MESSAGE

14 Balance Is Basic

WHAT IT TAKES TO BE MATHEMATICALLY PREPARED TODAY

> The changing needs of society, the explosion of the amount of quantitative data, and the availability of computers and calculators demand a redefining of the priorities for basic mathematics skills. . . . The present technological society requires daily use of such skills as estimating, problem solving, interpreting data, organizing data, measuring, predicting, and applying mathematics to everyday situations.
>
> —National Council of Supervisors of Mathematics
> (Position Paper on Basic Skills, 1977)

What constitutes a balanced mathematics program has been the subject of significant work in mathematics education for more than thirty years. The National Council of Supervisors of Mathematics statement quoted at the beginning of this message presented a vision of *new basics* in 1977, supporting the teaching of an expanded set of basic skills calling for (among other things) a focus on problem solving and the incorporation of appropriate uses of technology. Shortly after, in 1980, the National Council of Teachers of Mathematics (NCTM) produced its call to action entitled *An Agenda for Action*, which outlines priorities for school mathematics. These priorities center on problem solving and call for the use of tools like calculators and computers. NCTM took an even bolder step at the end of the same decade in launching their landmark publication, *Curriculum and Evaluation Standards for School Mathematics* in 1989, in which a comprehensive mathematics program for grades K–12 is outlined, elaborating what mathematical content and processes all students need. In 2000, NCTM further refined the agenda with the publication of *Principles and Standards for School Mathematics*. And, building on all of this work, in 2010, the Common Core State Standards for

Mathematics were released, offering another iteration of what a balanced mathematics program should include.

What Is Basic?

Even though much has changed about the nature of the mathematics students need and the depth of mathematical thinking called for today, the fundamentals of a balanced mathematics program remain much the same as they have for years. Essentially, we want all students to

1. *make sense of mathematics*: understand mathematical concepts and ideas so they can make sense of the mathematics they do;
2. *do mathematics*: know mathematical facts and perform mathematical skills; and
3. *use mathematics*: solve a wide range of problems in various contexts by reasoning, thinking, and applying the mathematics they have learned.

However, even though we may agree on the broad categories, arguments abound over what exactly each of these pieces comprises and over the relative weight and importance of each. People are often quick to categorize opposing views, applying labels like back to basics or reform. For example, some people fear that students in a program they label as reform might not receive the emphasis on computation they see in more traditional programs. Others fear that if students focus excessively on computational procedures without adequate understanding, their proficiency will be superficial and short-lived. Still others argue that if students cannot solve particular types of problems, ranging from routine word problems to complex, sometimes ill-defined modeling problems, they will not be prepared for the world outside of school. It's time to get past these arguments, move beyond the labels, and look at what we mean when we discuss the components of a balanced mathematics program.

Making Sense of Mathematics: Conceptual Understanding

Without conceptual understanding, learning skills is meaningless. Many students acquire skills (with varying degrees of success), only to find that they lack the understanding to apply what they have learned. For example, in solving a problem involving fractions, a student may know the rules: how to *go straight across* (when multiplying fractions), *turn upside down and then go straight across* (when dividing fractions), or *find a common denominator* (for adding and subtracting fractions with unlike denominators). But what if a student doesn't understand the operations the rules represent or how they relate to the problem at hand? Or what if the student doesn't even have a firm grasp of what a fraction really represents? In such cases, every encounter with fractions results in a guessing game: Which strategy should I use? Even students skilled in procedures will be lost if they don't understand the meanings of fractions or the meaning of the operations well enough to determine which procedure might be appropriate.

On the other hand, if a student has a strong understanding of what a fraction represents, and also a solid understanding of what it means to add, subtract, multiply, or divide, the act of adding fractions becomes far less complex—and less of a guessing game. Students with both skills and conceptual understanding are likely to more easily form mental images of mathematical concepts they might recognize in problem situations. As a result, a student can see in her mind how to put the fractions together for the situation a problem presents. She has no reason to be tempted to choose an incorrect rule because she knows what operation(s) might be helpful and which procedure(s) could therefore be useful. The same reasoning can be applied to the importance of learning with understanding for any skill, from basic computational algorithms to procedures for solving equations.

Doing Mathematics: Skills, Facts, and Procedures

Understanding mathematical ideas and concepts is powerful. But without also knowing basic facts or how to perform the necessary skills, understanding alone is insufficient. If taught well, proficiency with mathematical facts and skills can contribute to mathematical understanding. Both components—skills and understanding—are especially critical when tackling challenging problems.

In today's highly technological world, we can debate the extent of computational skills a student needs. We all want students to know how to add, subtract, multiply, and divide whole numbers, decimals, and fractions for both positive and negative numbers. But we must be realistic about when students should be able to compute mentally, when they should perform computation with a pencil, and when they might rely on a calculator or other technological tool. It may be that being able to use a pencil and paper to do a long-division exercise with a four-digit divisor is not as critical as it once was. Perhaps students are proficient enough with the tool of division if they can perform pencil-and-paper division exercises through two-digit divisors, for example, and longer or more complex division exercises with a calculator. Students might be expected to estimate results of most division exercises mentally, and even determine exact answers to certain types of basic division exercises in their heads. These shifts don't mean that computational skills are unimportant. Rather, the shifts help us set priorities and make room for the rest of a balanced program that includes the solid understanding described earlier and what is arguably the most critical feature of a balanced mathematics program—problem solving.

Using Mathematics: Problem Solving

If students have a sound understanding of mathematical ideas and some level of skills mastery, they have certainly gained important mathematical knowledge. But there is a difference in knowledge that is useful for an exercise in a mathematics textbook and knowledge that is needed to solve authentic, multifaceted problems found in mathematics and in other contexts. For a student's knowledge of mathematics to truly come together, we must focus on the third component of a balanced mathematics program: problem solving. Solving problems requires that students consider the many tools they have acquired and select which one(s) might be useful for a wide range of problems at varying levels of complexity and in many different contexts. Solving problems is the most visible indicator of how well a student has assimilated the knowledge he has acquired and how comprehensive a set of mathematical tools the student has built.

Beyond the Big Three

All of the documents listed at the beginning of this message reinforce the importance of a balanced program of conceptual understanding, skills, and problem solving, while also reflecting shifts in emphasis within each of these. Beyond the consistent vision of a mathematics program reflecting a balance of these three elements, these publications also recognize the growing importance of what we might consider the connective tissue of a coherent mathematics program—mathematical habits of mind that include flexible thinking skills and strong quantitative reasoning. These critical habits of mind mirror the skills now called for in report after report from the business and policy sectors.

In *Adding It Up* (Kilpatrick, Swafford, and Findell 2001) and its partner booklet, *Helping Children Learn Mathematics* (Kilpatrick and Swafford 2002), the authors use the metaphor of a rope to define mathematical knowledge. The five strands of the rope represent understanding, computing, applying, reasoning, and engaging. The first three of these strands reflect the three components of balanced mathematics discussed in this message. The last two—reasoning (and the various types of mathematical thinking associated with it) and engaging (including habits of mind such as persistence and a willingness to take on a challenging problem)—reflect the connective tissue of mathematical habits of mind.

What Can We Do?

Whether you prefer the metaphor of a rope or Marilyn Burns' (2015) description of a three-legged stool (representing three aspects of arithmetic: number sense, computation, and problem solving) or other models of a multifaceted, balanced mathematics curriculum offered over the years, the fact is that all students need it all. They need a balanced program of understanding, skills, and problem solving and they need a flexible set of thinking and reasoning tools they can call on to pull all of these pieces together. From a mathematical standpoint, each piece needs to support the other pieces. Students need to connect understanding with doing and using mathematics. They need to use tools of communication, representation, reasoning, and thinking to make mathematics useful beyond the classroom. Whenever we omit part of a balanced mathematics program for any student, whatever is left falls apart. Whether intentional or unintentional, whether guided by good intentions or low expectations, whether targeted at one student or at a group of students, the student without a balanced and comprehensive knowledge of mathematics has no foundation upon which to build future mathematical success.

It only makes sense, from a mathematical perspective and from a moral and ethical perspective, for schools to absolutely commit to providing all students a deep, connected, comprehensive, balanced mathematics program that will allow every student to meet the increasing demands for their future in society and the workplace.

Reflection and Discussion

FOR TEACHERS

- What issues or challenges does this message raise for you? In what ways do you agree with or disagree with the main points of the message?
- How can you connect the skills students learn with their understanding of what the skills represent?
- How can you help students develop mathematical habits of mind such as the willingness to take on challenging tasks or the perseverance to spend time on a hard problem?
- Which components of a balanced mathematics program do you feel you are most effectively providing for your students? Which components might you more effectively target in the future?

(*continued*)

FOR FAMILIES

- What questions or issues does this message raise for you to discuss with your son or daughter, the teacher, or school leaders?
- How can you work together with your daughter or son to communicate how important it is to understand or make sense of the procedures she or he is learning?
- What messages can you convey to your son or daughter about the importance of understanding the mathematics being learned? Make a point to ask your son or daughter to explain to you a skill he or she is learning in terms of what it means, not just how to do it.

FOR LEADERS AND POLICY MAKERS

- How does this message reinforce or challenge policies and decisions you have made or are considering?
- How balanced is your mathematics curriculum, and how well do your textbooks or instructional materials reflect a comprehensive and connected blend of understanding, skills, and problem solving? How well does your program incorporate the critical connective tissue that includes reasoning and mathematical thinking?
- How can you help your teachers teach in ways that help students develop mathematical habits of mind?

RELATED MESSAGES

Faster Isn't Smarter

- Message 4, "Good Old Days," looks back at several waves of mathematics reform that included differing views of what is basic.
- Message 1, "Math for a Flattening World," makes a case for changing how schools prepare students for today's world.
- Message 16, "Hard Arithmetic Isn't Deep Mathematics," advocates teaching more than computation as a way to increase rigor.
- Message 20, "Putting Calculators in Their Place," considers the role of calculators when teaching computation and discusses how much technology is appropriate in a math classroom.

Smarter Than We Think

- Message 31, "Developing Mathematical Habits of Mind," addresses the importance of focusing mathematics teaching and learning around mathematical thinking, reasoning, and sense-making.

- Message 13, "Clueless," looks at the dangers of teaching shallow mathematics without deep understanding.
- Message 32, "Problems Worth Solving—And Students Who Can Solve Them," discusses the critical role of problem solving in a mathematics program.
- Message 3, "He Doesn't Know His Facts," tells the story of a man who became successful in a mathematically dependent field without knowing all of his basic facts.

MORE TO CONSIDER

- *About Teaching Mathematics: A K–8 Resource, Fourth Edition* (Burns 2015) is a practical resource and comprehensive reference for teachers on helping children build a balanced understanding of and proficiency with mathematics, including a description of the three-legged stool model mentioned in this message.
- "Fluency Without Fear: Research Evidence on the Best Ways to Learn Math Facts" (Boaler, Williams, and Confer 2014) offers strategies for developing computational fluency with understanding as part of a balanced mathematics program.
- *Number Talks, Grades K–5: Helping Children Build Mental Math and Computation Strategies*, *Updated with Common Core Connections* (Parrish 2010, 2014) is a multimedia resource for orchestrating classroom discourse toward building computational fluency and understanding.
- Common Core State Standards for Mathematics (NGA Center and CCSSO 2010) are mathematics standards for kindergarten through high school that present one view of a mathematics program that balances concepts, computation, and problem solving. www.corestandards.org.
- *Helping Children Learn Mathematics* (Kilpatrick and Swafford 2002), the partner document to *Adding It Up* (Kilpatrick, Swafford, and Findell 2001), is a short booklet for a broad audience of readers that highlights the rope model described in this message and presents clear recommendations for teachers, families, and leaders to improve mathematics teaching and support mathematics learning at school and at home.
- *A Research Companion to Principles and Standards for School Mathematics* (Kilpatrick, Martin, and Schifter 2003) summarizes the research behind the recommendations for a balanced mathematics program as described in *Principles and Standards for School Mathematics* (NCTM 2000).
- "Contemporary Curriculum Issues: Organizing a Curriculum Around Mathematical Habits of Mind" (Cuoco, Goldenberg, and Mark 2010) suggests using mathematical habits of mind,

rather than content topics, as a way to organize a mathematics program.

- *Elementary and Middle School Mathematics: Teaching Developmentally* (Van de Walle, Karp, and Bay-Williams 2009) is a widely recognized elementary and middle school mathematics methods book for preservice teachers that includes a beautiful development of numbers and operations and also comprehensively develops other topics in a well-balanced mathematics program.
- "From the Inside Out" (Fillingim and Barlow 2010) describes the kind of mathematical thinking involved in helping children become "doers of mathematics" in and outside of school.
- *Thinking Mathematically: Integrating Arithmetic and Algebra in Elementary School* (Carpenter, Franke, and Levi 2003) offers background and strategies on how to focus elementary mathematics instruction on mathematical habits of mind that support the transition from numbers to symbols.
- *Principles and Standards for School Mathematics* (National Council of Teachers of Mathematics 2000) refines, clarifies, and extends NCTM's landmark 1989 standards and describes the richness and depth of a comprehensive and balanced mathematics program.
- "Math Homework Is Due Tomorrow—How Can I Help?" (NCTM 2006b) is a downloadable brochure on NCTM's Family Resources page (http://nctm.org/resources/families) that helps families make sense of the various aspects of math they see coming home from school.
- NCTM's Family Resources page (http://nctm.org/resources/families) includes links to documents that answer questions about the changing nature of mathematics today, calculator use in mathematics classrooms, and the use of timed skills tests.

WWW This message is also available in printable format at mathsolutions.com/fasterisntsmarter2ndedition