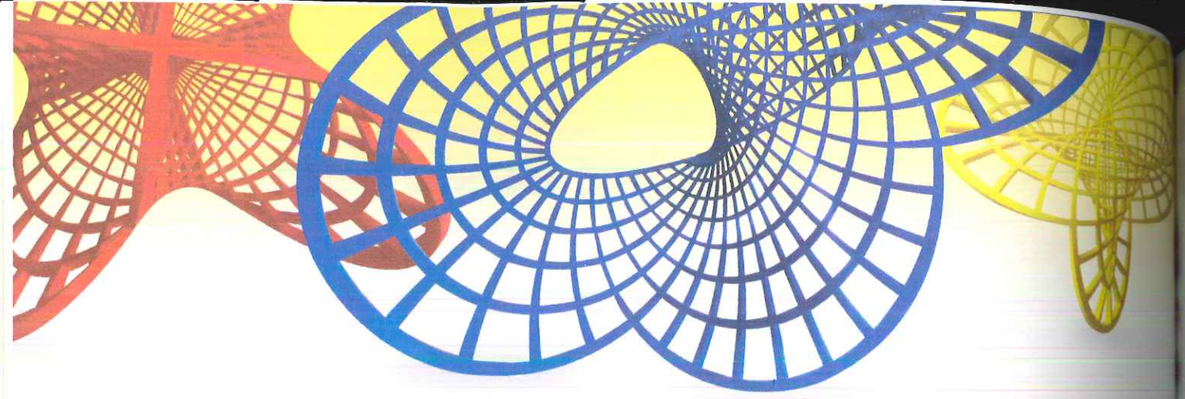
Developing Basic Fact Fluency



Developing Basic Fact Fluency

LEARNER OUTCOMES

After reading this chapter and engaging in the embedded activities and reflections, you should be able to:

1. Describe the phases for developing fluency of the basic facts.
2. Contrast different approaches to teaching basic facts, including effective teaching and assessment practices.
3. Illustrate strategies for helping students derive addition, subtraction, multiplication, and division facts.
4. Describe strategies for reinforcing and remediating basic fact mastery.

Basic facts for addition and multiplication are the number combinations where both addends or both factors are less than 10. Basic facts for subtraction and division are the corresponding combinations. Thus, $15 - 8 = 7$ is a subtraction fact because the corresponding addition parts are less than 10. The goal with basic facts is to develop **fluency.** The Common Core State Standards (CCSSO, 2010) describe this as students being able to (1) flexibly, (2) accurately, (3) efficiently, and (4) appropriately solve problems. Teachers plan their instruction for learning basic facts around "big ideas" related to developing fluency.

BIG IDEAS

- Students move through three phases in developing fluency with basic facts: counting, reasoning strategies, and mastery (Baroody, 2006). Instruction and assessment must help students through these phases without rushing them.
- Number relationships provide the foundation for strategies that help students remember basic facts. For example, knowing how numbers are related to 5 and 10 helps students master facts such as $3 + 5$ (think of a ten-frame) and $8 + 6$ (because 8 is 2 away from 10, take 2 from 6 to make $10 + 4 = 14$).
- When students are not fluent with the basic facts, they often need to drop back to earlier phases; more drill is not the answer.

Developmental Phases for Learning the Basic Facts

Developing fluency of addition and subtraction concepts begins in kindergarten and continues through grade 2, when students must know their addition facts from memory (i.e., have mastered the facts). Similarly, fluency with multiplication begins in grade 2, with knowing the facts from memory being a goal by the end of grade 3 (CCSSO, 2010).

Developing fluency with basic facts is a developmental process. Flash cards and timed tests are *not* the best way to develop fluency. Instead, focus on number sense (the four components of fluency). Research indicates that early number sense predicts school success more than other measures of cognition, like verbal, spatial, or memory skills or reading ability (Jordan, Kaplan, Locuniak, & Ramineni, 2007; Locuniak & Jordan, 2008; Mazzocco & Thompson, 2005).

Students progress from counting to eventually "just knowing" that $2 + 7$ is 9 or that 5×4 is 20. This developmental progression takes time and many experiences. Arthur Baroody, a mathematics educator who does research on basic facts, describes three phases of learning facts (2006, p. 22):

> *Phase 1: Counting strategies*: Using object counting (e.g., blocks or fingers) or verbal counting to determine the answer. (Example: $4 + 7 =$ ___. Student starts with 7 and counts on verbally 8, 9, 10, 11.)
>
> *Phase 2: Reasoning strategies*: Using known information to logically determine an unknown combination. (Example: $4 + 7$. Student knows that $3 + 7$ is 10, so $4 + 7$ is one more, 11.)
>
> *Phase 3: Mastery:* Producing answers efficiently (quickly and accurately). (Example: $4 + 7$. Student quickly responds, "It's 11; I just know it.")

Figure 1 outlines the developmental methods for solving basic addition and subtraction problems.

	Addition	Subtraction
Counting	Direct modeling (counting objects and fingers) • Counting all • Counting on from first • Counting on from larger Counting abstractly • Counting all • Counting on from first • Counting on from larger	Counting objects • Separating from • Separating to • Adding on Counting fingers • Counting down • Counting up Counting abstractly • Counting down • Counting up
Reasoning	Properties • $\alpha + 0 = \alpha$ • $\alpha + 1 =$ next whole number • Commutative property Known-fact derivations (e.g., $5 + 6 = 5 + 5 + 1$; $7 + 6 = 7 + 7 - 1$) Redistributed derived facts (e.g., $7 + 5 = 7 + (3 + 2) = (7 + 3) + 2 = 10 + 2 = 12$)	Properties • $\alpha - 0 = \alpha$ • $\alpha - 1 =$ previous whole number Inverse/complement of known addition facts (e.g., $12 - 5$ is known because $5 + 7 = 12$) Redistributed derived facts (e.g., $12 - 5 = (7 + 5) - 5 = 7 + (5 - 5) = 7$)
Retrieval	Retrieval from long-term memory	Retrieval from long-term memory

FIGURE 1 The developmental process for basic fact mastery for addition and subtraction.

Source: Henry, V. J., & Brown, R. S. (2008). "First-Grade Basic Facts: An Investigation into Teaching and Learning of an Accelerated, High-Demand Memorization Standard." *Journal for Research in Mathematics Education, 39*(2), p. 156. Reprinted with permission.

FORMATIVE ASSESSMENT Notes. When are students ready to work on reasoning strategies? When they are able to (1) use counting-on strategies (start with the largest and count up) and (2) see that numbers can be decomposed (e.g., that 6 is 5 + 1). Interview students by posing one-digit addition problems and ask how they solved it. For example, 3 + 8 (Do they count on from the larger?) and 5 + 6 (Do they see 5 + 5 + 1?). For multiplication, 3 × 8 (Do they know this is 3 eights? Do they see it as 2 eights and one more eight?). ■

Complete Self-Check 1: Developmental Phases for Learning the Basic Facts

Teaching and Assessing the Basic Facts

This section describes the different ways basic fact instruction has been implemented in schools, followed by a section describing effective strategies.

Different Approaches to Teaching the Basic Facts

Over the last century, three main approaches have been used to teach the basic facts. The pros and cons of each approach are briefly discussed in this section.

Memorization. This approach moves from presenting concepts of addition and multiplication straight to memorization of facts, not devoting time to developing strategies (Baroody, Bajwa, & Eiland, 2009). This approach requires students to memorize 100 separate addition facts (just for the addition combinations 0–9) and 100 multiplication facts (0–9). Students may even have to memorize subtraction and division separately—bringing the total to over 300 isolated facts! There is strong evidence that this method simply does not work. You may be tempted to respond that you learned your facts in this manner; however, as long ago as 1935 studies concluded that students develop a variety of strategies for learning basic facts in spite of the amount of isolated drill that they experience (Brownell & Chazal, 1935).

A memorization approach does not help students develop strategies that could help them master their facts. Baroody (2006) points out three limitations:

- *Inefficiency.* There are too many facts to memorize.
- *Inappropriate applications.* Students misapply the facts and don't check their work.
- *Inflexibility.* Students don't learn flexible strategies for finding the sums (or products) and therefore continue to count by ones.

Notice that a memorization approach works against the development of *fluency* (which includes being able to flexibly, accurately, efficiently, and appropriately solve problems). According to CCSS-M, students should have fluency with addition and subtraction facts (0–9) by the end of second grade and fluency with multiplication and division facts by the end of third grade (CCSSO, 2010). Compare grade-level standards at www.corestandards.org/math.

When taught basic facts via memorization, many struggling learners and students with disabilities continue to use counting strategies because they have not had explicit instruction on reasoning strategies (Mazzocco et al., 2008). In addition, drill can cause unnecessary anxiety and undermine student interest and confidence in mathematics.

Explicit Strategy Instruction. For more than three decades, explicit strategy instruction has been used in many classrooms. Students learn a strategy (e.g., combinations of 10). Students then explore and practice the strategies (e.g., using a ten-frame to see which facts equal 10). Research supports the use of explicit strategy instruction as effective in helping all students learn (and remember) their basic facts (e.g., Baroody, 1985; Bley & Thornton, 1995; Fuson, 1984, 1992; Rathmell, 1978; Thornton & Toohey, 1984).

Explicit strategy instruction is intended to *support* student thinking rather than give the students something new to remember. It is not effective to just memorize (for the same reason

that memorizing isolated facts doesn't work). A heavy focus on memorizing strategies results in students with lower number sense (Henry & Brown, 2008). The key is to help students see the possible strategies and then *choose* one that helps them solve the problem without counting. For example, seeing that 6 × 8 can be partitioned into (5 × 8) + (1 × 8). This chapter focuses heavily on strategies for learning the basic facts.

Guided Invention. Guided invention also focuses on strategies, but in a more open-ended manner. It is focused on having students select a strategy based on their knowledge of number relationships (Gravemeijer & van Galen, 2003).

A teacher might post the fact 6 + 7. One student may think of 6 + 7 as "double 6 is 12 and one more is 13." Another student sees it as 7 + 3 (to make 10) and then 3 more. Another student may take 5 from each addend to make 10 and then add the remaining 1 and 2. The key is that each student is using number combinations and relationships that make sense to them.

In *guided invention* the teacher may not explain a strategy, but carefully set up tasks where students notice number relationships. For example, in the 6 + 7 task, the teacher might ask students to place counters in two ten-frames and then think of different ways they can visually move the counters to combine the frames.

CCSS Standards for Mathematical Practice
MP7. Look for and make use of structure.

Teaching Basic Facts Effectively

Plan experiences that help students move through the three phases. In discussing student strategies, focus student attention on the methods that move students from phase 1 to phase 2. For example, ask students how they solved 7 + 4. Some will have used counting on (phase 1). Others will use the Making 10 strategy (7 + 3 is 10 and 1 more equals 11). Help the students who are counting on to see the connections to Making 10. To move from phase 2 to recall, continue to provide engaging and diverse experiences where students are using and talking about their strategies. Students will become quicker and eventually will "just know" more and more facts.

Use Story Problems. Research has found that when a strong emphasis is placed on solving problems, students not only become better problem solvers but also master more basic facts than students in a drill program (National Research Council, 2001). Story problems provide context that can help students understand the situation and apply flexible strategies for doing computation.

Some teachers are hesitant to use story problems with English language learners (ELLs) or students with disabilities because of the additional language or reading required, but because language supports understanding, story problems are important for all students. For addition, to work on the Making 10 strategy, you might use this story:

CCSS Standards for Mathematical Practice
MP1. Make sense of problems and persevere in solving them.

Rachel had 9 toy ponies in one barn and 6 ponies in another barn. How many ponies did she have altogether?

Pause & Reflect

How does this problem increase the likelihood that students will develop the Making 10 strategy?

The numbers and situation in this story lend to thinking of 9 + 6 as equivalent to 10 + 5 (one pony could be moved to the other barn).

Multiplication stories can focus on array situations. Arrays help students see how to decompose a fact (splitting the rows) and see the commutative property (e.g., 3 × 7 = 7 × 3). For example, consider that a class is working on the 7 facts. The teacher points to the calendar (an array) and poses the following question:

In 3 weeks we will be going to the zoo. How many days until we go to the zoo?

Standards for Mathematical Practice
MP3. Construct viable arguments and critique the reasoning of others.

Suppose that Aidan explains how she figured out 3 × 7 by starting with double 7 (14) and then adding 7 more. Ask students to explain what Aidan did and why it works.

Explore if doubling could be used for other 7s facts. Ask, "How might doubling help us figure out how many days in 4 weeks (4 × 7)? Give students time to work in small groups on this question. Students can discover how to think of a single fact as a combination of facts (e.g., that 4 × 7 is 2 × (2 × 7) or 7 doubled and doubled again), applying important properties of multiplication. Posing strategy-focused questions followed by a brief discussion of the strategies that students used can improve student accuracy and efficiency with basic facts (Rathmell, Leutzinger, & Gabriele, 2000).

Explicitly Teach Reasoning Strategies. In addition to using story situations, explicitly teach reasoning strategies. A lesson may examine a collection of facts for which a particular strategy is appropriate. For example, students must know their Combinations of 10 addition facts before they are ready to learn the facts that result in numbers greater than 10 (so that they can use this strategy). You can give partners a ten-frame and a deck of cards numbered 0 to One student draws a card and places that many counters on the ten-frame. Without counting, their partner tells how many more to fill the frame (i.e., equal 10). Discuss how to figure out combinations that equal 10. Use other games and activities to be sure all students know their Combinations of 10.

The "big idea" behind teaching reasoning strategies is for students to make use of *known facts* and relationships to *derive unknown facts*. Students might use one of their Combinations of 10 strategies, like 7 + 3, to solve an unknown fact, like 7 + 5, noticing that 7 + 5 = 7 + 3 + 2. Keep this "big idea" in mind as you review each of the reasoning strategies described in this section. Watch how Connor, Myrna, and Miguel use known facts to solve 6 + ___ = 13.

Don't expect to have a strategy introduced and understood with just one lesson or activity. Students need lots of opportunities to make a strategy their own. Start with the most basic strategies and build from there. Plan many activities and make games and interactive activities part of daily work at school and home.

It is a good idea to display reasoning strategies for students to reference. Give the strategies names that make sense so that students know when to apply them (e.g., "Strategy for × 3s: Double and add one more set. Ex: 3 × 7 = (2 × 7) + 7 = 14 + 7 = 21").

Assessing Basic Facts Effectively

Why do we use assessment strategies? To figure out what students know and what they do not know so that we can design instruction. Why, then, is assessment of basic facts often limited to timed tests? We must do better if we are going to ensure that all students learn their basic facts.

What Is Wrong with Timed Tests? First, timed tests do not assess the four elements of fluency. You gain no insights into which strategies students are using, nor if they are flexible in using those strategies. You have a little insight into how efficient they are, but you don't really know much here either, because they might have used very inefficient strategies for some facts while going quickly through others. So, at best, you get a sense of which facts they are getting correct (accuracy). Second, timed tests negatively affect students' number sense and recall of facts (Boaler, 2012, 2014; Henry & Brown, 2008; Ramirez, Gunderson, Levine, & Beilock, 2013). Third, timed tests are not needed for students to master their facts (Kling, 2011), so they take up time that could be used in meaningful and more palatable learning experiences.

How Might I Assess Basic Fact Fluency? Think about each of the aspects of fluency and ask yourself, "How can I determine if each of my students is able to do that for this set of facts?" Table 1 offers a few ideas for each component of fluency (based on Kling & Bay-Williams, 2014).

Standards for Mathematical Practice
MP2. Reason abstractly and quantitatively.

As you assess, remember there is no one "best" strategy for any fact. For example, 7 + 8 could be solved using Making 10 or Near-Doubles. The more you emphasize choice, the more

TABLE 1 EFFECTIVE STRATEGIES FOR ASSESSING BASIC FACT FLUENCY

Aspects of Fluency	Observation	Interview Probes	Writing (Journals or Tests)
Appropriate strategy selection	As they play a game, are they picking a strategy that makes sense for that fact? For example, for 9 + 2 they might count on, but not for 9 + 6.	Nicolas solved 6 + 8 by changing it in his mind to 4 + 10. What did he do? Is this a good strategy? Tell why or why not.	Review the multiplication table. Write which facts are your "toughies." Next to each one, tell a strategy that you want to remember to use.
Flexibility	As for strategy selection, do they pick Making 10 for 9 + 6? Do they notice that 8 × 3 is also 3 × 8?	Solve 6 × 7 using one strategy. Now try solving it using a different strategy.	Explain how you think about these two problems: 13 − 3 = 12 − 9 =
Efficiency	How long does it take to select a strategy? Are they quick to use doubles? Does efficiency vary with certain facts, like facts over 10 (add) or the 7s facts (multiply)?	Go through this stack of cards and sort by the ones you *just know* and the ones you *use a strategy*	Solve these [basic fact] problems (provide a set of 10). If you *just knew* the answer, circle it. If you *used a strategy,* write the strategy's name (e.g., Close Fact).
Accuracy	Which facts are they consistently getting correct?	What is the answer to 7 × 8? How do you know it is correct (how might you check it)?	Review your [3s facts] with your partner. Make a stack of the ones when you were correct and not correct. Record which facts you have "down pat" and which you are still learning.

students will be able to find strategies that work for them, and that will lead to their own fact fluency.

Activity 1 is good for assessing students' flexibility and ability to select an appropriate strategy for a fact.

Activity 1

CCSS-M: 1.OA.C.6; 2.OA.B.2; 3.OA.B.5; 3.OA.C.7

If You Didn't Know

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Pose the following task: If you did not know the answer to 8 + 5 (or any fact that you want students to think about), how could you figure it out without counting? Encourage students to come up with more than one way (hopefully using the strategies suggested previously). ELLs and reluctant learners benefit from first sharing their ideas with a partner and then with the class.

The more students are engaged in activities and games, the more chance you have to use observations and interviews to monitor which *strategies* students know and don't know and which *facts* they know and don't know (games are discussed in the Reinforcing Basic Fact Mastery section). Then, you can adapt the games and instruction to address their needs.

Complete Self-Check 2: Teaching and Assessing the Basic Facts

Reasoning Strategies for Addition Facts

Basic fact mastery depends on progressing through three phases. The second phase, reasoning strategies, warrants significant attention; too often students are asked to go from counting (phase 1) to memorization (phase 3). Therefore a significant part of this chapter is devoted to what reasoning strategies are important to teach and how to teach them well.

Reasoning strategies for addition facts are directly related to one or more number relationships. In kindergarten and first grade, significant time should be devoted to decomposing and composing numbers and exploring number combinations (e.g., combinations that equal 5 and combinations that equal 10). In grade 2, students continue to develop reasoning strategies until they know their addition facts from memory. It takes many experiences over many months for students to move from using strategies to just knowing their facts. Note: no memorization needed—just many activities like the one shared here! The first four strategies listed are foundational to the later strategies. Watch John A. Van de Walle discuss reasoning strategies for addition.

One More Than and Two More Than

Each of the 36 facts highlighted in the following chart has at least one addend of 1 or 2. Being able to count on, then, is a necessary prerequisite to being able to apply this strategy (Baroody et al., 2009).

+	0	1	2	3	4	5	6	7	8	9
0		1	2							
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3		4	5							
4		5	6							
5		6	7							
6		7	8							
7		8	9							
8		9	10							
9		10	11							

Story problems in which one of the addends is a 1 or a 2 are easy to make up. For example, *Seven children were waiting for the slide. Then 2 more children got in line. How many children were waiting for the slide?* Ask different students to explain how they got the answer of 9. Some will count on from 7. Some may still need to count 7 and 2 and then count all. Others will say they knew that 2 more than 7 is 9. Helping students see the connection between counting on and adding two will help students move from counting strategies to reasoning strategies.

Activity 2

CCSS-M: 1.OA.A.1; 1.OA.C.6; 2.OA.B.2

How Many Feet in the Bed?

Read *How Many Feet in the Bed?* by Diane Johnston Hamm. On the second time through the book, ask students how many more feet are in the bed when a new person gets in. Ask students to record the equation (e.g., 6 + 2) and tell how many. Two less can be considered as family members get out of the bed. Find opportunities to make the connection between counting on two and adding two using a number line or ten-frame. For ELLs, be sure that they know what the phrases "two more" and "two less" mean (and clarify the meaning of *foot*, which is also used for measuring). Acting out with students in the classroom can be a great illustration for both ELLs and students with disabilities.

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The different responses will provide you with a lot of information about students' number sense. As students are ready to use the two-more-than idea without "counting all," they can begin to practice with activities such as the following.

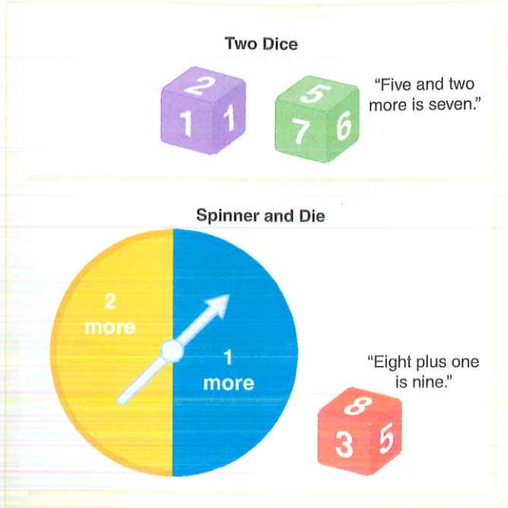

FIGURE 2 One-more and two-more activities.

Figure 2 illustrates the ideas in Activity 3. Notice that activities such as these can be modified for almost all of the strategies in the chapter.

Activity 3

CCSS-M: 1.OA.C.5; 1.OA.C.6; 2.OA.B.2

One More Than and Two More Than with Dice and Spinners

Make a die labeled +1, +2, +1, +2, "one more," and "two more." Use with another die labeled 3, 4, 5, 6, 7, and 8 (or whatever values students need to practice). After each roll of the dice, students should say the complete fact: "Four and two more is six." Alternatively, roll one die and use a spinner with +1 on one half and +2 on the other half. For students with disabilities, you may want to start with a die that just has +1 on every side and then another day move on to a +2 die. This will help emphasize and practice one approach. Similarly, in Expanded Lesson Two More Than/Two Less Than, students use dot cards to connect the idea of more and less to adding and subtracting.

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Adding Zero

Nineteen addition facts have zero as one of the addends. Though adding 0 is generally easy, some students overgeneralize the idea that answers to addition problems are bigger than the addends. They also may have a harder time when the 0 comes first (e.g., 0 + 8). Use story problems involving zero and use drawings that show two parts with one part empty.

+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1									
2	2									
3	3									
4	4									
5	5									
6	6									
7	7									
8	8									
9	9									

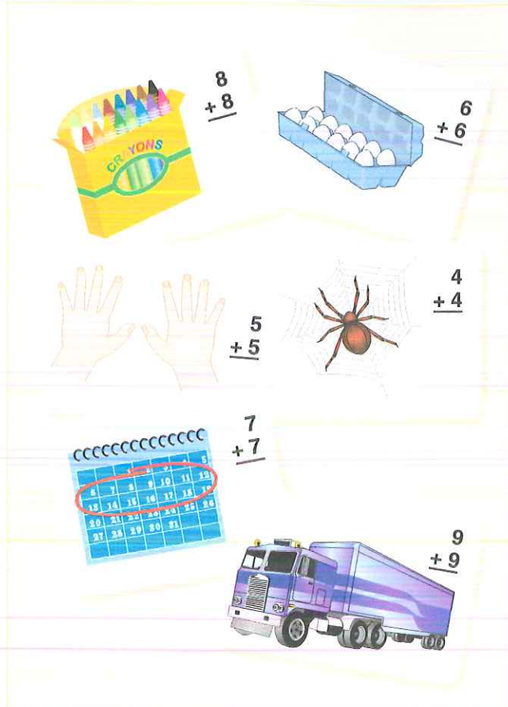

FIGURE 3 Situations for doubles facts.

Asking students to generalize from a set of problems is a good way to reinforce reasoning and avoid overgeneralization. You can write about 10 zero facts on the board, some with the zero first and some with the zero second. Discuss how the equations are alike. Ask students to create their own stories and/or to illustrate the problems.

Doubles

There are ten doubles facts from 0 + 0 to 9 + 9, as shown here. Students often know doubles, perhaps because of their rhythmic nature. These facts can be anchors for other facts.

+	0	1	2	3	4	5	6	7	8	9
0	0									
1		2								
2			4							
3				6						
4					8					
5						10				
6							12			
7								14		
8									16	
9										18

Students with disabilities or difficulties with memorizing can benefit from picture cards for each of the doubles, as shown in Figure 3. Story problems can focus on pairs of like addends: "Alex and Zack each found 7 seashells at the beach. How many did they find together?"

Activity 4

CCSS-M: 1.OA.C.5; 1.OA.C.6; 2.OA.B.2

Double Magic

A double machine is a fun concept for students and a good connection to algebraic thinking. Read *Two of Everything* (Hong, 1993), a Chinese folktale in which a couple (the Haktaks) find a magic pot that doubles everything that goes into it. Use a plastic cauldron (easily purchased around Halloween) or any container. Click here to access a recording page for students. Make a set of cards with an "input number" on the front side and the double of the number on the reverse. The card is flipped front to back as it comes out of the pot. You can do this as a whole class, having students write the double on a personal whiteboard, or have students work in partners, with one student flipping the card and the other stating the fact.

Activity 5

CCSS-M: 1.OA.C.6; 2.OA.B.2

Calculator Doubles

Students work in pairs with a calculator. Students enter the "double maker" (2×) into the calculator. One student says a double—for example, "Seven plus seven." The other student presses 7, says what the double is, and then press = to see the correct double (14) on the display. The students then switch roles and reset the calculator (2×). For ELLs who are just learning English, invite them to say the double in their native language or in both their native language and English. (Note that the calculator is also a good way to practice + 1 and + 2 facts.)

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Combinations of 10

Perhaps the most important strategy for students to know is the combinations that equal 10. It is a foundational fact from which students can derive many facts (Kling, 2011). Consider story situations such as the following and ask students to tell possible answers.

There are ten boys and girls on the bus. How many girls and how many boys might be on the bus?

The ten-frame is a very useful tool for creating a visual image for students.

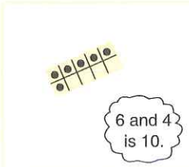


FIGURE 4 Combinations of 10 on a ten-frame.

Activity 6

CCSS-M: 1.OA.B.4; 1.OA.C.6; 2.OA.B.2

How Much to Equal 10?

Place counters on one Ten-Frame (see Figure 4) and ask, "How many more to equal 10?" This activity can be repeated using different start numbers. Eventually, display a blank ten-frame and say a number less than 10. Students start with that number and complete the "10 fact." If you say, "four," they say, "four plus six equals ten." This can be completed as whole class or with students working with a partner. Students who are still in phase 1 of learning the facts (using counting strategies) or students with disabilities may need additional experience or one-on-one time working on this process.

Making 10

All of the basic facts with sums greater between 11 and 20 can be solved by using the Making 10 strategy. Students use their known facts that equal 10 and then add the rest of the number onto 10. For example, to solve 6 + 8, a student might start with the larger number (8), see that 8 is 2 away from 10; therefore, they take 2 from the 6 to make 10 and then add on the remaining 4 to get 14. Making 10 is also aptly called Break Apart to Make Ten (BAMT) (Sarama & Clements, 2009) and Up over 10 (the CCSS-M uses the phrase Making 10). Listen to student explanations as they use the Making 10 strategy.

+	0	1	2	3	4	5	6	7	8	9
0										
1										
2										11
3									11	12
4								11	12	13
5							11	12	13	14
6						11	12	13	14	15
7					11	12	13	14	15	16
8				11	12	13	14	15	16	17
9			11	12	13	14	15	16	17	18

This reasoning strategy is extremely important and is heavily emphasized in high-performing countries (Korea, China, Taiwan, and Japan) where students learn facts sooner and more accurately than U.S. students (Henry & Brown, 2008). Yet this strategy is not emphasized enough in the United States. A study of California first graders found that this strategy contributed more to developing fluency than using doubles (even though using doubles had been emphasized by teachers and textbooks in the study) (Henry & Brown, 2008).

The Making 10 strategy can also be applied to larger numbers. For example, for 28 + 7, students can make 30, seeing that 28 + 7 = 30 + 5. Thus, this reasoning strategy deserves significant attention in teaching addition (and subtraction) facts.

Double Ten-Frame (see Blackline Master 15) can help students visualize the Making 10 strategy. For example, cover two ten-frames with a problem, like 6 + 8. Ask students to visualize moving counters from one frame (e.g., the one with 6 in it) to fill the other ten-frames (e.g., the one with 8). Ask, "How many moved?" "How many remain in the unfilled frame?" After students have found a total, have students share and record the equations. Activities 7 and 8 are designed for this purpose.

Activity 7

CCSS-M: 1.OA.B.3; 1.OA.C.6; 2.OA.B.2

Move It, Move It

This activity is designed to help students see how to go Making 10. Click here to access the Activity Page for Move It, Move It and a mat with Double Ten-Frame (Blackline Master 15). Flash cards are placed next to the ten-frames, or a fact can be given orally. The students cover each frame with counters to represent the problem (9 + 6 would mean covering nine places on one frame and six on the other). Ask students to "move it"—to decide a way to move the counters so that they can find the total without counting. Get students to explain what they did and connect to the new equation. For example, 9 + 6 may have become 10 + 5 by moving one counter to the first ten-frame. Emphasize strategies that are working for that student (5 as an anchor and/or Combinations of 10 and/or Making 10).

Activity 8

CCSS-M: 1.OA.B.3; 1.OA.C.6; 2.OA.B.2

Frames and Facts

STUDENTS *with* SPECIAL NEEDS

Make Little Ten-Frame Cards and display them to the class on a projector. Show an 8 (or 9) card. Place other cards beneath it one at a time as students respond with the total. Have students say aloud what they are doing. For 8 + 4, they might say, "Take 2 from the 4 and put it with 8 to make 10. Then 10 and 2 left over is 12." Move to harder cards, like 7 + 6. The activity can be done independently with the little ten-frame cards. Ask students to record each equation, as shown in Figure 5. Especially for students with disabilities, highlight how they should explicitly think about filling in the little ten-frame starting with the higher number. Show and talk about how it is more challenging to start with the lower number as a counterexample.

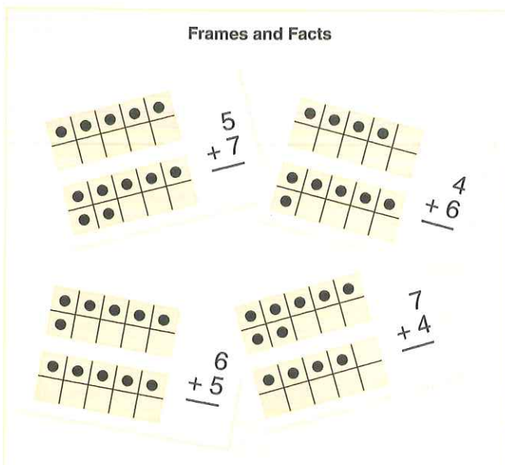


FIGURE 5 Frames and facts activity.

Using 5 as an Anchor

The use of an anchor (5 or 10) is a reasoning strategy that builds on students' knowledge of number relationships and is therefore a great way to both reinforce number sense and learn the basic facts. Using 5 as an anchor means looking for fives in the numbers in the problem. For example, in 7 + 6, a student may see that 7 is 5 + 2 and that 6 is 5 + 1. The student would add 5 + 5 and then the extra 2 from the 7 and the extra 1 from the 6, adding up to 13.

Ten-frames can help students see numbers as 5 and some more. And because the ten-frame is a visual model, it may be a strategy that visual learners and students with disabilities find particularly valuable.

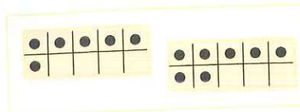
Near-Doubles

Near-doubles are also called "doubles-plus-one" or "doubles-minus-one" facts and include all combinations where one addend is one more or less than the other. This is a strategy that uses a known fact to derive an unknown fact. The strategy

Activity 9

CCSS-M: 1.OA.B.3; 1.OA.C.5; 1.OA.C.6; 2.OA.B.2

Flash



Project a Double Ten-Frame (Blackline Master 15) on the board. Without letting students see, place counters on each—for example, six on one and seven on the other—so that the top row is full (five counters) and the extras are in the next row of each ten-frame. Flash (uncover) for about three to five seconds and recover. Ask students how many counters they saw. Then uncover and have students explain how they saw it. You can also have students use the Little Ten-Frames to do this activity with partners.

is to double the smaller number and add 1 or to double the larger and then subtract 1. Therefore, students must know their doubles before they can work on this strategy.

+	0	1	2	3	4	5	6	7	8	9
0		1								
1	1		3							
2		3		5						
3			5		7					
4				7		9				
5					9		11			
6						11		13		
7							13		15	
8								15		17
9									17	

To introduce near-doubles, write a doubles fact and a near-doubles fact right under it, as illustrated here.

5 + 5
5 + 6

Ask students how the first equation might help them solve the second equation. Activity 10 elaborates on this idea.

Activity 10

CCSS-M: 1.OA.B.3; 1.OA.C.6; 2.OA.B.2

On the Double!

Create a display (on the board or on paper) that illustrates the doubles (see Figure 6). Prepare cards with near-doubles (e.g., 4 + 5). Ask students to find the doubles fact that could help them solve the fact they have on the card and place it on that spot. Ask students if there are other doubles that could help.

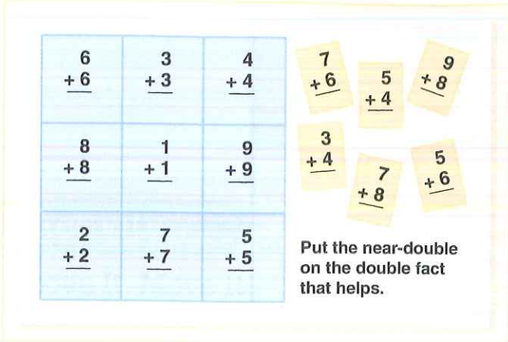


FIGURE 6 Near-doubles fact activity.

Near-doubles can be more difficult for students to recognize and therefore may not be a strategy that all students find useful. In that case, do not force it.

Complete Self-Check 3a: Reasoning Strategies for Addition Facts

Reasoning Strategies for Subtraction Facts

Subtraction facts prove to be more difficult than addition. This is especially true when students have been taught subtraction through a "count-count-count" approach; for 13 − 5, *count* 13, *count* off 5, *count* what's left. As discussed earlier in the chapter, counting is the first phase in reaching basic fact mastery.

Figure 1 at the beginning of the chapter lists the ways students might subtract, moving from counting to mastery. Without opportunities to learn and use reasoning strategies, students continue to rely on counting strategies to come up with subtraction facts, a slow and often inaccurate approach. Therefore, spend sufficient time working on the reasoning strategies outlined here to help students move to phase 2 and eventually on to mastery (phase 3).

Think-Addition

As the label implies, in this strategy students use known addition facts to produce the unknown quantity or part of the subtraction (see Figure 7). If this important relationship between parts and the whole—between addition and subtraction—can be made, subtraction facts will be much easier for students to learn. As with addition facts, it is helpful to begin with facts that have totals of 10 or less (e.g., 8 − 3, 9 − 7) before working on facts that have a total (minuend) higher than 10 (e.g., 13 − 4).

The value of think-addition cannot be overstated; however, if think-addition is to be used effectively, it is essential that addition facts be mastered first. Evidence suggests that students learn very few, if any, subtraction facts without first mastering the corresponding addition facts. In other words, mastery of 3 + 5 can be thought of as prerequisite knowledge for learning the facts 8 − 3 and 8 − 5.

Story problems that promote think-addition are those that sound like addition but have a missing addend: *join, initial part unknown; join, change unknown;* and *part-part-whole, part unknown*. Consider this problem:

Janice had 5 fish in her aquarium. Grandma gave her some more fish. Then she had 12 fish. How many fish did Grandma give Janice?

Notice that the action is *join*, which suggests addition. There is a high probability that students will think, "Five and how many more equals 12?" In the discussion in which you use problems such as this, your task is to connect this thought process with the subtraction fact, 12 − 5. Students may use an Making 10 strategy to solve this, just as they did with addition facts ("It takes 5 to get to 10 and 2 more to 12 is . . . 7").

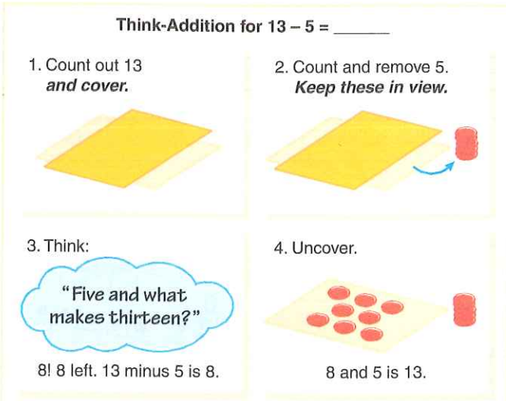


FIGURE 7 Using a think-addition for subtraction facts.

Pause & Reflect

Before reading further, look at the three subtraction facts shown here, and reflect on what thought process you use to get the answers. Even if you "just know them," think about what a likely process might be.

14	12	15
−9	−6	−6

What stories might you tell that will help students "think addition?"

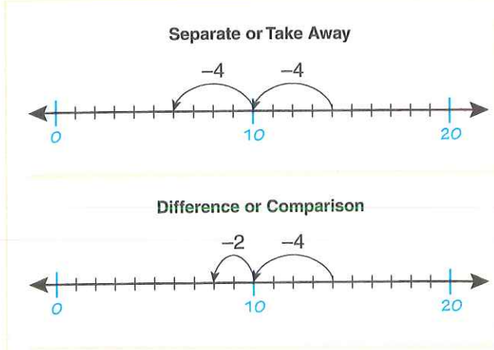


FIGURE 8 Down Under 10 illustrated on the number line for 14 – 8.

Down Under 10

There are two ways to think about Down Under 10. The first is to think about it as a "separate" situation. For 14 − 8, the thinking is to first take away 4 to get to 10, then take away 4 more to get the answer of 6. Another way to think about this is as a "comparison," finding the difference" or distance between the two numbers. How far apart are 14 and 8? Jump down 4 to the 10 and two more to the 8—they are 6 apart. These two interpretations are illustrated in Figure 8.

Down Under 10 is a derived fact strategy: Students use what they know (that 14 minus 4 is 10) to derive a related fact (14 − 5). Like the Combinations of 10 and Making 10 strategies discussed previously, this strategy is emphasized in high-performing countries but not emphasized enough in the United States (Fuson & Kwon, 1992).

To develop the Down Under 10 strategy, write pairs of facts in which the difference for the first fact is 10 and the second fact is either 8 or 9: 16 − 6 and 16 − 7; 14 − 4 and 14 − 6, and so on. Have students solve each problem and discuss their strategies. If students do not naturally see the relationship, ask them to think about how the first fact can help solve the second. Illustrate on a number line. Use story problems such as these:

Becky walks 16 blocks to school. She has walked 7. How many more blocks does Becky have left? (Separate)

Becky walks 16 blocks to school; Corwin walks 9 blocks. How many more blocks does Becky walk? (Comparison)

Activity 11

CCSS-M: 1.OA.B.4; 1.OA.C.6; 2.OA.B.2

Apples in the Trees

Project a Double Ten-Frame (Blackline Master 15) as a display with chips covering the first ten-frame and some of the second (e.g., for 16, cover 10 in the first frame and 6 on the second frame). Tell students some apples have fallen to the ground—you will tell them how many and they will tell you how many are still in the trees. Repeat the activity, asking students to explain their thinking. For ELLs or culturally diverse students, you can change to a context that is familiar or timely.

ENGLISH LANGUAGE LEARNERS

Take from 10

This excellent strategy is not as well known or commonly used in the United States but is consistently used in high-performing countries. It takes advantage of students' knowledge of

the combinations that make 10, taking the initial value apart into 10 and ones. This is how it works for 15 − 8:

(1) Think: 10 + 5
− 8

(2) Take from the 10: 10 − 8 = 2

(3) Add 5 back on: 2 + 5 = 7

Try it on these examples:

15 − 8 = 17 − 9 = 14 − 6 =

If you have students from other countries, they may know this strategy and can share it with others. It can be used for all subtraction facts having minuends greater than 10 (the "toughies") by just knowing how to subtract from 10 and knowing addition facts with sums less than 10.

Activity 12

CCSS-M: 1.OA.B.3; 1.OA.B.4; 1.OA.C.6; 2.OA.B.2

Apples in Two Trees

Adapting Activity 11, explain that each ten-frame is a different tree. Tell students you will tell them how many apples fall out of the "full" tree and they will tell you how many apples are left (on both trees). Each time, ask students to explain their thinking.

In the discussion of addition and subtraction strategies, you have seen a lot of activities. Activities and games provide a low-stress approach to practicing strategies and working toward fluency. More games and activities for all operations can be found later in this chapter.

Complete Self-Check 3b: Reasoning Strategies for Subtraction Facts

Reasoning Strategies for Multiplication and Division Facts

Using a problem-based approach and focusing on reasoning strategies are just as important, if not more so, for developing mastery of the multiplication and related division facts (Baroody, 2006; Wallace & Gurganus, 2005). As with addition and subtraction facts, start with story problems as you develop reasoning strategies.

Understanding the commutative property cuts the basic facts to be memorized in half! For example, a 2 × 8 array can be described as 2 rows of 8 or 8 rows of 2. In both cases, the answer is 16.

Foundational Facts: 2, 5, 0, 1

For some reason, multiplication is often approached in numerical order (1s, 2s, 3s, . . . up through 9s). (Note: in some settings, multiplications facts go through 10 or even 12, but in the CCSS, basic facts are limited to single-digit factors, which is how they are addressed in this section). A more effective approach is to start with the facts that build on students' strengths and prior knowledge. A good place to start? 2s and 5s! These facts connect to students'

experiences with skip counting and addition doubles (Heege, 1985; Kamii & Anderson, 2003; Watanabe, 2006). This work can begin at the end of second grade or at the beginning of third grade. Next, 0s and 1s are foundational facts. Be sure these are understood, not just memorized. The sections here share strategies for helping students learn the foundation facts: 2, 5, 0, and 1. Watch John A. Van de Walle discuss reasoning strategies for multiplication.

Twos. Facts that have 2 as a factor are equivalent to the addition doubles and should already be known by students. Help students realize that 2 × 7 is the same as 7 + 7. Use story problems in which 2 is the number of sets. Later, use problems in which 2 is the size of the sets, helping students recognize the commutative property of multiplication.

Standards for Mathematical Practice
MP7. Look for and make use of structure.

George was making sock puppets. Each puppet needed 2 buttons for eyes. If George makes 7 puppets, how many buttons will he need for the eyes?

×	0	1	2	3	4	5	6	7	8	9
0			0							
1			2							
2	0	2	4	6	8	10	12	14	16	18
3			6							
4			8							
5			10							
6			12							
7			14							
8			16							
9			18							

Fives. Practice skip counting by fives. Keep track of how many fives have been counted (If we jump by 5s four times on the number line, where will we land?). Use arrays that have rows with 5 dots. Point out that such an array with six rows is a model for 6 × 5, eight rows is 8 × 5, and so on. Time is also a good context for fives because of the way analog clocks are made.

Activity 13

CCSS-M: 3.OA.A.1; 3.OA.C.7

Clock Facts

Focus on the minute hand of the clock. When it points to a number, how many minutes after the hour is it? See Figure 9(a). Connect this idea to multiplication facts with 5. Hold up a flash card as in Figure 9(b), and then point to the number on the clock corresponding to the other factor. In this way, the fives facts become the "clock facts."

×	0	1	2	3	4	5	6	7	8	9
0						0				
1						5				
2						10				
3						15				
4						20				
5	0	5	10	15	20	25	30	35	40	45
6						30				
7						35				
8						40				
9						45				

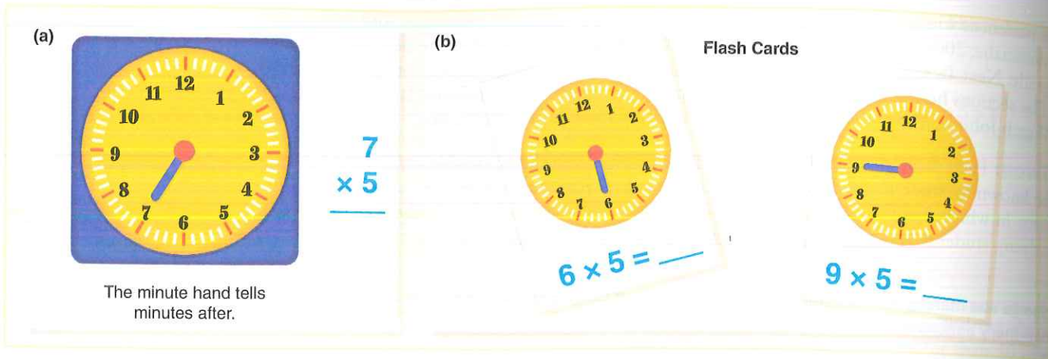


FIGURE 9 Using clocks to help learn fives facts.

Zeros and Ones. Thirty-six facts have at least one factor that is either 0 or 1. These facts, though apparently easy on a procedural level, tend to get confused with "rules" that some students learned for addition—for example, the fact $6 + 0$ stays the same, but 6×0 is always zero, or that $1 + n$ is the next counting number, but $1 \times n$ stays the same. The concepts behind these facts can be developed best through story problems. For example, invite students to tell stories to match a problem.

$6 \times 0 =$ ____ . There are six bowls for raisins. Each bowl is empty. How many raisins?
$0 \times 6 =$ ____ . You worked 0 hours babysitting at $6 an hour. How much money did you make?

Avoid rules that aren't conceptually based, such as "Any number multiplied by zero is zero."

Illustrate ones using arrays to show commutativity ($8 \times 1 = 1 \times 8$) and use stories like the ones for zero. With 0 and 1, help students generalize what it means to have $n \times 0$, $0 \times n$, $1 \times n$ and $n \times 1$ without just memorizing these properties.

×	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2								
3	0	3								
4	0	4								
5	0	5								
6	0	6								
7	0	7								
8	0	8								
9	0	9								

Nifty Nines

Standards for Mathematical Practice
MP2. Reason abstractly and quantitatively.

Nines are in a category by themselves. They aren't used for deriving the other facts, but there are several reasoning strategies and patterns specific to 9s. Nines can actually be derived from 10s. For example, 7×9 can be found by finding 7×10 and removing one set of 7, or $70 - 7$. Because students often can multiply by 10 and subtract from a decade value, this strategy is effective. You might introduce this idea by showing a set of bars such as those in Figure 10 with only the end cube a different color. After explaining that every bar has 10 cubes, ask students if they can think of a good way to figure out how many are yellow.

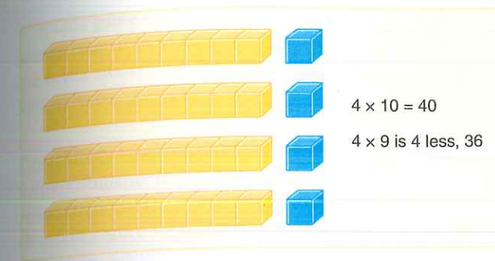
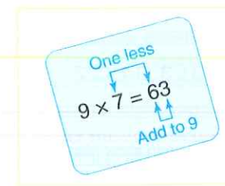


FIGURE 10 Using tens to think of the nines.

FIGURE 11 Nifty nines using fingers to show 4×9.

Nines facts have interesting patterns that can help to find the products: (1) the tens digit of the product is one less than the non-nine factor and (2) the sum of the two digits in the product equals 9. For $7 \times 9 = 63$, the tens digit is one less than 7 and $6 + 3 = 9$. Ask students to explore and discover nines patterns and write down patterns they notice.



×	0	1	2	3	4	5	6	7	8	9
0										0
1										9
2										18
3										27
4										36
5										45
6										54
7										63
8										72
9	0	9	18	27	36	45	54	63	72	81

After discussing all the patterns, ask students how these patterns can be used to figure out a product to a nines fact. Challenge students to think about why this pattern works. (*Warning:* This strategy, grounded in the base-ten system, can be useful, but it also can cause confusion because the conceptual connection is not easy to see.)

A tactile way to help remember the nifty nines is to use fingers—but not for counting. Here's how: Hold up both hands. Starting with the pinky on your left hand, count over for which fact you are doing. For example, for 4×9, you move to the fourth finger (your pointer). Bend it down. Look at your fingers: You have three to the left of the folded finger representing 3 tens and six to the right—36! (Barney, 1970). See Figure 11.

Derived Multiplication Fact Strategies

The following chart shows the remaining 25 multiplication facts. Notice if students recognize the commutative property, there are only 15 facts to learn. These remaining facts can be learned by using foundational facts. (Note: students must first know their foundational facts!)

×	0	1	2	3	4	5	6	7	8	9
0										
1										
2										
3				9	12		18	21	24	
4				12	16		24	28	32	
5										
6				18	24		36	42	48	
7				21	28		42	49	56	
8				24	32		48	56	64	
9										

Standards for Mathematical Practice

MP5. Use appropriate tools strategically.

Arrays. Arrays are powerful thinking tools for deriving multiplication facts. Figure 12(a) illustrates a Multiplication Array with lines through a 10 by 10 array to show fives (known facts). Students can use this hint to derive multiplication facts. For example, to see 7×7 as $(5 \times 7) + (2 \times 7)$, or $35 + 14$ (Figure 12(b)).

There are numerous games that use arrays that can help students derive facts. Activity 14 is one example of a game using arrays.

Activity 14

CCSS-M: 3.OA.A.1; 3.OA.B.5; 3.OA.C.7

Strive to Derive

STUDENTS *with* SPECIAL NEEDS

You will need Multiplication Array Cards (e.g., for 3s, 4s, 6s, and 9s), a straight thin stick (like uncooked spaghetti), and two dice, one labeled with 3, 3, 6, 6, 9, 9; one with 0, 1, 4, 6, 7, 8. To make Multiplication Array Cards, use 2-Centimeter Grid Paper (Blackline Master 5) and cut out each possible size. In marker, write the product both ways (e.g., cut out a 5-by-6 array and write 5×6 and 6×5 on the array). Spread the cards on the table so they can be seen. Player 1 rolls the dice and selects the related array card. Player 1 then places the stick to partition the array into known facts. If Player 1 rolled a 3 and a 6, she would pick the 3-by-6 array. She can partition it as 2×6 and 1×6 or as 3×5 and 3×1. If Player 1 can solve the fact using a derived fact, she scores 1 point. Return array card to the collection. Player 2 repeats the process. Continue to 10 points. Initially, or to modify for students who struggle, focus on one foundation set of facts, for example, Strive to Derive from 5.

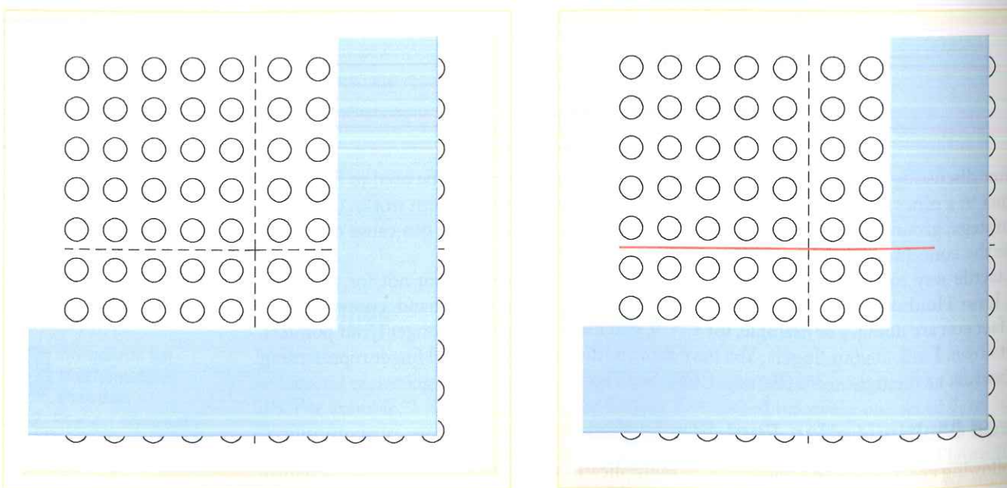


FIGURE 12 A multiplication array can illustrate how to partition a fact into two or more known facts.

This game (and others like it) helps students look for known facts in an unknown fact problem. The more they play, the better students become at partitioning unknown facts into known facts.

Doubling. Doubling is a very effective reasoning strategy in helping students learn the difficult facts (Flowers & Rubenstein, 2010/2011). The Double and Double Again strategy shown in Figure 13(a) is applicable to all facts with 4 as one of the factors. Remind students that the idea works when 4 is the second factor as well as when it is the first. For 4×8, double 16 is also a difficult addition. Help students with this by noting, for example, that $15 + 15$ is 30, and $16 + 16$ is 2 more, or 32. Adding $16 + 16$ on paper defeats the development of reasoning.

The Double and One More Set strategy shown in Figure 13(b) is a way to think of facts with one factor of 3. With an array or a set picture, the double part can be circled, and it is clear that there is one more set. Two facts in this group involve more difficult mental additions: 8×3 and 9×3. Using doubling and one more, you can generate any fact. For the fact 6×7, think of 2×7 (14), then double it to get 4×7 (28), then add two more sets of 7 ($28 + 14 = 42$).

If either factor is even, a Half Then Double strategy as shown in Figure 13(c) can be used. Select the even factor and cut it in half. If the smaller fact is known, that product is doubled to get the new fact.

Close Facts. Many students prefer to go to a fact that is "close" and then add one more set to this known fact, as shown in Figure 14.

For example, think of 6×7 as 6 sevens. Five sevens is close: That's 35. Add one more seven to get 42. Using set language "5 sevens" is helpful in remembering that one more 7 is needed (not one more 5). This "close" fact reasoning strategy is critically important because it reinforces number sense and it can be used to derive any unknown fact.

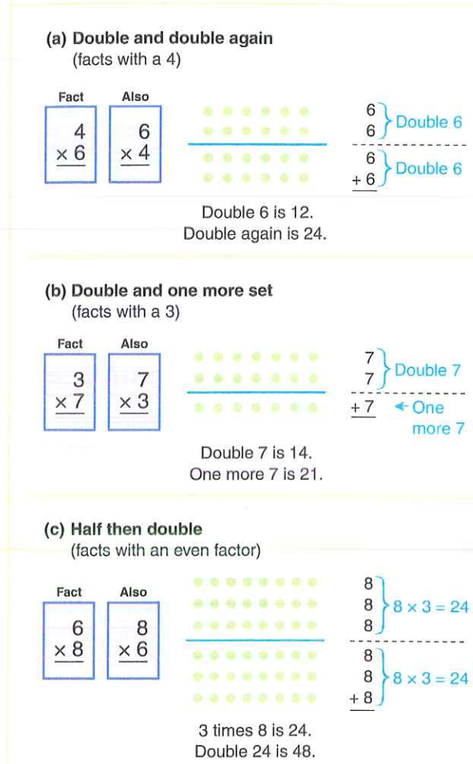


FIGURE 13 Using doubles (known facts) to derive unknown facts.

Division Facts

Mastery of multiplication facts and connections between multiplication and division are key elements of division fact mastery. For example, to solve $36 \div 4$, we tend to think, "Four times what is thirty-six?" In fact, because of this, the reasoning facts for division are to (1) think multiplication and then (2) apply a multiplication reasoning fact, as needed. Missing factor stories can assist in making this connection.

Analea is creating bags of mini muffins for a bake sale. She puts four in a bag and fills up bags until she has all 36 mini muffins in bags. How many bags did she fill?

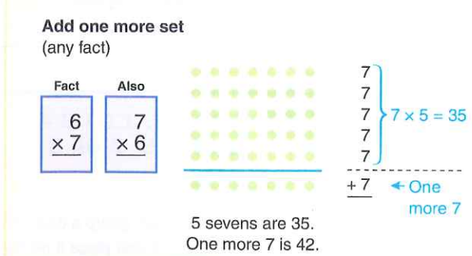


FIGURE 14 Using a close (known) fact to derive an unknown fact.

Notice this story can be represented as $4 \times \_\_ = 36$ (missing factor) or as $36 \div 4 = \_\_$ (division). The double and double again reasoning strategy can be used to solve the missing factor (What number doubled and doubled again equals 36?). Seeing a situation as both a multiplication and a division problem can help students to think multiplication when encountering division facts.

Pause & Reflect

Select what you consider a difficult fact to remember and see how many of the derived fact strategies you can use to derive the fact.

Complete Self-Check 3c: Reasoning Strategies for Multiplication and Division Facts

Reinforcing Basic Fact Mastery

When students "just know" a fact, or can apply a reasoning strategy so fast they almost don't realize they have done it (e.g., Making 10), they have reached phase 3: mastery. CCSS-M uses the phrase "know from memory" (CCSS0, 2010, pp. 19, 23). Repeated experiences with reasoning strategies are effective in committing facts to memory; memorizing is not. Therefore, games or activities that focus on reasoning strategies are more effective than drilling with flash cards (and more palatable for students) . We must use these effective strategies to ensure all students become fluent with facts and abandon long-existing strategies that do not work! If students do not become fluent with their basic facts, they will certainly struggle with multidigit computation. In addition, students who do not know their facts often struggle to understand higher mathematics concepts because their cognitive energy must focus too much on computation when it should be focusing on the more sophisticated concept being developed (Forbringer & Fahsl, 2010).

Games to Support Basic Fact Fluency

Games are fun to play over and over again and therefore are an excellent way to provide repeated experiences for students to learn their facts. Playing games that infuse reasoning strategies helps students be able to flexibly select strategies, decide which strategy is most appropriate for the given problem, and become more efficient and accurate in finding the answer. This is what it takes to become *fluent* with the basic facts! In addition, games increase student involvement, encourage student-to-student interaction, and improve communication—all of which are related to improved academic achievement (Bay-Williams & Kling, 2014; Forbringer & Fahsl, 2010; Kamii & Anderson, 2003; Lewis, 2005).

As you use games, remember to focus on related clusters of facts and on what individual students need to practice. Also, encourage students to self-monitor—they can create their own game board/game, including the facts they are working on (their personal "toughies").

Activity 15

CCSS-M: 1.OA.B.4; 1.OA.C.6; 2.OA.B.2; 3.OA.B.5; 3.OA.B.6; 3.OA.C.7

Salute!

Place students in groups of three, and give each group a deck of cards (omitting face cards and using aces as ones). Two students draw a card without looking at it and place it on their forehead facing outward (so the others can see it). The student with no card tells the sum (or product). The first of the other two to correctly say what number is on their forehead "wins" the card set. For ELLs, students with disabilities, and reluctant learners, speed can inhibit participation and increase anxiety. Speed of response can be removed as a variable by having students write down the card they think they have (within five seconds) and getting a point if they are correct. This can be differentiated by including only certain cards (e.g., addition facts using only the numbers 1 through 5).

ENGLISH LANGUAGE LEARNERS

STUDENTS *with* SPECIAL NEEDS

Activity 16

CCSS-M: 1.OA.B.4; 1.OA.C.6; 2.OA.B.2; 3.OA.B.5; 3.OA.B.6; 3.OA.C.7

What's Under My Thumb?

Create a set of circle cards with fact families for each pair of students or have students create their own based on the facts they need to be practicing (see Figure 15(a)). You can begin this activity as a whole class and then move to partners. Hold up a card with your thumb over one number. Ask, "What is under my thumb?" Call on a student to share the answer and how they reasoned to get the answer. Place students in partners with their sets of cards and play. Groups can switch decks with other groups for more experiences. Individuals can explore cards like the ones in Figure 15(b), with the answer on the back. Alternatively, you can use strips rather than circles (see Figure 15(c)).

A Missing-Part Worksheet can be used as a follow-up to Activity 16. Fill in two of the three numbers for a set of facts, differentiated for students. An example for addition is shown in Figure 16. Have students write an equation for each missing-number card. This is important because students need to connect the missing addend to subtraction.

Table 2 offers some ideas for how classic games can be adapted to focus on basic fact mastery (reflects ideas from Forbringer & Fahsl, 2010, and Kamii & Anderson, 2003). Also included are ideas for differentiating the games.

When all facts are learned, continued reinforcement through occasional games and activities is important. Consider the following activity that engages students in creatively applying all four operations.

CCSS Standards for Mathematical Practice

MP4. Model with mathematics.

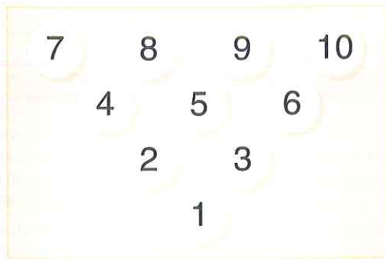
Activity 17

CCSS-M: 2.OA.B.2; 3.OA.C.7; 5.OA.A.1

Bowl a Fact

In this activity (suggested by Shoecraft, 1982), you draw circles placed in triangular fashion to look like bowling pins, with the front circle labeled 1 and the others labeled consecutively through 10. Click here for an activity page that includes lines for recording equations. For culturally diverse classrooms, be sure that students are familiar with bowling. (If they are not, consider showing a YouTube clip or photographs.)

ENGLISH LANGUAGE LEARNERS



Take three dice and roll them. Students use the numbers on the three dice to come up with equations that result in answers that are on the pins. For example, if you roll 4, 2, and 3, they can "knock down" the 5 pin with $4 \times 2 - 3$. If they can produce equations to knock down all 10 pins, they get a strike. If not, roll again and see whether they can knock the rest down for a spare. After doing this with the whole class, students can work in small groups.

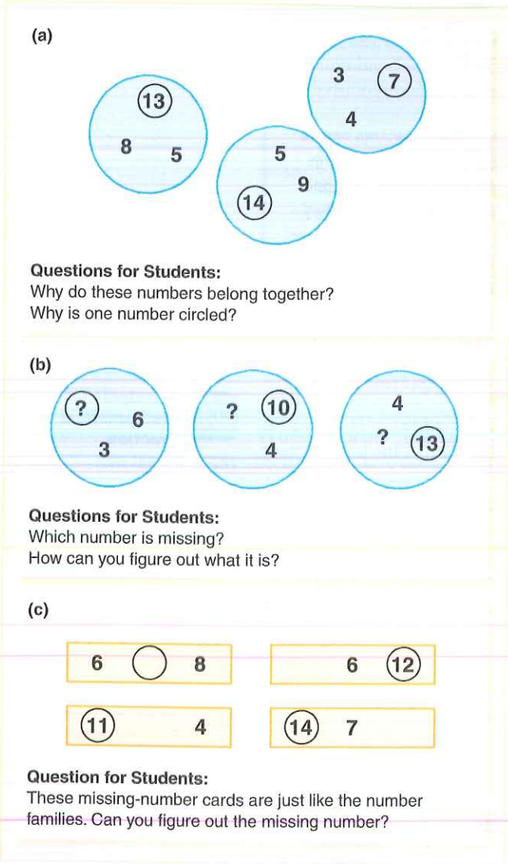


FIGURE 15 Introducing missing-number cards. Note: These are shown for addition/subtraction but work well for multiplication/division, too.

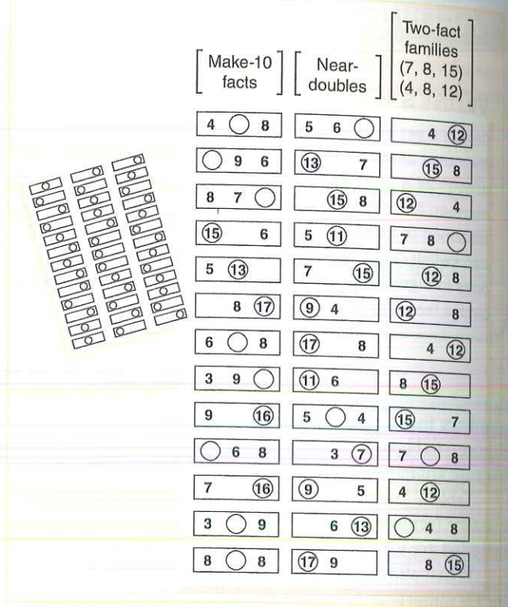


FIGURE 16 Example missing-number worksheets. Labels (in brackets) are not included on student pages.

TABLE 2 CLASSIC GAMES ADAPTED TO BASIC FACT MASTERY

Classic Game	How to Use It with Basic Fact Mastery	Suggestions for Differentiation
Bingo	Each bingo card has a fact problem (e.g., 2 × 3) in each box. The same fact will be on multiple bingo cards but in different locations on each card. You will call out an answer (e.g., 6), and the students will find a matching problem (or more than one problem) on their card.	Create bingo boards that focus on different clusters of facts (e.g., doubles or doubles +1 on some boards, and Making 10 on other boards). Be sure that the answers you call out are an even mix of the clusters so that everyone has the same chance to win.
Concentration	Create cards that have a fact problem (e.g., 3 × 5) on one half and the answers (e.g., 15) on the other half. Shuffle the cards and turn them face-down in a 6 × 4 grid. (If you like, you can make the grid larger to use more cards.)	Select cards that focus on a particular cluster of facts (e.g., +1 and × 5 facts) for each round of the game. Multiple groups can play the game simultaneously—each group will use the parts of the deck that contain the facts they are working on. Also, consider making cards that show the ten-frames below the numbers to help provide a visual for students.

(Continued)

TABLE 2 *(Continued)*

Classic Game	How to Use It with Basic Fact Mastery	Suggestions for Differentiation
Dominoes	Create (or find online) dominoes that have a fact on one side and an answer (not to that fact) on the other. Each student gets the same number of dominoes (around eight). On his or her turn, they can play one of the dominoes in their hand only if they have an answer or a fact that can connect to a domino on the board.	As with other games, select the dominoes that focus on a particular clusters of facts.
Four in a Row	Create a 6 × 6 square game board with a sum (or product) written on each square. Below, list the numbers 0 through 9. Each of the two players has counters of a different color to use as their game pieces. On the first turn, Player 1 places a marker (paper clip) on two addends/factors and then gets to place his or her colored counter on the related answer. (If you have repeated the same answer on different squares of the board, the player only gets to cover one of them.) Player 2 can only move one paper clip and then gets to place his or her colored counter on the related answer. The first player to get four in a row wins.	Rather than list all the values below the chart, just list the related addends or factors. For example, use 1, 2, 6, 7, 8, 9 if you want to work on +1 and +2, or use 3, 4, 5, 6 if you are working on these multiplication facts.
Old Maid (retitled as Old Dog)	Create cards for each fact and each answer. Add one card that has a picture of an old dog (or use your school mascot). Shuffle and deal cards. On each player's turn, they draw from the person on their right, see whether that card is a match to a card in their hand (a fact and its answer), and, if so, lay down the pair. Then the person to their left draws from them. Play continues until all matches are found and someone is left with the Old Dog. Winner can be the person with (or not with) the Old Dog, or the person with the most pairs.	See Concentration (above).

Source: Based on ideas from Forbringer & Fahsl, 2010, and Kamii & Anderson, 2003.

***FORMATIVE ASSESSMENT* Notes.** As students are engaged in games and activities, interview students to find out whether they are using counting strategies, reasoning strategies, or quick recall. Ask students to tell what strategy they just used. If you observe counting, ask the student to try a reasoning strategy. If many students are counting, more experiences (with ten-frames, for example) are needed. ■

About Drill

Drill—repetitive non-problem-based activity—in the absence of reasoning has repeatedly been demonstrated as ineffective. However, drill can strengthen memory and retrieval capabilities (Ashcraft & Christy, 1995). Drill is only appropriate after students know strategies and have moved from phase 2 to phase 3. Drill should also be low-stress and engaging. The many games and activities in this chapter can continued to be played even after students know the facts from memory. Students will smile when they see cards coming out for another game of *Salute!*

Too often, drill includes too many facts too quickly, and students become frustrated and overwhelmed. Also, students progress at different paces—gifted students tend to be good memorizers, whereas students with intellectual disabilities have difficulty memorizing (Forbringer & Fahsl, 2010). Rather than work on all facts, identify a group of facts (e.g, 3s) and look at patterns within that set. Students can create their own cards with each fact written both ways (e.g., 3 × 7 and 7 × 3), create a dot array, and record the answer on the back. They can work on these at home, with a partner, and keep track of the ones they "just know" and the ones for which they use a strategy.

A plethora of websites and software programs provide opportunities to drill on the basic facts (see Figure 17). Though currently none exist that work on strategy development, these programs can support students who are near mastery or maintaining mastery. One disadvantage of most of these sites is that they focus on all the facts at one time. Two exceptions (sites that organize drill by fact family) are *Fun 4 the Brain* and *Math Fact Café*.

CCSS Standards for Mathematical Practice

MP8. Look for and express regularity in repeated reasoning.

Online Resources for Mastery of Basic Math Facts

Name	Description
Fun 4 the Brain: Math Games	This site offers over a dozen games for addition facts and ten for multiplications facts. Pick a game (e.g., Snowy's Friend) and then pick the fact families you would like to explore.
Math Fact Café	Here you will find a lot of downloadable practice: pre-made fact sheets, flashcards, and practice pages, or create your own practice pages, selecting the number of practice problems, the level of difficulties, and which fact families.
NCTM Illuminations *Deep Sea Duel*	Play this applet: The first person to choose a set of digit cards with a specified sum wins. You can choose how many cards, what types of numbers, and the level of strategy.
NCTM Illuminations *Factorize*	In this applet, students visually explore the concept of factors by creating rectangular arrays. Choose your own number or one randomly selected.
NCTM Illuminations *Let's Learn Those Facts*	Six complete lessons for addition facts are provided, including links to resources and student recording sheets.
NCTM Illuminations *Multiplication: It's in the Cards Lessons*	These four lessons, including links to resources and student recording sheets, use the properties of multiplication to help students master the multiplication facts. See also "Six and Seven as Factors."
NCTM Illuminations *The Product Game*	Four lessons provide guidance on using the engaging and effective games "Factor Game" and "Product Game" to help students see the relationship between products and factors.
NCTM's Calculation Nation *Factor Dazzle and Times Square*	These multiplication games are among a number of interactive, fun games to play. Registered players can play against others from all over the world.
BBC *Cross the Swamp*	This British applet asks students to supply a missing operation (+/− or ×/÷) and a number to complete an equation (e.g., 4 __ __ = 12). There are five questions in a set, each with three levels of difficulty.
NLVM *Diffy*	Diffy is a classic mathematics puzzle that involves finding the differences of given numbers. Here it is presented as an applet.
IXL Learning	IXL contains interactive practice tools to monitor student progress toward basic fact mastery. Connections with the Common Core State Standards, Department of Defense Education Activity standards, and existing state standards are provided.

FIGURE 17 Websites and applets for support in teaching the basic facts.

Fact Remediation

Students who have not mastered their addition facts by third grade or their multiplication facts by fourth grade (or beyond) are in need of interventions that will help them master the facts. More drill is not an intervention! Students who do not know their facts may be stuck back in phase 1 (counting strategies) and likely lack number sense and reasoning strategies (phase 2). Effective remediation first requires figuring out which facts a student knows and which ones he or she does not. Second, effective remediation requires a focus on the three phases—determining where a student is and explicitly teaching reasoning strategies (phase 2) in order to reach mastery (phase 3). Review the ideas offered in the Assessing Basic Facts section to figure out what students do and do not know. Then, use these ideas to help students master all of their facts.

1. *Explicitly teach reasoning strategies.* Students' fact difficulties are due to a failure to develop or connect concepts and relationships such as those that have been discussed in this chapter. They need instruction focused at phase 2, not phase 3. In a remediation program, students may not have the benefit of class discussion. Share with them strategies that you have "seen other students use." Be certain that they have a conceptual understanding of the strategy and are able to use it.

   For example, if a student knows his addition facts within 10, but struggles with the ones that sum to 11–19, then you know which facts to target. Determine if he knows the Combinations of 10 strategy. Practice it until he is fluent with it. Then, explicitly teach the Making 10 strategy. For multiplication, you might notice that a student is very good at doubling. Help her write the way to solve her "toughies" using doubles. For example, to multiply by 4, she can double and double again. For 8, she can double, double, double. Doubling has been found to be very effective in helping middle school students master multiplication facts, increasing their reasoning skills as well as their confidence (Flowers & Rubenstein, 2010/2011).
2. *Provide hope.* As noted in the discussion of timed tests, students' confidence can be affected. Students may feel they are doomed to use finger counting forever. Let these students know that they will explore strategies that will help them with the facts. Turn off the timers. Shorten (or eliminate) the quizzes.
3. *Inventory the known and unknown facts.* Find out which facts are known quickly and comfortably and which are not (see assessment ideas discussed previously). Invite students to do this for themselves as well. Provide sheets of mixed multiplication facts and ask students to answer the ones they "just know" and circle the facts where they have to pause to count or use a strategy. Review the results with them and discuss which strategies and facts you will work on.
4. *Build in success.* Begin with easier and more useful reasoning strategies like Combinations of 10 for addition. Success builds success! Have students find all the facts that can be solved with a newly learned strategy. Use fact charts to show the set of facts you are working on. It is surprising how the chart quickly fills up with mastered facts.
5. *Provide engaging activities.* Use the many games and activities in this chapter to work on phase 2 and phase 3. As students play, ask which strategies they are using. Deemphasize competition and emphasize collaboration. Prepare take-home versions of a game and assign students to play the game at home at least once. Invite parents in for a Math Night and teach them games that they can enjoy (e.g., *Salute!*) and teach families to focus on reasoning strategies over memorization as they play the games.

What to Do When Teaching Basic Facts. We close this chapter with some important reminders in effectively teaching the basic facts. This is such an important life skill for all learners that it is important that we, as teachers, use what research suggests are the most effective practices. The following list of recommendations can support the development of quick recall.

1. *Ask students to self-monitor.* The importance of this recommendation cannot be overstated. Across all learning, having a sense of what you don't know and what you need to learn is important. It certainly holds true with memorizing facts. Students should be able to identify their "toughies" and continue to work on reasoning strategies to help them derive those facts.
2. *Focus on self-improvement.* Help students notice that they are getting quicker or learning new facts or strategies. For example, students can keep track of how long it took them to go through their "fact stack" and then, two days later, pull out the same stack and see whether they are quicker (or more accurate or use a new strategy) compared to the last time.
3. *Limit practice to short time segments.* You can project numerous examples of double ten-frames in relatively little time. Or you can do a story problem a day—taking five minutes to share strategies. You can also have each student pull a set of flash cards, pair with another student, and go through each other's set in two minutes. Long periods (ten minutes or more) are not effective. Using the first five to ten minutes of the day, or extra time just before lunch, can provide continued support on fact development without taking up mathematics instructional time better devoted to other topics.

CCSS Standards for Mathematical Practice

MP1. Make sense of problems and persevere in solving them.

4. *Work on facts over time.* Rather than do a unit on fact memorization, work on facts over months and months, working on one reasoning strategy or set of facts until it is learned, then moving on. Be sure foundational facts come first and are down pat before teaching derived fact strategies.
5. *Involve families.* Share the big plan of how you will work on learning facts over the year. One idea is to let parents or guardians know that during the second semester of second grade (or third grade), for example, you will have one or two "Take Home Facts of the Week." Ask family members to help students by using reasoning strategies when they don't know a fact.
6. *Make fact practice enjoyable.* There are many games (including those in this chapter) designed to reinforce facts that are not competitive or anxiety inducing.
7. *Use technology.* When students work with technology, they get immediate feedback and reinforcement, helping them to self-monitor.
8. *Emphasize the importance of knowing their facts.* Without trying to create pressure or anxiety, emphasize to students that in real life and in the rest of mathematics, they will be recalling these facts all the time—they really must learn them and learn them well.

What Not to Do When Teaching Basic Facts. The following list describes strategies that may have been designed with good intentions but work against student recall of the basic facts.

1. *Don't use timed tests.* As we have mentioned, little insight is gained from timed tests and they can potentially negatively affect students. Turn the timers off!
2. *Don't use public comparisons of mastery.* You may have experienced the bulletin board that shows which students are on which step of a staircase to mastering their multiplication facts. Imagine how the student who is on the step 3 feels when others are on step 6. It is great to celebrate student successes, but avoid comparisons between students.
3. *Don't proceed through facts in order from 0 to 9.* Work on foundational facts first, then move to the tougher facts.
4. *Don't work on all facts all at once.* Select a strategy (starting with easier ones) and then work on memorization of that set of facts (e.g., doubles). Be sure these are really learned before moving on. Differentiation is needed!
5. *Don't expect quick recall too soon.* This has been addressed throughout the chapter but is worth repeating. Quick recall or mastery can be obtained only after significant time has been spent on reasoning strategies.
6. *Don't use facts as a barrier to good mathematics.* Mathematics is not solely about computation. Mathematics is about reasoning, using patterns, and making sense of things. Mathematics is problem solving. There is no reason that a student who has not yet mastered all basic facts should be excluded from real mathematical experiences (allow calculators so that students don't get bogged down on computation while working on more complex tasks).
7. *Don't use fact mastery as a prerequisite for calculator use.* Requiring that students master the basic facts before they can use a calculator has no foundation. Calculator use should be based on the instructional goals of the day. If your lesson goal is for students to discover the pattern (formula) for the perimeter of rectangles, then students might be recording the length, width, and perimeter for different rectangles and looking for patterns. Using a calculator can quicken the computation in this lesson and keep the focus on measurement.

Complete Self-Check 4: Reinforcing Basic Fact Mastery

REFLECTIONS

WRITING TO LEARN

Click here to assess your understanding and application of chapter content.

1. Describe advantages of a developmental approach to helping students develop fluency with their basic facts.
2. How might you decide which addition facts a student only solves by counting, for which ones they are using reasoning strategies, and which ones they just know?
3. What is meant by "Making 10"? What strategies might you use to help students learn this reasoning strategy?
4. For the fact 6×7, describe two derived fact strategies a student might use (assume the student knows their foundation facts).
5. Why are games and interactive software important in supporting basic fact fluency?
6. This chapter suggests an approach to teaching the basic facts that may be quite different than your own experience. Describe three things that are highlighted in this chapter that were new to you and that you hope to remember to use.

FOR DISCUSSION AND EXPLORATION

- Explore a web-based program for drilling basic facts. What features does the program have that are good? Not so good? How would you use such programs in a classroom with only one or two available computers? How would you differentiate it to address those who are working on different fact strategies?
- Assume you are teaching a grade that expects mastery of facts (grade 2 for addition and subtraction or grade 3 or 4 for multiplication and division). How will you design fact mastery across the semester or year? Include timing, strategy development, involvement of families, use of games, and so forth.

RESOURCES

LITERATURE CONNECTIONS

One Less Fish *Toft and Sheather (1998)*

This beautiful book with an important environmental message starts with 12 fish and counts back to zero fish. On a page with 8 fish, ask, "How many fish are gone?" and "How did you figure it out?" Encourage students to use the Down Over 10 strategy. Any counting-up or counting-back book can be used in this way!

The Twelve Days of Summer *Andrews and Jolliffe (2005)*

You will quickly recognize the style of this book with five bumblebees, four garter snakes, three ruffed grouse, and so on. The engaging illustrations and motions make this a wonderful book. Students can apply multiplication facts to figure out how many of each item appear by the end of the book. (For example, three ruffed grouse appear on days 3, 4, 5, and so on.)

RECOMMENDED READINGS

Articles

Baroody, A. J. (2006). Why children have difficulties mastering the basic fact combinations and how to help them. *Teaching Children Mathematics, 13*(1), 22–31.

Baroody suggests that basic facts are developmental in nature and contrasts "conventional wisdom" with a number-sense view. Great activities are included as exemplars.

Boaler, J. (2014). Research suggests that timed tests cause math anxiety. *Teaching Children Mathematics 20*(8), 469–474.

This is a wonderful article to challenge the longstanding practice of timed-tests. Just because we have always done this, doesn't mean it's a good idea!

Buchholz, L. (2004). Learning strategies for addition and subtraction facts: The road to fluency and the license to think. *Teaching Children Mathematics, 10*(7), 362–367.

A second-grade teacher explains how her students developed and named their strategies and even extended them to work with two-digit numbers. She found her "lower ability" students were very successful using reasoning strategies.

Crespo, S., Kyriakides, A. O., & McGee, S. (2005). Nothing "basic" about basic facts: Exploring addition facts with fourth graders. *Teaching Children Mathematics, 12*(2), 60–67.

This article provides evidence of the critical importance of addressing remediation through a focus on reasoning strategies and number sense.

Kling, G. (2011, September). Fluency with basic addition. *Teaching Children Mathematics, 18*, 80–88.

Kling offers many research-based ideas for developing fluency. She emphases the need to begin with doubles and combinations of 10 and build from there.Developing Basic Fact Fluency

APPENDIX

ACTIVITIES AT A GLANCE

This table lists the named and numbered activities in the chapter. In addition to providing an easy way to find an activity, the table provides the main mathematics content stated as succinctly as possible and the related Common Core State Standards. Remember that this is a chapter about teaching mathematics and not a chapter of activities. It is extremely important not to take any activity as a suggestion for instruction without reading the full chapter.

Developing Basic Fact Fluency

Activity		Mathematics Content	CCSS-M
1	If You Didn't Know	Use known facts to determine unknown facts	1.OA.C.6; 2.OA.B.2; 3.OA.B.5; 3.OA.C.7
2	How Many Feet in the Bed?	Practice facts for +2 and −2	1.OA.A.1; 1.OA.C.6; 2.OA.B.2
3	One More Than and Two More Than with Dice and Spinners	Practice addition facts for +1 and +2	1.OA.C.5; 1.OA.C.6; 2.OA.B.2
4	Double Magic	Practice addition doubles facts	1.OA.C.5; 1.OA.C.6; 2.OA.B.2
5	Calculator Doubles	Practice addition doubles facts	1.OA.C.6; 2.OA.B.2
6	How Much to Equal 10?	Practice combinations of 10 facts	1.OA.B.4; 1.OA.C.6; 2.OA.B.2
7	Move It, Move It	Develop Making 10 strategy	1.OA.B.3; 1.OA.C.6; 2.OA.B.2
8	Frames and Facts	Develop Making 10 strategy	1.OA.B.3; 1.OA.C.6; 2.OA.B.2
9	Flash	Practice using 5 and 10 as anchors	1.OA.B.3; 1.OA.C.5; 1.OA.C.6; 2.OA.B.2
10	On the Double!	Practice near-doubles addition facts	1.OA.B.3; 1.OA.C.6; 2.OA.B.2
11	Apples in the Trees	Practice subtraction to 20	1.OA.B.4; 1.OA.C.6; 2.OA.B.2
12	Apples in Two Trees	Develop missing-value concept, relating addition to subtraction	1.OA.B.3; 1.OA.B.4; 1.OA.C.6; 2.OA.B.2
13	Clock Facts	Develop minute intervals on the clock as a strategy for fives multiplication facts	3.OA.A.1; 3.OA.C.7

Developing Basic Fact Fluency (*Continued*)

Activity		Mathematics Content	CCSS-M
14	Strive to Derive	Use known facts to determine unknown facts	3.OA.A.1; 3.OA.B.5; 3.OA.C.7
15	Salute!	Identify the missing addend or factor	1.OA.B.4; 1.OA.C.6; 2.OA.B.2; 3.OA.B.5; 3.OA.B.6; 3.OA.C.7
16	What's Under my Thumb?	Connect addition and subtraction facts or multiplication and division facts	1.OA.B.4; 1.OA.C.6; 2.OA.B.2; 3.OA.B.5; 3.OA.B.6; 3.OA.C.7
17	Bowl a Fact	Practice creating equations in addition, subtraction, multiplication, and division	2.OA.B.2; 3.OA.C.7; 5.OA.A.1

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Credits are listed in order of appearance.

List, Based on Baroody, A. J. (2006). Why children have difficulties mastering the basic number combinations and how to help them, *Teaching Children Mathematics*, 13(1), 22–31. Common Core Standards, © Copyright 2010. National Governors Association Center for Best Practices and Council of Chief State School Officers. All rights reserved;

Pop-Ups

Select Activity Pages and Expanded Lessons from: Bay-Williams (2013) *Field Experience Guide, Resources for Teachers of Elementary and Middle School Mathematics.*

list, Based on Baroody, A. J. (2006). Why children have difficulties mastering the basic number combinations and how to help them. *Teaching Children Mathematics*, 13(1), 22–31. Table 2, Based on ideas from Forbringer & Fahsl, 2010, and Kamii & Anderson, 2003.

Developing Whole-Number Place-Value Concepts