MATH 503. Functional Analysis

Spring 2017. McAllister 106, MWF 11:15 –12:05.

Instructor: Alexei Novikov. Office hours: by appointment.

Textbook: A.Bressan, Lecture Notes on Functional Analysis, with Applications to Linear Partial Differential Equations. American Mathematical Society Graduate Studies in Mathematics, Vol. 143, Providence RI, 2013.

The course covers the topics required for the Analysis C qualifying exam.

Prerequisites: Math 501 (Real Analysis), and Math 535 (Linear Algebra).

Grade: The final grade will be determined based on:

60 % homeworks (collected weekly in class, every Friday),

10 % midterm quiz (given in class),

30 % final exam.

The lowest score among the homework assignments will be dropped.

Topics covered in the course:

1. Normed spaces. Banach spaces. Linear operators. Examples.

2. Spaces of bounded linear operators. The uniform boundedness principle and the open mapping theorem.

3. Bounded linear functionals. Dual spaces. The Hahn-Banach extension theorem. Separation of convex sets.

4. Spaces of continuous functions. Ascoli's theorem, Stone-Weierstrass' theorem. Spaces of Holder continuous functions and of k-times differentiable functions.

5. Hilbert spaces. Perpendicular projections. Orthonormal bases. Self-adjoint operators.

6. Compact operators on a Hilbert space. Fredholm's alternative. Spectrum and eigenfunctions of a compact, self-adjoint operator. Applications to Sturm-Liouville boundary value problems.

7. Weak derivatives. Sobolev spaces. Embedding theorems. Applications to linear elliptic equations.

8. Linear semigroups. Generators, resolvents. Applications to linear parabolic equations.