Faster Isn't Smarter

SECOND EDITION

Messages About Math, Teaching, and Learning in the 21st Century

A resource for teachers, leaders, policy makers, and families

CATHY L. SEELEY

Foreword by Marilyn Burns

Math Solutions
Sausalito, California, USA

Math Solutions
One Harbor Drive, Suite 101
Sausalito, California, USA 94965
www.mathsolutions.com

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MESSAGE

16 Hard Arithmetic Isn't Deep Mathematics

THE IMPORTANCE OF DEPTH IN TEACHING MATHEMATICS

> U.S. teachers do what we ask them to do. We hand them fragmented, inclusive curricula—curricula "a mile wide and ***an inch deep.***" We provide as their fundamental resources encyclopedic, inclusive textbooks filled with brief coverage of many topics and with a preponderance of low-demand tasks for students.
>
> —William H. Schmidt, Curtis C. McKnight, and Senta A. Raisen in *A Splintered Vision*

A mile wide and an inch deep. Now that essentially every state has implemented standards that focus on fewer topics—tackling the "mile-wide" curriculum—the challenge is to use instructional time gained to help students reach strong and lasting mathematical understanding that goes beyond "an inch deep." The kind of mathematics high school graduates need today goes far beyond what once was sufficient. In the past, it might have been enough to know how to read, write, and do the basic measurement and arithmetic used in everyday life. In the past, it might also have been enough for students who were going to college to master a set of algebraic tools that enabled them to enroll in higher-level mathematics or science courses. But today's world is characterized by rapid change and pervasive technology. More of our students are likely to participate in some kind of postsecondary education than ever before. Many jobs in demand now didn't even exist five years ago. And most of those jobs call for far more sophisticated thinking and reasoning than in the past.[1]

In this environment, how do we raise the bar on the mathematical proficiency we expect of all students, regardless of what path they might pursue after high school? What kind of mathematics do all students

[1]See also Message 15, "Less Can Be More," which deals with the "mile-wide" curriculum.

need? And how likely is it that all students can achieve the goals that we set? These questions cannot be answered by simply asking students to do more of what they've been expected to do in the past—to add, subtract, multiply, and divide bigger or uglier numbers. If we are to offer students the opportunity to understand mathematics at more than an "inch-deep" level, we need to do more than teach harder arithmetic at earlier grades.

The Meaning of Depth

We can begin to raise the bar on mathematical proficiency by eliminating some of the endless repetition in our curriculum, especially related to teaching arithmetic skills. Mathematics standards today allow us to focus on fewer topics at each grade level, assigning the focus on important mathematical ideas to different grades, rather than superficially teaching a little bit about many topics in each grade. In this way, students have a chance to learn mathematics deeply, so that they don't need to be taught the same topics year after year. Depth means that students know a lot about multiplication as well as knowing an algorithm for performing multiplication. Depth means that when we introduce fractions, we teach students what fractions represent, in what kinds of situations fractions might be useful, how fractions compare to one another, how they relate to what students know about whole numbers, what it means when the numerator or denominator increases or decreases, and so on. Depth means that before students confront the rules for operating with fractions—what students remember as going straight across, turning upside down, or finding a common denominator—we ensure that they know a lot about fractions and a lot about operations. Depth means that when students study ratios, they go well beyond solving proportions to recognize and utilize proportional relationships in many apparently different settings (like scale drawings, percent increase, sales tax, maps, multiplicative growth and even early notions of mathematical functions), in the process helping them connect mathematical ideas from prekindergarten through grade 12. Depth also means that students earning credit for a high school algebra course not only know how to solve equations but also have built a strong base of algebraic thinking and have developed a diverse set of algebraic tools with which they can represent, model, and solve many kinds of problems—both within and outside of mathematics.

WHAT DEPTH IS NOT

Depth does not mean making all students master arithmetic procedures faster, earlier, or with more digits. A school, school system, or state that calls for students to master fraction operations earlier than another

system does not necessarily have a more rigorous program. Depth does not mean narrowing our curriculum to numbers and operations alone at the expense of measurement, geometry, and data analysis, areas where those numbers and operations are actually used and where students learn critical non-numerical ways of thinking as well. Depth does not necessarily mean more exercises. Focusing on more or longer arithmetic procedures at the expense of deeper explorations and problem solving is not the same as raising our expectations for all students. And depth does not have to be painful or boring.

WHERE DEPTH IS SHALLOW

Some of the students least likely to experience deep mathematics are those identified as members of special populations. In many cases, students whose first language is not English, students who have learning disabilities, students with physical limitations, students in special education programs, and students who lag behind for whatever reason may receive the narrowest mathematics teaching we have to offer, often limited to numerical facts and procedures or only what will appear on the state test. Compassionate teachers may decide that students who have difficulty dealing with language should be limited to working on strictly numerical exercises involving no words. Consequently, many of these students never see a challenging problem or an engaging context simply because their teachers have determined that it would be too frustrating for them. We cannot be surprised when such students perform poorly on tests of higher-level thinking if their experience has never prepared them to think. These students, especially, need strong support and nurturing that goes well beyond arithmetic, even hard arithmetic.

What Can We Do?

In schools I visit, I continue to find wonderful examples of classrooms where students are learning mathematics in depth. In these classrooms, students experience mathematics taught with strong understanding. They are actively engaged in meaningful and challenging mathematics that may include but go well beyond arithmetic. In these schools, the door is open for every student to meet rigorous mathematics expectations, and students are naturally more engaged in the problems they see and the mathematics they learn.

Depth is not the same as hard arithmetic. Teaching for depth means that students need to wrestle with mathematical ideas in problems that take some time to solve, and they need opportunities to represent and communicate what they learn.

Mathematics standards today are more focused and balanced than ever, no longer presenting a "mile-wide" curriculum. If we take the

opportunity provided by more focused standards to also make shifts in how we structure our classrooms and engage our students, we can ensure that all students will understand mathematics deeply, well beyond "an inch deep," and that they can use this deep and lasting understanding to solve any problems they encounter in mathematics classes or beyond.

Reflection and Discussion

FOR TEACHERS

- What issues or challenges does this message raise for you? In what ways do you agree with or disagree with the main points of the message?
- If your state or school system is shifting its curriculum and standards toward deep mathematics rather than hard arithmetic, how can your teaching support this shift?
- What will it take to help all students meet high standards for mathematics? What barriers might keep students from reaching these standards, and how can you tackle these barriers?

FOR FAMILIES

- What questions or issues does this message raise for you to discuss with your son or daughter, the teacher, or school leaders?
- In what ways can you support your daughter or son at home in learning more than hard arithmetic?
- What kind of problems might you present to your son or daughter that require the use of thinking and reasoning skills?
- How can you support the teacher or school in teaching deep mathematics?

FOR LEADERS AND POLICY MAKERS

- How does this message reinforce or challenge policies and decisions you have made or are considering?
- How well does your mathematics program incorporate more than hard arithmetic, especially for students who are behind or who have special needs?
- What kinds of professional learning opportunities and support can you provide to help teachers strengthen their knowledge of mathematics and expand their teaching repertoire to go beyond numerical skills?

RELATED MESSAGES

Faster Isn't Smarter

- Message 15, "Less Can Be More," looks at the problems with our "mile-wide" mathematics curriculum.
- Message 32, "Yes, but . . . ," considers the reasons why we think some students do not reach high standards.

Smarter Than We Think

- Message 16, "Let It Go . . . ," reminds us that in order to have time for teaching deep mathematics, we may need to let go of topics and skills that are no longer as important for a particular grade level or course.
- Message 45, "Math Is *Supposed* to Make Sense," emphasizes the importance of students making sense of the mathematics they learn based on deep understanding.
- Message 12, "Upside-Down Teaching," offers a teaching model focused on student engagement and discourse around rich problems as a means of developing deep and lasting understanding.
- Message 31, "Developing Mathematical Habits of Mind," describes the kind of deep mathematical thinking advocated in this message.

MORE TO CONSIDER

- *About Teaching Mathematics, A K–8 Resource, Fourth Edition* (Burns 2015) offers a vision of a balanced mathematics program in which computational facility is one part, along with developing number sense and solving problems.
- "Evidence of Deeper Learning Outcomes: Report #3 Findings from the Study of Deeper Learning: Opportunities and Outcomes" (Zeiser et al. 2014) considers how to help students achieve deep, lasting learning of mathematics, rather than temporary learning.
- *The Schools Our Children Deserve: Moving Beyond Traditional Classrooms and "Tougher Standards"* (Kohn 2000b) addresses issues related to recent trends to focus on basics and tests, including a chapter with the subtitle "Confusing Harder with Better."
- *What's Your Math Problem? Getting to the Heart of Teaching Problem Solving* (Gojak 2011) considers the importance of giving students rich, nonroutine problems without having first taught students exactly how to solve them and offers classroom strategies for helping students learn mathematics meaningfully through their work with such problems.
- "Takeaways from Math Methods: How Will You Teach Effectively?" (Bay-Williams 2014) offers three big ideas for teaching toward student thinking.

- *How to Differentiate Your Math Instruction: Lessons, Ideas, and Videos with Common Core Support, Grades K–5* (Dacey, Bamford Lynch, and Salemi 2013) and *Math for All: Differentiating Instruction, Grades 6–8* (Dacey and Gartland 2009) are two resources that offers tasks, strategies, and research-based guidance for teaching deep mathematics to students of varying abilities, interests, and learning styles.
- *Accessible Mathematics: Ten Instructional Shifts That Raise Student Achievement* (Leinwand 2009) suggests classroom shifts teachers can make that lead to deeper student learning in mathematics.
- *Beyond Arithmetic: Changing Mathematics in the Elementary Classroom* (Mokros, Russell, and Economopoulos 1995) helps teachers examine and improve their mathematics teaching around richer problems that engage students more than traditional exercises, including guidance on evaluating or assessing this different kind of teaching and learning.
- "Why Is Teaching with Problem Solving Important to Student Learning?" (Cai and Lester 2010) summarizes research findings about the role of problem solving in the mathematics classroom.
- "Effective Teaching for the Development of Skill and Conceptual Understanding of Number: What Is Most Effective?" (Hiebert and Grouws 2007) presents a summary of research on developing understanding and proficiency with numbers across the grades, including suggestions for effective research-based practices for the classroom.
- *That Used to Be Us: How America Fell Behind in the World It Invented and How We Can Come Back* (Friedman 2011) makes a renewed call for investing in education that prepares workers of the future to think, analyze, and create.
- See also Appendices A and B on finding, selecting and evaluating in-depth tasks and Appendix C for several resources on teaching around problem solving listed as part of the Essential Library.