Shapes That Fit Many Categories

Analyzing student responses  
With commentary added by Andrew Baxter

(Adapted from pp 66-67 in the “Picturing Polygons” book in the “Investigations in Number, Data, and Space” series. Intended for Grade 5.)

How can we tell how well a student understands something? Ask them! But then what do we do with their answer?

Consider following questions (asked of Grade 5), and analyze their responses for quality. Discuss your impression with your group. Some responses are complete, and some are incomplete. Incomplete answers may be too broad, too narrow, too vague, or too unclear (perhaps due to imprecise language).

**Question 1. A square is a kind of rhombus. How can this be?**

1. Because it has four equal sides.  
   *Grammar issues aside, it is an excellent sign this student is writing in terms of the appropriate properties.*
2. The sides on the rhombus are tilted  
   *The student appears to give a reason for why a rhombus is different from a square, not why a square is a kind of rhombus.*
3. Both have parallel and equal sides  
   *This student is also writing in terms of appropriate properties, which is a good sign they are understanding at the appropriate level.*
4. The way it’s made is how a rhombus is made  
   *Too vague. There may be a seed of understanding that can be brought to bloom if one can pick apart what they mean by “made.”*

**Question 2. Name all the shapes in the picture below that are parallelograms. How can they be parallelograms and have other names as well.**



1. Square, rhombus, rectangle. Everything is the same except the angles are different.  
   *It is strange they left out parallelogram from being a parallelogram. It is unclear what “everything is the same” means.*
2. Rectangle and rhombus because they have two equal sides.  
   *They excluded the square from being a parallelogram, although it is not clear whether this was an oversight or intentional. Their reasoning for why a quadrilateral (“two equal sides”) is not enough to conclude the shape is a parallelogram - there are plenty of shapes with two equal sides, or even two pairs of equal sides, that are not parallelograms, e.g., a kite.*
3. All of them. The opposite sides are parallel like a parallelogram.  
   *They are writing in terms of appropriate properties. They have identified the most important feature that makes a shape a parallelogram (parallel sides). A truly complete answer would address the second half of the question, explaining why these shapes also have other names.*
4. Rectangle. A parallelogram looks like a rectangle but it is tipped looking.  
   *Like reply B in the previous question, this student appears to be giving a reason for why the rectangle is not a parallelogram since they insist on its being “tipped” looking. The student does not appear to be thinking in terms of appropriate properties of the shapes, but is still basing everything on the visual level.*
5. Rhombus because it’s tilted.  
   *This student is perhaps thinking in terms of properties (“tilted”), but not appropriate mathematical properties. Other shapes could be described as tilted that are not parallelograms, and there are parallelograms that are do not look tilted.*

**Question 3. An equilateral triangle is isosceles. How can this be?**

1. Because maybe they have long sides or the triangle is thinner.  
   *This appears to explain why an isosceles triangle might not be an equilateral triangle, rather than what the question asks.*
2. They both have at least two equal sides.  
   *The student is writing in terms of appropriate properties.*
3. Because an isosceles is just a little bit taller than an equal triangle.  
   *This appears to explain why a isosceles triangle is cannot be an equilateral (=”equal”?) triangle (because the isosceles is taller). This student is thinking in purely visual terms and not thinking about properties.*
4. They both have equal sides.  
   *While the phrasing leaves something to be desired, the student appears to be thinking in terms of the appropriate property of “pairs of equal sides.”*

**Question 4. Some obtuse triangles are scalene. Some obtuse triangles are isosceles. Sketch examples of each.**

1.   
   *A perfectly minimal and well-labeled demonstration of examples of each class.*
2.   
   *Yes they have examples of each requested type, but they also have this third triangle that is neither. Since nothing is labeled, it is hard to know what they student intended.*
3.   
   *This student has just drawn two different scalene obtuse triangles. Perhaps the student thinks that all obtuse triangles are scalene, which is a common student misconception (or a limit of imagination).*

**Question 5. Obtuse triangles cannot be equilateral. Explain why this is true.**

1. Since all triangles have to have three sides, if you made an equilateral triangle it would not connect. [Includes picture below.]  
      
   *This student is thinking in terms of properties. They know that the attempted equilateral triangle must have three equal sides and starts with those. They then create an obtuse triangle with two of them and find the third side unable to connect and close the triangle.*
2. An obtuse triangle has no equal sides but an equilateral does.  
   *If it were true that obtuse triangles had no equal sides then the reasoning would hold up. The problem is that obtuse triangles can have equal sides, just not three of them. Perhaps this student thinks all obtuse triangles are scalene.*
3. An equilateral triangle is where the vertices are all the same. An obtuse angle cannot fit more than once in a triangle.  
   *This student mixes up equilateral with equiangular, although that is perhaps forgivable in triangles where those two ideas always coincide. The student shows an understanding of the angle sum of a triangle, which is a good sign.*
4. Obtuse triangles must have at least one side that is longer. [Includes picture below.]  
     
   *This student shows a good understanding of how properties interact. They shows an understanding of how larger angles make the side opposite longer. They then know that such a longer side is a different length than others, which contradicts equilateral.*
5. Because obtuse is one angle bigger than 90°, there can’t be 2 bigger than 90°, and an equilateral triangle has all the same angles.  
   *This is essentially the same argument as C above.*