m-\tau_0}|Z_n(\tau(n),m)|\frac{nQ(n)}{\mu} \le \frac{v^*}{\lambda^{\tau_0}(1-\lambda)^{\tau_0}},$$

hence, the series is absolutely convergent. Moreover, for $m \ge \tau(n)$, it follows that $\lambda^{m-\tau_0}(1-\lambda)^{n-1-m-\tau_0}$ is strictly increasing in $\lambda \in (0,1/2)$ and converges to 0 as $\lambda \to 0$. As a result, there exists $\bar{\lambda} \in (0,1)$ such that, for all $\lambda \in (0,\bar{\lambda})$,

$$\sum_{n=n_0+1}^{\infty}\sum_{m=\tau(n)}^{n-1}\mathcal{R}_n(\tau(n),m)\lambda^{m-\tau_0}(1-\lambda)^{n-1-m-\tau_0} \le \frac{1}{2}\binom{n_0-1}{\tau_0}|Z_0(\tau_0,\tau_0)|\frac{n_0Q(n_0)}{\mu}$$

Therefore, for all $\lambda \in (0,\bar{\lambda})$,

$$\begin{aligned}\Pi(0,\lambda) &= \sum_{n=n_0}^{\infty}\sum_{m=\tau(n)}^{n-1}\mathcal{R}_n(\tau(n),m)\lambda^m(1-\lambda)^{n-1-m} \\ &\le \frac{1}{2}\lambda^{\tau_0}(1-\lambda)^{\tau_0}\binom{n_0-1}{\tau_0}Z_0(\tau_0,\tau_0)\frac{n_0Q(n_0)}{\mu} < 0.\end{aligned}$$

Let λ'_ε be the unique solution to $\sum_{n=0}^{\infty}(1-\lambda)^nQ(n) = 1-\varepsilon$, and $\lambda_\varepsilon = \min\{\bar{\lambda},\lambda'_\varepsilon\}$. Then, for all $\lambda \in (0,\lambda_\varepsilon)$, σ^0 is an equilibrium in which p_* wins with probability exceeding $1-\varepsilon$.

Case 2: Suppose that $V_0^G(\tau_0) > 0$ but $\sum_{\kappa=0}^{\tau_0}\theta^\kappa\binom{\tau_0}{\kappa}Z_0(\kappa,\kappa) < 0$ for some $\theta \in \mathbb{R}_{++}$.

For any $\lambda \in (0,\frac{\theta}{1-\theta})$, let $\alpha_\lambda := \frac{\lambda}{\theta(1-\lambda)}$; then, $\alpha_\lambda \in (0,1)$, is strictly increasing in λ, and converges to 0 as $\lambda \to 0$.

Since $\lim_{n\to\infty} nQ(n) = 0$,

$$\sum_{n=n_0+1}^{\infty} \sum_{g=0}^{\tau(n)} \sum_{m=g}^{g+\tau(n)} \mathcal{R}_n(g,m)\lambda^m(1-\lambda)^{n-n_0-m}\alpha_\lambda^{\tau(n)-\tau_0-g}(1-\alpha_\lambda)^{\tau(n)-\tau_0+g-m} \le \frac{v^*}{\alpha_\lambda^{\tau_0}(1-\alpha_\lambda)^{\tau_0}}$$

and so the series on the left-hand side is absolutely convergent for any $\lambda \in (0, \frac{\theta}{1+\theta})$. Moreover,

$$\lambda^m(1-\lambda)^{n-n_0-m}\alpha_\lambda^{\tau(n)-\tau_0-g}(1-\alpha_\lambda)^{\tau(n)-\tau_0+g-m}$$
$$= \theta^{-n+n_0+m}(\lambda(1-\lambda)\theta - \lambda^2)^{\tau(n)-\tau_0}\left(\frac{\lambda}{\theta(1-\lambda)-\lambda}\right)^{m-g},$$

which is strictly increasing in $\lambda \in (0, \frac{\theta}{2(1+\theta)})$ and converges to 0 as $\lambda \to 0$. As a result, there exists $\bar{\lambda} \in (0, \frac{\theta}{2(1+\theta)})$ such that, for all $\lambda \in (0, \bar{\lambda})$,

$$\sum_{n=n_0+1}^{\infty} \sum_{g=0}^{\tau(n)} \sum_{m=g}^{g+\tau(n)} |\mathcal{R}_n(g,m)|\lambda^m(1-\lambda)^{n-n_0-m}\alpha_\lambda^{\tau(n)-\tau_0-g}(1-\alpha_\lambda)^{\tau(n)-\tau_0+g-m}$$
$$\le \frac{1}{4}\binom{n_0-1}{\tau_0}\frac{n_0Q(n_0)}{\mu}\sum_{\kappa=0}^{\tau_0}\theta^\kappa\binom{\tau_0}{\kappa}|Z_0(\kappa,\kappa)|$$

Moreover,

$$\sum_{g=0}^{\tau_0} \sum_{m=g+1}^{g+\tau_0} |\mathcal{R}_0(g,m)|\lambda^m(1-\lambda)^{-m}\alpha_\lambda^{-g}(1-\alpha_\lambda)^{g-m}$$
$$= \sum_{g=0}^{\tau_0} \sum_{m=g+1}^{g+\tau_0} |\mathcal{R}_0(g,m)|\theta^m\left(\frac{\lambda}{\theta(1-\lambda)-\lambda}\right)^{m-g},$$

which converges to 0 as $\lambda \to 0$. Therefore, there exists $\bar{\lambda}' \in (0, \bar{\lambda})$ such that

$$\sum_{g=0}^{\tau_0} \sum_{m=g+1}^{g+\tau_0} |\mathcal{R}_0(g,m)|\lambda^m(1-\lambda)^{-m}\alpha_\lambda^{-g}(1-\alpha_\lambda)^{g-m}$$
$$\le \frac{1}{4}\binom{n_0-1}{\tau_0}\frac{n_0Q(n_0)}{\mu}\sum_{\kappa=0}^{\tau_0}\theta^\kappa\binom{\tau_0}{\kappa}|Z_0(\kappa,\kappa)|$$

for all $\lambda \in (0, \bar{\lambda}')$. Therefore, for all $\lambda \in (0, \bar{\lambda}')$,

$$\begin{aligned}\Pi(\alpha_\lambda, \lambda) &= \sum_{n=n_0}^{\infty} \sum_{g=0}^{\tau(n)} \sum_{m=g}^{g+\tau(n)} |\mathcal{R}_n(g,m)| \lambda^m (1-\lambda)^{n-1-m} \alpha_\lambda^{\tau(n)-g} (1-\alpha_\lambda)^{\tau(n)+g-m} \\ &\leq \frac{1}{2} \alpha_\lambda^{\tau_0} (1-\alpha_\lambda)^{\tau_0} (1-\lambda)^{n_0-1} \binom{n_0-1}{\tau_0} \frac{n_0 Q(n_0)}{\mu} \sum_{\kappa=0}^{\tau_0} \theta^\kappa \binom{\tau_0}{\kappa} Z_0(\kappa,\kappa) < 0.\end{aligned}$$

Finally, analogous to the argument in Case 1, $V_0^G(\tau_0) > 0$ implies that there exists $\bar{\lambda}'' \in (0, \bar{\lambda}')$ such that $\Pi(0, \lambda) > 0$ for all $\lambda \in (0, \bar{\lambda}'')$. As a result, for any $\lambda \in (0, \bar{\lambda}'')$ there exists $\alpha'_\lambda \in (0, \alpha_\lambda)$ such that $\Pi(\alpha'_\lambda, \lambda) = 0$; hence an equilibrium.

Now let λ'_ε be the unique solution to $\sum_{n=n_0}^{\infty} \left(\frac{\theta(1-\lambda)-\lambda}{\theta}\right)^n Q(n) = 1-\varepsilon$ when $\varepsilon \leq \theta^{-1}$ and 1 otherwise, and let $\lambda_\varepsilon = \min\{\bar{\lambda}'', \lambda'_\varepsilon\}$. Then, for all $\lambda \in (0, \lambda_\varepsilon)$, $\sigma^{\alpha'_\lambda}$ is an equilibrium in which p_* wins with probability exceeding $1-\varepsilon$.

Now, suppose $\mathcal{K}_*(n_0) > 0$ and fix $\varepsilon \in (0,1)$. The advantageous correlation condition implies that $Z_0(\tau_0, \tau_0), Z_0(0,0) > 0$, and therefore there exists $\delta > 0$ such that $\sum_{\kappa=0}^{\tau_0} \theta^\kappa \binom{\tau_0}{\kappa} Z_0(\kappa,\kappa) > \delta$ for all $\theta \in \mathbb{R}_{++}$.

Let ν_ε be the unique solution in $(0,1)$ to $\sum_{n=n_0}^{\infty} \nu^n Q(n) = 1-\varepsilon$, and let $\bar{\lambda} = 1-\sqrt{\nu_\varepsilon}$ and $\bar{\alpha} = \sqrt{\nu_\varepsilon}$. Then, for any $\alpha \in (\bar{\alpha}, 1]$ and $\lambda \in (0, \bar{\lambda})$, p^* wins with probability exceeding

$$\sum_{n=n_0}^{\infty} \bar{\alpha}^n (1-\bar{\lambda})^n Q(n) = \sum_{n=n_0}^{\infty} \nu_\varepsilon^n Q(n) = 1-\varepsilon$$

in the strategy profile σ^α. It therefore suffices to show that there exists $\lambda_\varepsilon \in (0, \bar{\lambda})$ such that, for all $\lambda \in (0, \lambda_\varepsilon)$ and $\alpha \in [0, \bar{\alpha}]$, σ^α is not an equilibrium. We do this by first showing that there exists $\bar{\lambda}_0 \in (0,1)$ such that σ^0 is not an equilibrium for all $\lambda \in (0, \bar{\lambda}_0)$ (step 1), and then showing that there exists $\lambda^* \in (0,1)$ such that, for all $\lambda \in (0, \lambda^*)$, σ^α is not an equilibrium for any $\alpha \in (0, \bar{\alpha})$ (step 2).

Step 1: Since $\lim_{n\to\infty} nQ(n) = 0$,

$$\sum_{n=n_0+1}^{\infty} \sum_{m=\tau(n)}^{n-1} |\mathcal{R}_n(\tau(n), m)| \binom{n-1}{m} \lambda^{m-\tau_0} (1-\lambda)^{n-1-m-\tau_0} \leq \frac{v^*}{\lambda^{\tau_0}(1-\lambda)^{\tau_0}}$$

and so the series is absolutely convergent. Moreover, for $m \geq \tau(n) > \tau_0$, $\lambda^{m-\tau_0}(1-\lambda)^{n-1-m-\tau_0}$ is strictly increasing in $\lambda \in (0, 1/2)$ and converges to 0 as $\lambda \to 0$. As a result, there exists $\lambda_0 \in (0,1)$ such that

$$\sum_{n=n_0+1}^{\infty} \sum_{m=\tau(n)}^{n-1} |\mathcal{R}_n(\tau(n), m)| \lambda^{m-\tau_0}(1-\lambda)^{n-1-m-\tau_0} \leq \frac{1}{4}\binom{n_0-1}{\tau_0} Z_0(\tau_0, \tau_0)\frac{n_0 Q(n_0)}{\mu}$$

Moreover, since $\lambda^{m-\tau_0}(1-\lambda)^{\tau_0-m}$ is increasing in $\lambda \in (0,1)$ and converges to 0 as $\lambda \to 0$, there exists $\lambda_0' \in (0,1)$ such that

$$\sum_{m=\tau_0+1}^{n_0-1} \binom{n_0-1}{m} \lambda^{m-\tau_0}(1-\lambda)^{\tau_0-m}|Z_0(\tau_0, m)| \leq \frac{1}{4}\binom{n_0-1}{\tau_0} Z_0(\tau_0, \tau_0)$$

Let $\bar{\lambda}_0 = \min\{\lambda_0, \lambda_0'\}$; then for all $\lambda \in (0, \bar{\lambda}_0)$,

$$\begin{aligned}\Pi(0,\lambda) &= \sum_{n=n_0} \sum_{m=\tau(n)} \mathcal{R}_n(\tau(n), m)\binom{n-1}{m}\lambda^m(1-\lambda)^{n-1-m} \\ &\geq \frac{1}{2}\binom{n_0-1}{\tau_0}\lambda^{\tau_0}(1-\lambda)^{\tau_0} Z_0(\tau_0, \tau_0)\frac{n_0 Q(n_0)}{\mu} > 0,\end{aligned}$$

and so σ^0 is not an equilibrium.

Step 2: It remains to show that there exists $\lambda^* \in (0,1)$ such that, for all $\lambda \in (0, \lambda^*)$, σ^α is not an equilibrium for any $\alpha \in (0, \bar{\alpha})$. We show this by establishing a contradiction. Suppose that, for any $\lambda^* \in (0,1)$, there exists $\lambda \in (0, \lambda^*)$ and $\alpha_\lambda \in (0, \bar{\alpha})$ such that $\Pi(\alpha_\lambda, \lambda) = 0$; hence, there exists a sequence $(\lambda_t, \alpha_t)_{t=1}^\infty$ such that $\lambda_t \to 0$ and, for all $t \geq 1$, $\alpha_t \in (0, \bar{\alpha})$, and $\Pi(\alpha_t, \lambda_t) = 0$, where

$$\Pi(\alpha_t, \lambda_t) = \sum_{n=n_0}^{\infty} \sum_{g=0}^{\tau(n)} \sum_{m=g}^{g+\tau(n)} \mathcal{R}_n(g, m)\lambda_t^m(1-\lambda_t)^{n-1-m}\alpha_t^{\tau(n)-g}(1-\alpha_t)^{\tau(n)+g-m}. \tag{4}$$

We consider three collectively exhaustive cases: (i) there is a subsequence such that $\frac{\alpha_t(1-\lambda_t)}{\lambda_t} \to 0$, (ii) there is a subsequence such that $\alpha_t \to 0$ but $\frac{\alpha_t(1-\lambda_t)}{\lambda_t} \geq \gamma$ for some $\gamma > 0$, and (iii) there is a subsequence such that $\alpha_t \geq \gamma$ for some $\gamma > 0$.

Case (i). In this case, there is a subsequence such that $\lambda_t, \alpha_t, \frac{\alpha_t(1-\lambda_t)}{\lambda_t}, \lambda_t(1-\lambda_t)(1-\alpha_t), \frac{\lambda_t}{(1-\lambda_t)(1-\alpha_t)}$ are all decreasing, and converge to 0. From $\Pi(\alpha_t, \lambda_t) = 0$, it follows that (for all t, with the subscript suppressed for convenience),

$$-\binom{n_0-1}{\tau_0}\lambda^{\tau_0}(1-\lambda)^{\tau_0}(1-\alpha)^{\tau_0} Z_0(\tau_0, \tau_0)\frac{n_0 Q(n_0)}{\mu}$$

$$
\begin{aligned}
&= \sum_{g=0}^{\tau_0-1} \mathcal{R}_0(g,g)\lambda^g(1-\lambda)^{n_0-1-g}\alpha^{\tau_0-g}(1-\alpha)^{\tau_0} \\
&+ \sum_{g=0}^{\tau_0}\sum_{m=g+1}^{g+\tau_0} \mathcal{R}_0(g,m)\lambda^m(1-\lambda)^{n_0-1-m}\alpha^{\tau_0-g}(1-\alpha)^{\tau_0+g-m} \\
&+ \sum_{n=n_0+1}^{\infty} \mathcal{R}_n(\tau(n),\tau(n))\lambda^{\tau(n)}(1-\lambda)^{\tau(n)}(1-\alpha)^{\tau(n)} \\
&+ \sum_{n=n_0+1}^{\infty}\sum_{g=0}^{\tau(n)-1} \mathcal{R}_n(g,g)\lambda^g(1-\lambda)^{n-1-g}\alpha^{\tau(n)-g}(1-\alpha)^{\tau(n)} \\
&+ \sum_{n=n_0+1}^{\infty}\sum_{g=0}^{\tau(n)}\sum_{m=g+1}^{g+\tau(n)} \mathcal{R}_n(g,m)\lambda^m(1-\lambda)^{n-1-m}\alpha^{\tau(n)-g}(1-\alpha)^{\tau(n)+g-m}
\end{aligned}
$$

Therefore (dividing both sides by $[\lambda(1-\lambda)(1-\alpha)]^{\tau_0}$),

$$
\begin{aligned}
&-\binom{n_0-1}{\tau_0} Z_0(\tau_0,\tau_0)\frac{n_0 Q(n_0)}{\mu} \\
&\quad= \sum_{g=0}^{\tau_0-1} \mathcal{R}_0(g,g)\left(\frac{\alpha(1-\lambda)}{\lambda}\right)^{\tau_0-g} \\
&\quad+ \sum_{g=0}^{\tau_0}\sum_{m=g+1}^{g+\tau_0} \mathcal{R}_0(g,m)\left(\frac{\alpha(1-\lambda)}{\lambda}\right)^{\tau_0-g}\left(\frac{\lambda}{(1-\lambda)(1-\alpha)}\right)^{m-g} \\
&\quad+ \sum_{n=n_0+1}^{\infty} \mathcal{R}_n(\tau(n),\tau(n))[\lambda(1-\alpha)(1-\lambda)]^{\tau(n)-\tau_0} \\
&\quad+ \sum_{n=n_0+1}^{\infty}\sum_{g=0}^{\tau(n)-1} \mathcal{R}_n(g,g)\left(\frac{\alpha(1-\lambda)}{\lambda}\right)^{\tau(n)-g}[\lambda(1-\alpha)(1-\lambda)]^{\tau(n)-\tau_0} \\
&\quad+ \sum_{n=n_0+1}^{\infty}\sum_{g=0}^{\tau(n)}\sum_{m=g+1}^{g+\tau(n)} \mathcal{R}_n(g,m)\left(\frac{\alpha(1-\lambda)}{\lambda}\right)^{\tau(n)-g}\left(\frac{\lambda}{(1-\lambda)(1-\alpha)}\right)^{m-g}[\lambda(1-\alpha)(1-\lambda)]^{\tau(n)-\tau_0}.
\end{aligned}
$$

By Lemma 2, the left-hand side converges to 0 but the right-hand side is constant and bounded away from 0.

Case (ii). In this case, there is a subsequence such that $\lambda_t, \alpha_t, \frac{\alpha_t}{1-\alpha_t}, \alpha_t(1-\lambda_t)^2(1-\alpha_t)$ are all decreasing and converge to 0. From $\Pi(\alpha_t,\lambda_t) = 0$ it follows that (t subscript suppressed)

$$
\begin{aligned}
&-\sum_{g=0}^{\tau_0}\binom{n_0-1}{g}\binom{n_0-1-g}{\tau_0-g}\lambda^g(1-\lambda)^{n_0-1-g}\alpha^{\tau_0-g}(1-\alpha)^{\tau_0}Z_0(g,g)\frac{n_0Q(n_0)}{\mu}\\
=&\sum_{g=0}^{\tau_0}\sum_{m=g+1}^{\tau_0}\mathcal{R}_0(g,m)\lambda^m(1-\lambda)^{n_0-1-m}\alpha^{\tau_0-g}(1-\alpha)^{\tau_0+g-m}\\
&+\sum_{n=n_0+1}^{\infty}\sum_{g=0}^{\tau(n)}\mathcal{R}_n(g,g)\lambda^g(1-\lambda)^{n-1-g}\alpha^{\tau(n)-g}(1-\alpha)^{\tau(n)}\\
&+\sum_{n=n_0+1}^{\infty}\sum_{g=0}^{\tau(n)}\sum_{m=g+1}^{g+\tau(n)}\mathcal{R}_n(g,m)\lambda^m(1-\lambda)^{n-1-m}\alpha^{\tau(n)-g}(1-\alpha)^{\tau(n)+g-m}
\end{aligned}
$$

Therefore (dividing both sides by $\alpha^{\tau_0}(1-\alpha)^{\tau_0}(1-\lambda)^{n_0-1}$),

$$
\begin{aligned}
&-\binom{n_0-1}{\tau_0}\frac{n_0Q(n_0)}{\mu}\sum_{g=0}^{\tau_0}\binom{\tau_0}{g}\left(\frac{\lambda}{\alpha(1-\lambda)}\right)^g Z_0(g,g)\\
=&\sum_{g=0}^{\tau_0}\sum_{m=g+1}^{\tau_0}\mathcal{R}_0(g,m)\left(\frac{\lambda}{\alpha(1-\lambda)}\right)^m\left(\frac{\alpha}{1-\alpha}\right)^{m-g}\\
&+\sum_{n=n_0+1}^{\infty}\sum_{g=0}^{\tau(n)}\mathcal{R}_n(g,g)\left(\frac{\lambda}{\alpha(1-\lambda)}\right)^g[\alpha(1-\alpha)(1-\lambda)^2]^{\tau(n)-\tau_0}\\
&+\sum_{n=n_0+1}^{\infty}\sum_{g=0}^{\tau(n)}\sum_{m=g+1}^{g+\tau(n)}\mathcal{R}_n(g,m)\left(\frac{\lambda}{\alpha(1-\lambda)}\right)^m\left(\frac{\alpha}{1-\alpha}\right)^{m-g}[\alpha(1-\alpha)(1-\lambda)^2]^{\tau(n)-\tau_0}\\
:=&\tilde{\Pi}(\alpha,\lambda)
\end{aligned}
$$

If there exists a further subsequence such that $\frac{\lambda_t}{\alpha_t(1-\lambda_t)}$ is decreasing, then the left-hand side converges to 0 by Lemma 2 while the right-hand side is constant. Otherwise, there exists a subsequence such that $\frac{\lambda_t}{\alpha_t(1-\lambda_t)}$ converges up to some $\theta^*>0$. For each t in that subsequence, let $\alpha_t^*=\frac{\lambda_t}{\theta^*(1-\lambda_t)}$. Eventually, $\alpha_t^*\in(0,1)$, and so

$$
\begin{aligned}
&\sum_{g=0}^{\tau_0}\sum_{m=g+1}^{\tau_0}\mathcal{R}_0(g,m)\,(\theta^*)^m\left(\frac{\alpha^*}{1-\alpha^*}\right)^{m-g}\\
&+\sum_{n=n_0+1}^{\infty}\sum_{g=0}^{\tau(n)}\mathcal{R}_n(g,g)\,(\theta^*)^g\,[\alpha^*(1-\alpha^*)(1-\lambda)^2]^{\tau(n)-\tau_0}
\end{aligned}
$$

$$+\sum_{n=n_0+1}^{\infty}\sum_{g=0}^{\tau(n)}\sum_{m=g+1}^{g+\tau(n)}\mathcal{R}_n(g,m)\,(\theta^*)^m\left(\frac{\alpha^*}{1-\alpha^*}\right)^{m-g}[\alpha^*(1-\alpha^*)(1-\lambda)^2]^{\tau(n)-\tau_0}$$

is absolutely convergent. Since, for each t there exists $t'\geq t$ such that $\frac{\alpha_{t'}}{1-\alpha_{t'}}\leq\frac{\alpha_t}{1-\alpha_t}$ and $\alpha_{t'}(1-\alpha_{t'})(1-\lambda_{t'})^2\leq\alpha_t^*(1-\alpha_t^*)(1-\lambda_t)^2$, it follows that

$$\begin{aligned}&\sum_{g=0}^{\tau_0}\sum_{m=g+1}^{\tau_0}\mathcal{R}_0(g,m)\,(\theta^*)^m\left(\frac{\alpha}{1-\alpha}\right)^{m-g}\\&+\sum_{n=n_0+1}^{\infty}\sum_{g=0}^{\tau(n)}\mathcal{R}_n(g,g)\,(\theta^*)^g\,[\alpha(1-\alpha)(1-\lambda)^2]^{\tau(n)-\tau_0}\\&+\sum_{n=n_0+1}^{\infty}\sum_{g=0}^{\tau(n)}\sum_{m=g+1}^{g+\tau(n)}\mathcal{R}_n(g,m)\,(\theta^*)^m\left(\frac{\alpha}{1-\alpha}\right)^{m-g}[\alpha(1-\alpha)(1-\lambda)^2]^{\tau(n)-\tau_0}\end{aligned}$$

is eventually absolutely convergent, and then converges 0 by Lemma 2.

Case (iii). In this case, there is a subsequence such that λ_t is decreasing and, since $\alpha\in(0,\bar{\alpha})$, there exists some $\gamma\in(0,1/2)$ such that $\alpha_t\in[\gamma,(1-\gamma)]$ for all t. From $\Pi(\alpha_t,\lambda_t)=0$ it follows that (t subscript suppressed)

$$\begin{aligned}&-\sum_{n=n_0}^{\infty}\binom{n-1}{\tau(n)}(1-\lambda)^{n-1}\alpha^{\tau(n)}(1-\alpha)^{\tau(n)}Z_n(0,0)\frac{nQ(n)}{\mu}\\&=\sum_{n=n_0}^{\infty}\sum_{m=1}^{\tau(n)}\mathcal{R}_n(0,m)\lambda^m(1-\lambda)^{n-1-m}\alpha^{\tau(n)}(1-\alpha)^{\tau(n)-m}\\&+\sum_{n=n_0}^{\infty}\sum_{g=1}^{\tau(n)}\sum_{m=g}^{g+\tau(n)}\mathcal{R}_n(g,m)\lambda^m(1-\lambda)^{n-1-m}\alpha^{\tau(n)-g}(1-\alpha)^{\tau(n)+g-m}\end{aligned}$$

Since $\alpha\in[\delta,1-\delta]$ it follow that $\alpha^{\tau(n)}(1-\alpha)^{\tau(n)}\geq\gamma^{\tau(n)}$, and so the left-hand side is greater $\sum_{n=n_0}^{\infty}\binom{n-1}{\tau(n)}(1-\lambda)^{n-1}\delta^{\tau(n)}Z_n(0,0)\frac{nQ(n)}{\mu}$, which converges to

$$\sum_{n=n_0}^{\infty}\binom{n-1}{\tau(n)}\delta^{\tau(n)}Z_n(0,0)Q(n)>0,$$

while the right-hand side converges to 0 by Lemma 2. ■

B.3 The Role of Elites (Section 5.3)

Preliminaries: For any state ω, we denote by $G_E(\omega)$ the number of elites who receive good news, $M_E(\omega)$ the number of elites who receive informative signals, $G_N(\omega) = G(\omega) - G_E(\omega)$ and $M_N(\omega) = M(\omega) - M_E(\omega)$, with typical realizations of these random variables denoted, respectively, by g_E, m_E, g_N, and m_N.

For $g_E \in \{0, ..., |\mathcal{E}|\}$, $g_N \in \{0, ..., |\mathcal{NE}|\}$, $m_E \in \{g_e, ..., |\mathcal{E}|\}$, and $m_N \in \{g_e, ..., |\mathcal{NE}|\}$,

$$Z_i(g_E, g_N, m_E, m_N) := P(g_E, g_N | S_i = s^0, m_E, m_N) V_i(S_i = s^0, g_E, g_N, m_E, m_N).$$

Proof of Proposition 6

Fix $\varepsilon \in (0,1)$ and let $\sigma^* \in \Sigma^*$ with $\sigma_i^*(s^0) = \mathbb{1}[i \in \mathcal{E}]$. Since $|\mathcal{E}| \leq \tau$, p_* wins for the strategy profile σ^* in the event $\{S = s_0\}$, and therefore wins with probability exceeding $1 - \varepsilon$ for all $\lambda \in (0, 1 - (1-\varepsilon)^{\frac{1}{n}})$. Hence, it is sufficient to show that σ^* is a strict equilibrium for λ sufficiently small.

If $i \in \mathcal{E}$ receives signal s^0, then

$$\Pi_i(\sigma^*, \lambda) = \lambda^{\tau - |\mathcal{E}|+1} \sum_{g_E=0}^{|\mathcal{E}|-1} \sum_{m_E=g_E}^{|\mathcal{E}|-1} \sum_{m_N=\hat{g}(m_E,m_N)}^{|\mathcal{NE}|} \binom{|\mathcal{E}|-1}{m_E} \binom{|\mathcal{NE}|}{m_N} \lambda^{m_E+m_N-\tau+|\mathcal{E}|-1} (1-\lambda)^{n-1-m_E-m_N} Z_i(g_E, \hat{g}(m_E, m_N), m_E, m_N),$$

where $\hat{g}(m_E, m_N) = \tau - (|\mathcal{E}| - 1 - (m_e - g_e))$. Since

$$\lim_{\lambda \to 0} \Pi_i(\sigma^*, \lambda) \lambda^{-\tau+|\mathcal{E}|-1} = \binom{|\mathcal{NE}|}{\tau - |\mathcal{E}|+1} Z_i(0, \tau - |\mathcal{E}|+1, 0, \tau - |\mathcal{E}|+1),$$

which is strictly positive by elite-adverse correlation, there exists $\bar{\lambda}_E \in (0,1)$ such that $\Pi_i(\sigma^*, \lambda) > 0$ for all elites who receive the signal s^0 for all $\lambda \in (0, \bar{\lambda}_E)$.

If $i \in \mathcal{NE}$ receives signal s^0, then

$$\Pi_i(\sigma^*, \lambda) = \lambda^{\tau - |\mathcal{E}|} \sum_{g_E=0}^{|\mathcal{E}|} \sum_{m_E=g_E}^{|\mathcal{E}|} \sum_{m_N=\hat{g}(m_E,m_N)-1}^{|\mathcal{NE}|-1} \binom{|\mathcal{E}|}{m_E} \binom{|\mathcal{NE}|-1}{m_N} \lambda^{m_E+m_N-\tau+|\mathcal{E}|} (1-\lambda)^{n-1-m_E-m_N} Z_i(g_E, \hat{g}(m_E, m_N)-1, m_E, m_N).$$

Since

$$\lim_{\lambda\to 0} \Pi_i(\sigma^*,\lambda)\lambda^{-\tau+|\mathcal{E}|} = \binom{|\mathcal{NE}|}{\tau-|\mathcal{E}|} Z_i(0,\tau-|\mathcal{E}|,0,\tau-|\mathcal{E}|),$$

which is strictly negative by elite-adverse correlation, there exists $\bar{\lambda}_{NE} \in (0,1)$ such that $\Pi_i(\sigma^*,\lambda) < 0$ for all non-elites who receive the signal s^0 for all $\lambda \in (0,\bar{\lambda}_{NE})$.

As a result, σ^* is an equilibrium for all $\lambda \in (0,\min\{\bar{\lambda}_E,\bar{\lambda}_{NE}\})$. ■

Proof of Proposition 7

For some (P_W, v_W, v_L, e), let $i \in \mathcal{NE}$ and $w \in \{\tau+1,...,n\}$. Then,

$$\begin{aligned} &P(G_N = \tau - e | S_i = s^0, M = M_N = \tau - e, W = w) \\ &\quad = \left(\frac{w-e}{n-e}\right) \frac{\binom{w-e-1}{\tau-e}\binom{n-1-e-(w-e-1)}{0}}{\binom{n-1-e}{\tau-e}} + \left(\frac{n-w}{n-e}\right) \frac{\binom{w-e}{\tau-e}\binom{n-1-e-(w-e)}{0}}{\binom{n-e-1}{\tau-e}} = \frac{\binom{w-e}{\tau-e}}{\binom{n-e}{\tau-e}} \\ &P(G_N = \tau - e | S_i = s^0, M = M_N = \tau - e, W = \tau) = \frac{\binom{\tau-e}{\tau-e}}{\binom{n-e}{\tau-e}} \end{aligned}$$

Therefore, for any $w \in \{\tau,...,n\}$,

$$P(W = w | E_0(e)) = \frac{\binom{w-e}{\tau-e} P_W(w)}{\sum_{w'=\tau}^{n} \binom{w'-e}{\tau-e} P_W(w')}.$$

where $E_0(e) = \{S_i = s^0, G = M = M_N = \tau - e\}$ for $i \in \mathcal{NE}$. Since

$$P(W_i | E_0(e), W = w) = \frac{w - e - (\tau - e)}{n - e - (\tau - e)} = \frac{w-\tau}{n-\tau},$$

it follows that

$$V_i(E_0(e)) = \sum_{w=\tau}^{n} \left(\left(\frac{w-\tau}{n-\tau}\right) v_W - \left(\frac{n-w}{n-\tau}\right) v_L \right) \frac{\binom{w-e}{\tau-e} P_W(w)}{\sum_{w'=\tau}^{n} \binom{w'-e}{\tau-e} P_W(w')}.$$

For $0 \le e < e' \le \tau$ and $\tau \le w < w' \le n$,

$$\binom{w'-e'}{\tau-e'}\binom{w-e}{\tau-e} < \binom{w-e'}{\tau-e'}\binom{w'-e}{\tau-e}$$

and so $P_W(.|E_0(e)) \succcurlyeq_{LR} P_W(.|E_0(e'))$. Since $P(W_i|E_0(e), W = w)$ is strictly increasing in w for $i \in \mathcal{NE}$, it follows that $V_i(E_0(e)) \geq V_i(E_0(e'))$. ■