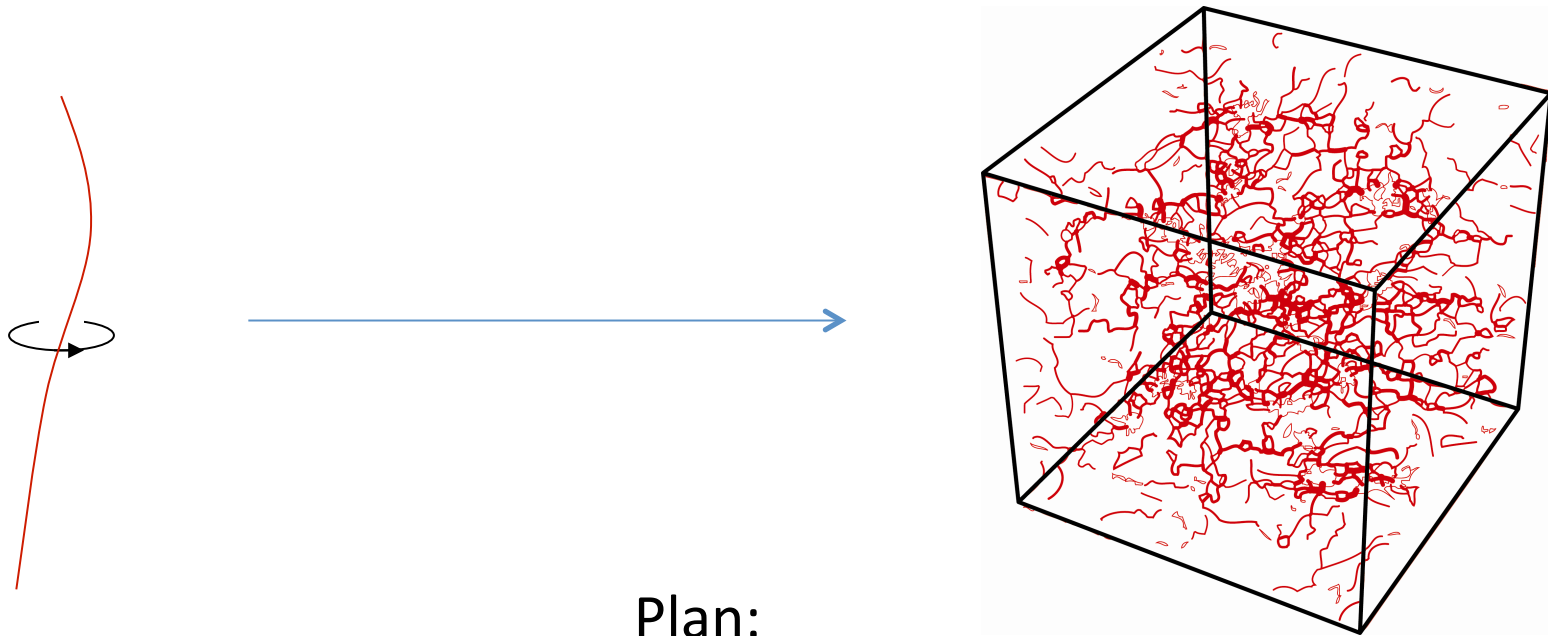


Quantum Turbulence (QT) in the $T=0$ limit ($T < 0.5\text{K}$ for ^4He , $T < 0.1\text{-}0.3\text{mK}$ for $^3\text{He-B}$)

Challenge: with known properties of a vortex line at $T=0$, *understand* behaviour of a vortex tangle



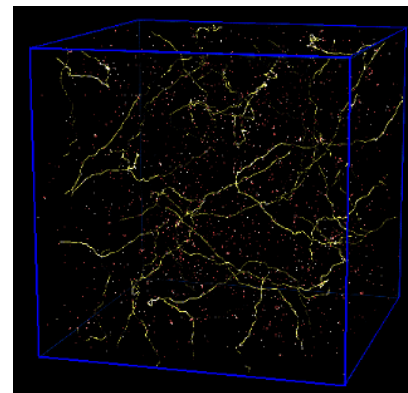
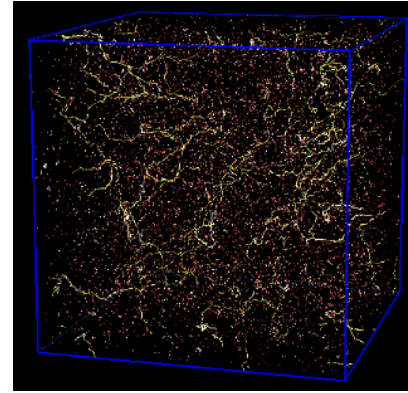
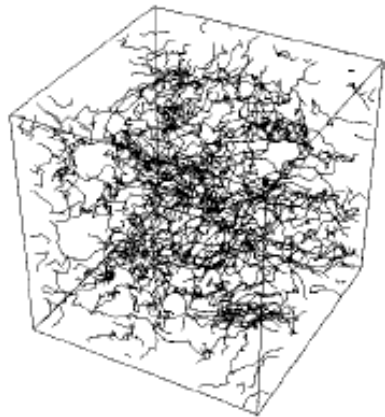
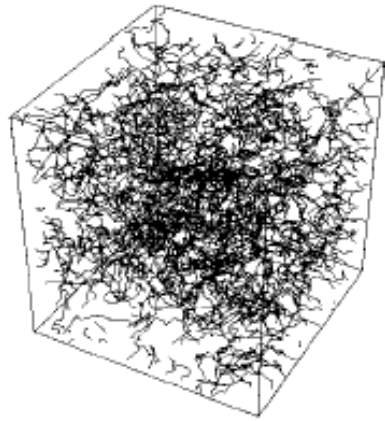
Plan:

1. Outline current understanding and its *loose ends*;
2. Discuss existing and *desired* experimental techniques.

Motivation: a new type of turbulence

Relevance to other problems:

- QT can mimic classical turbulence (e.g. large Re)
- QT has features of wave turbulence (Kelvin waves)
- Analogies with other line defects: dislocations, Abrikosov vortices, cosmic strings



Computer simulations of the evolution of a vortex tangle (Tsubota, Araki, Nemirovskii)

Computer simulations of the evolution of cosmic strings (Cambridge)

Descriptions of vortex dynamics

$R \approx \xi$. Microscopic level

Gross-Pitaevskii equation for superfluid

$$i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2M} \nabla^2 \Psi + V_0 |\Psi|^2 \Psi - E_0 \Psi$$

$R \gg \xi$. Intermediate level (filament model)

$$s(\zeta, t) \quad s' = ds/d\zeta$$

- Vortex lines move with local \mathbf{u}_s
- \mathbf{u}_s is function of the configuration of the tangle
- Reconnections ad-hoc

$$\frac{ds}{dt} = \mathbf{u}_{si}$$

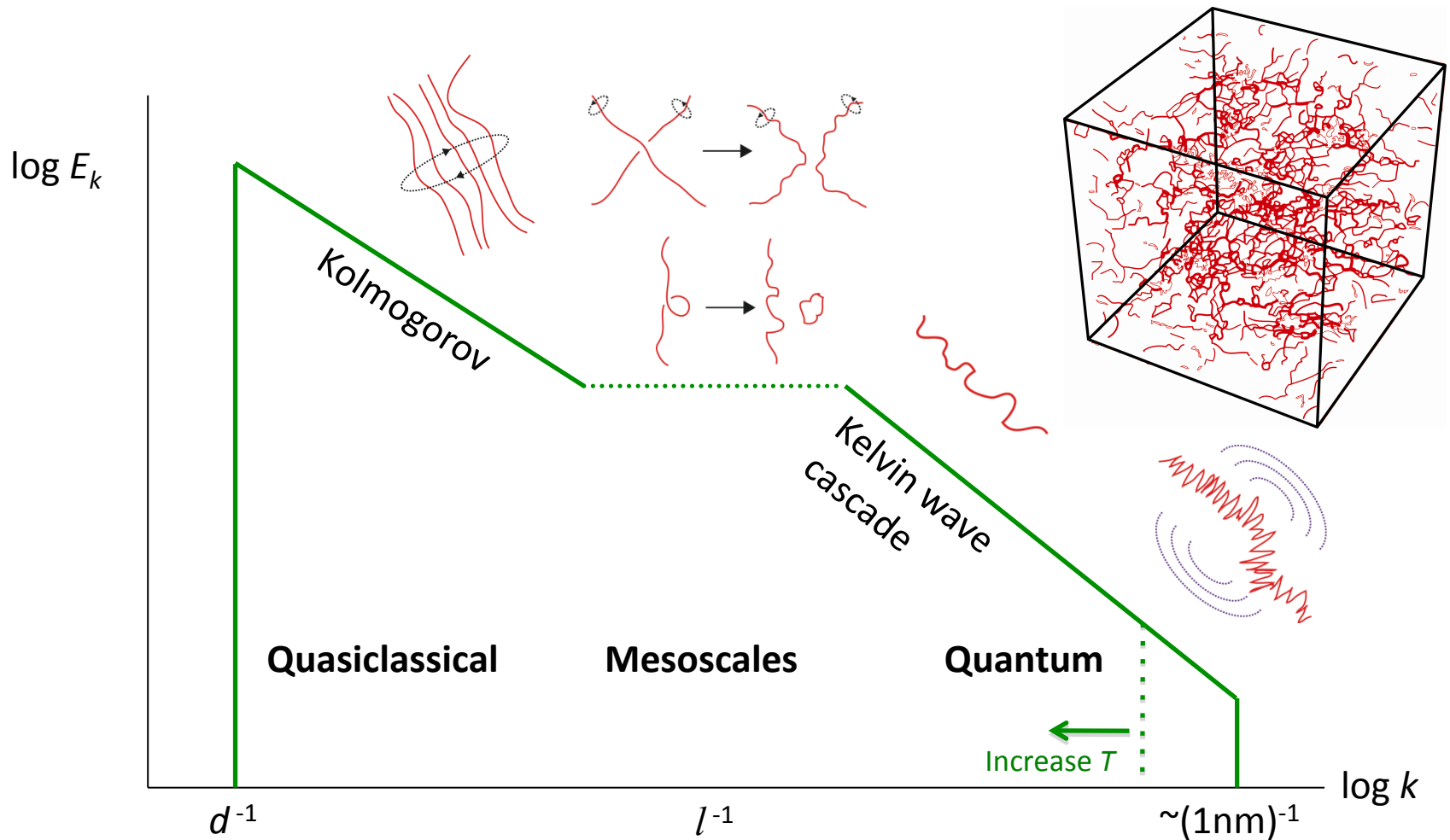
$$\mathbf{u}_{si}(\mathbf{s}) = \frac{\kappa}{4\pi} \oint_{\mathcal{L}} \frac{(\mathbf{s}_1 - \mathbf{s}) \times d\mathbf{s}_1}{|\mathbf{s}_1 - \mathbf{s}|^3}$$

$R \gg \ell$. Macroscopic level,

- Coarse-grained equations
- Cannot account for quantum cascade

$$\frac{\partial \mathbf{v}_s}{\partial t} + (\mathbf{v}_s \cdot \nabla) \mathbf{v}_s = -\frac{1}{\rho} \nabla p + f(L, ?)$$

QT at $T=0$ was studied since 2000. Our *beliefs* after 15 years:

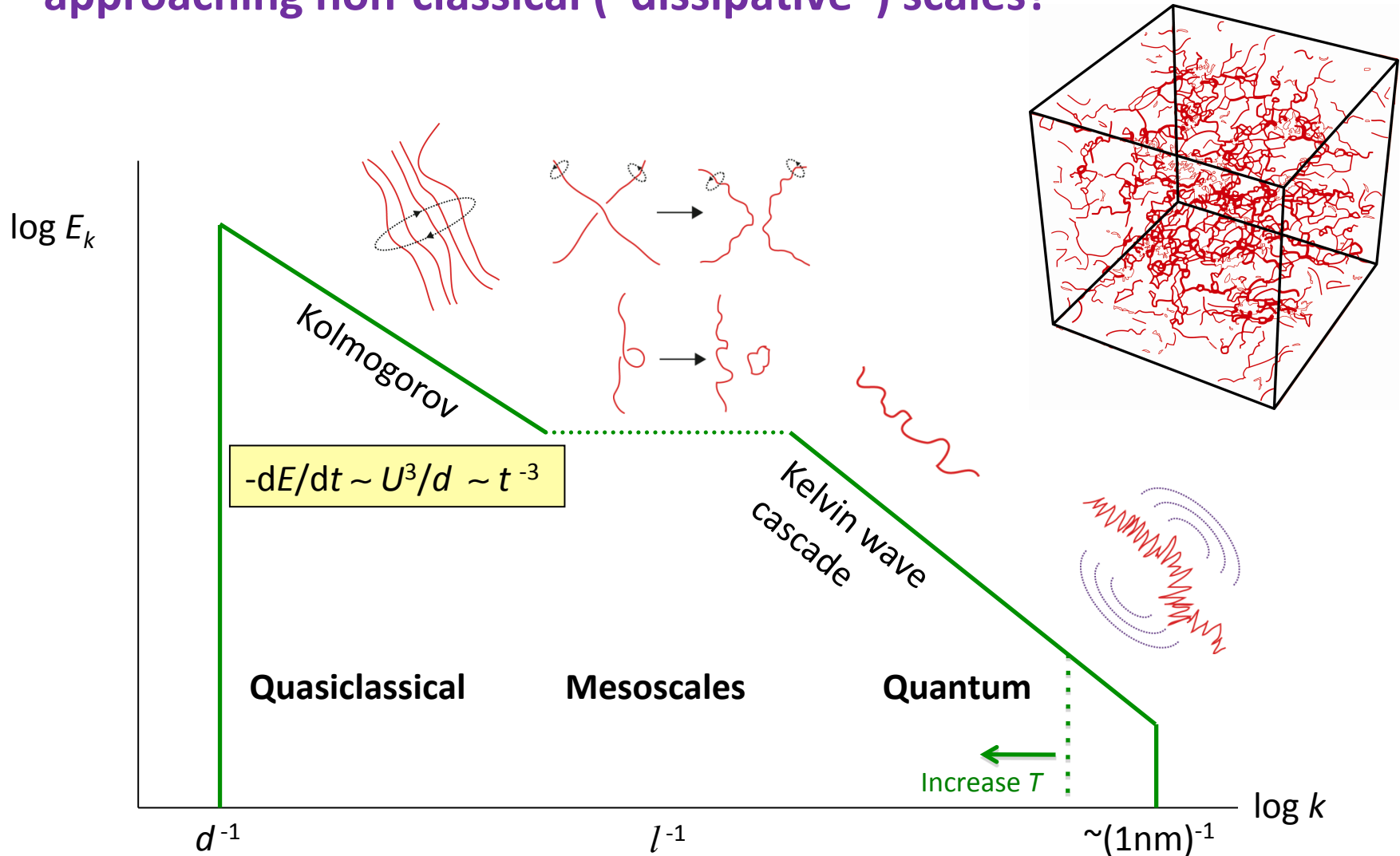


Assumed energy spectrum of QT (HIT)

1. Quasi-classical behaviour:

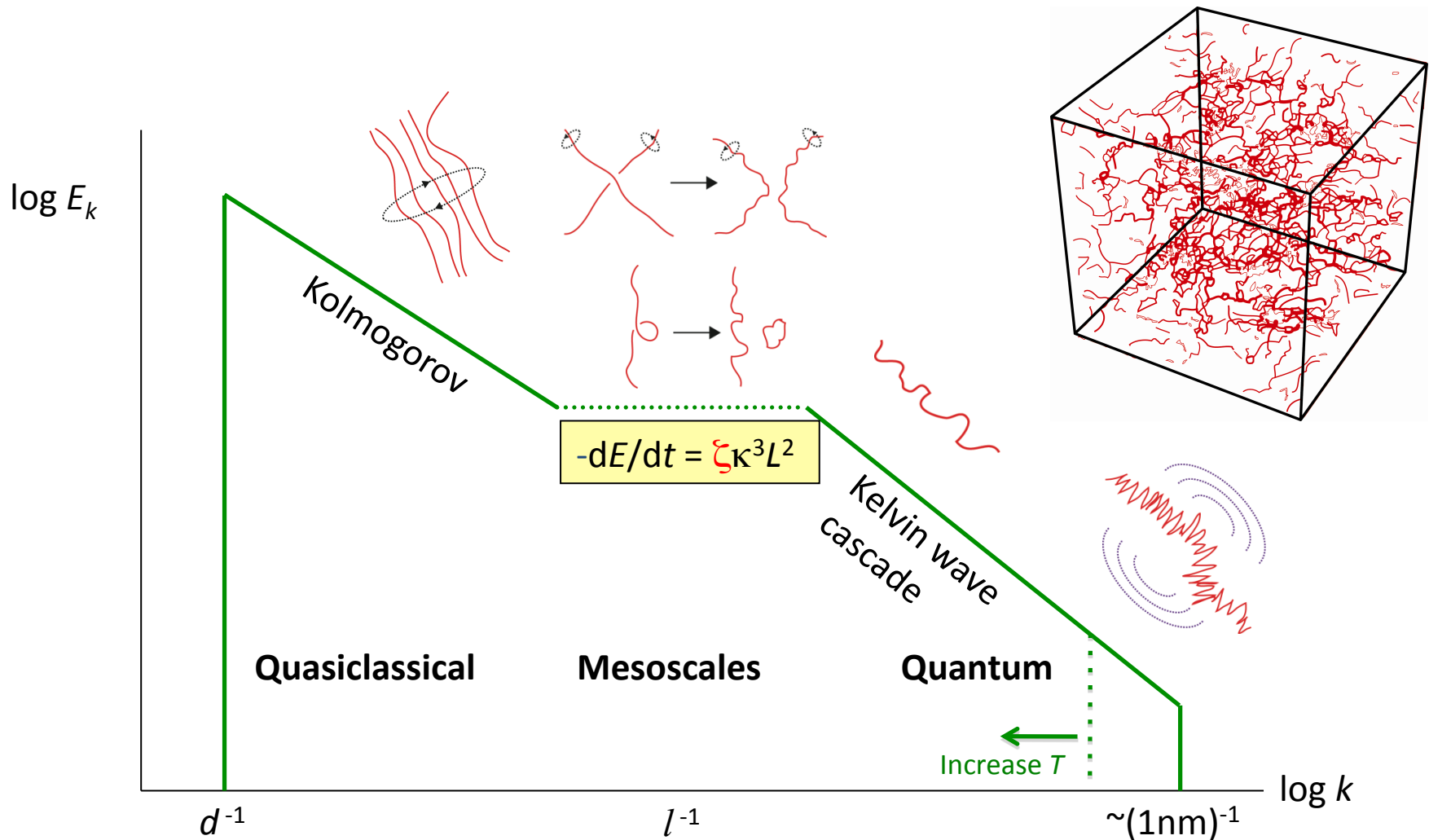
Can coarse-grained flow be described by quasi-classical equations:

- at the generation stage (vortex nucleation/multiplication)?
- approaching non-classical ("dissipative") scales?



2. QT at quantum length scales

- Which processes maintain energy cascade to non-classical scales?
- Dominant mechanism of energy removal?
- Derivation of the formula for the rate of energy removal?



Dissipation rate: Classical vs. Quantum

Classical fluid:

Inertial cascade; energy flux determined by Navier-Stokes equation

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} = -\frac{1}{\rho} \nabla p + \nu \nabla^2 \mathbf{v}$$

Dissipation at small lengths through shear viscosity (where ν is a material parameter):

$$\epsilon_c = \nabla \cdot (v_i \sigma_{ik}) + \sigma_{ij} \frac{\partial v_i}{\partial x_j} = \nabla \cdot (v_i \sigma_{ik}) + \frac{\nu}{2} \left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right)^2 \quad -\dot{\mathcal{E}}_c = \int \epsilon_c dV = \nu \int_V \omega^2 dV$$

$$dE/dt = -\nu (\text{curl } \mathbf{v})^2$$

Superfluid quasi-classical flow:

Inertial cascade; energy flux is, perhaps, determined by “quasi-Navier-Stokes” equation

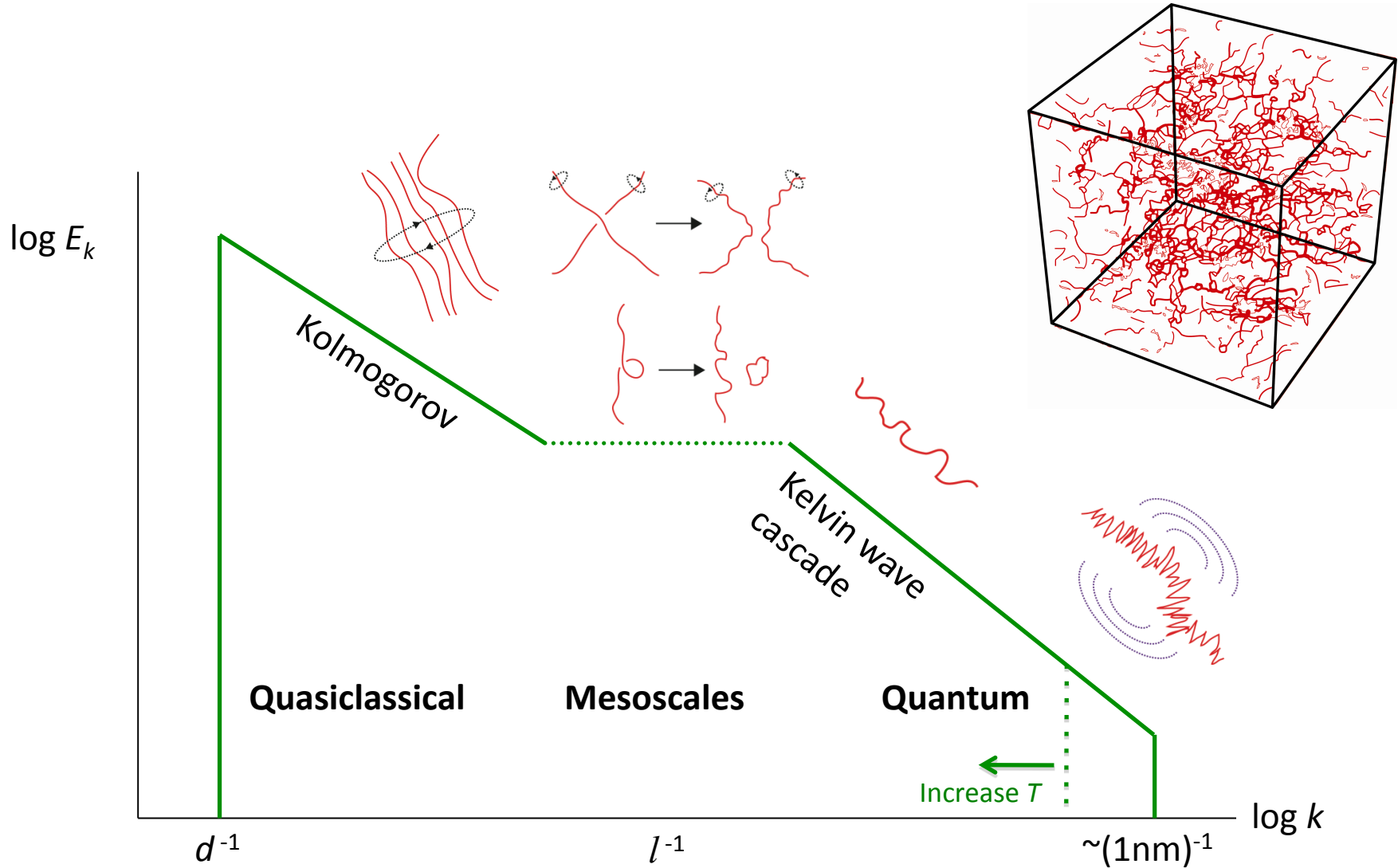
$$\frac{\partial \mathbf{v}_s}{\partial t} + (\mathbf{v}_s \cdot \nabla) \mathbf{v}_s = -\frac{1}{\rho} \nabla p + f(L, ?)$$

Energy flux, maintained by vortex lines (where $\nu' = \zeta \kappa$ is an effective parameter, $\zeta \sim 1$):

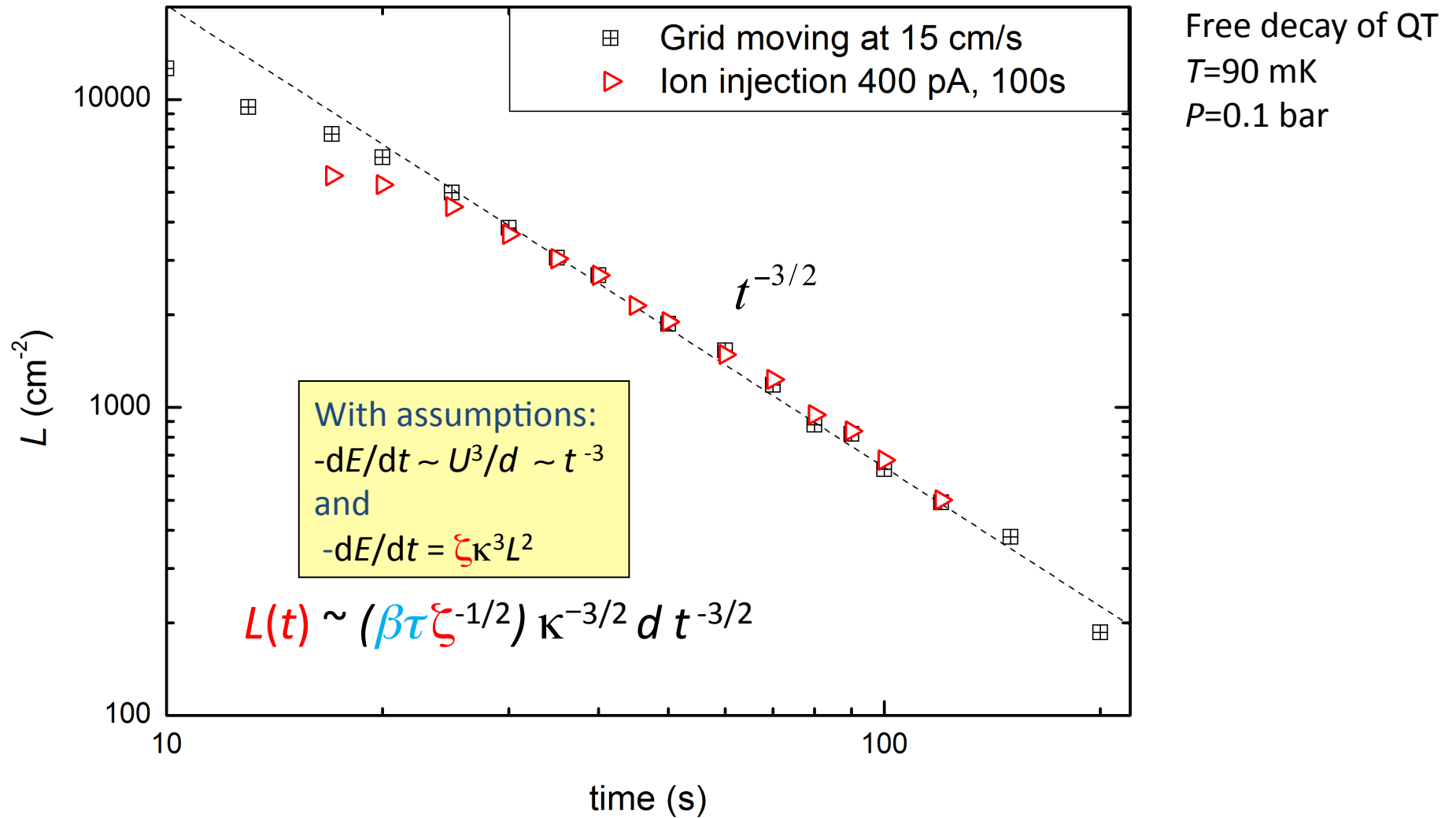
$$\text{Assumption: } dE/dt = -\nu' \kappa^2 L^2$$

3. Role of solid boundaries in confined QT?

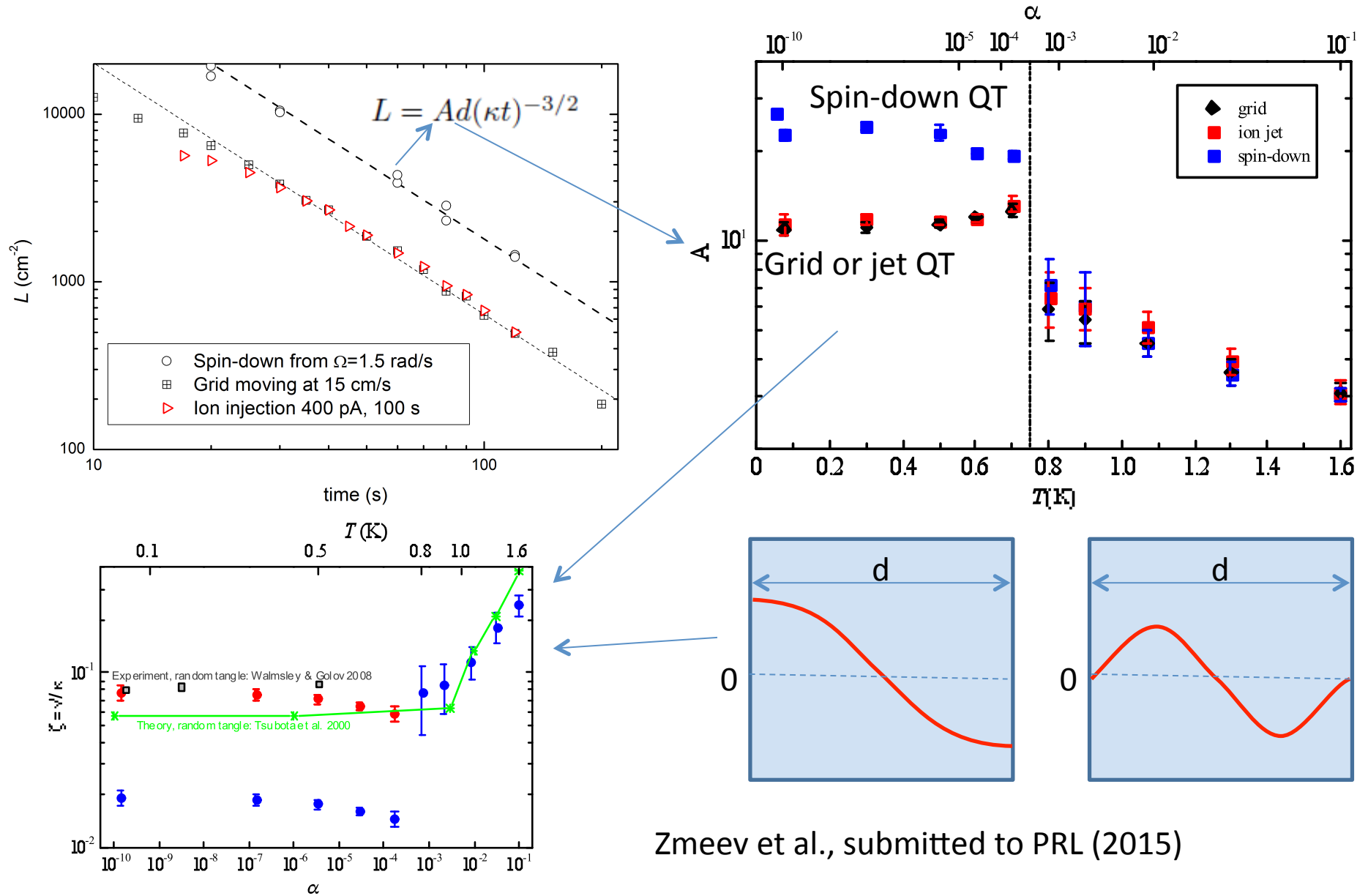
- Effective boundary conditions: 'slip' or 'no-slip'?



Circumstantial evidence for quasi-classical cascade



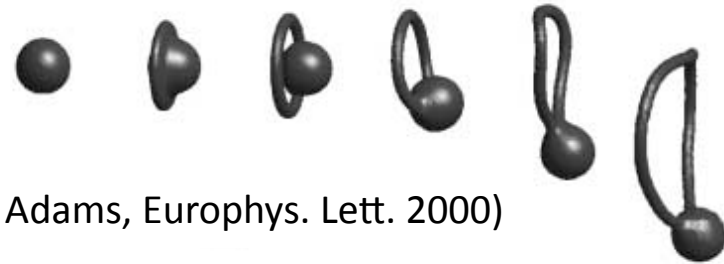
At $T < 0.8\text{K}$, spin-down turbulence differs from HIT: long-lived angular momentum



Zmeev et al., submitted to PRL (2015)

Our Detection Method: Scattering of Charged Vortex Rings off Vortex Lines

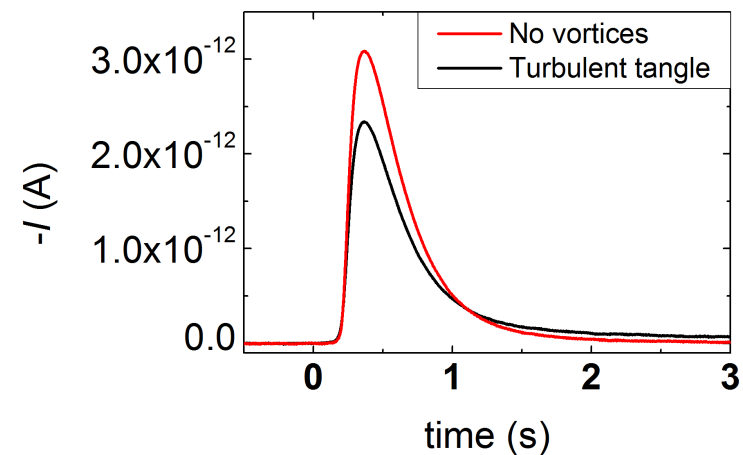
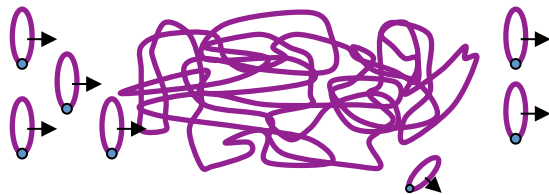
In helium at $T < 0.7\text{K}$, an electron (inside bubbles of $R \sim 20\text{\AA}$) nucleates a vortex rings and travels with it.



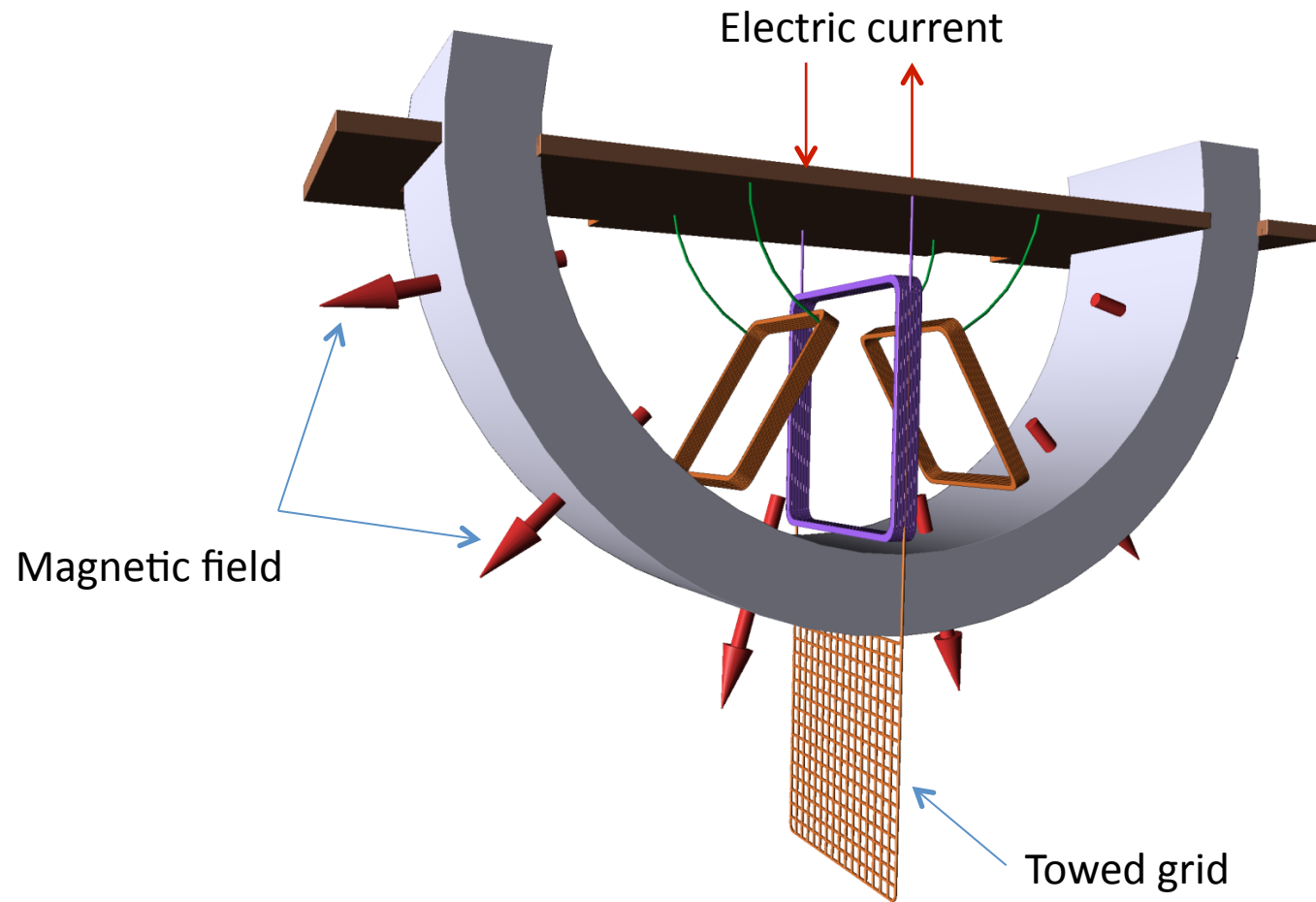
(image from Winiecki and Adams, Europhys. Lett. 2000)

Charged vortex rings of suitable radius ($\sim 1\ \mu\text{m}$) are used as detectors of vortex length L

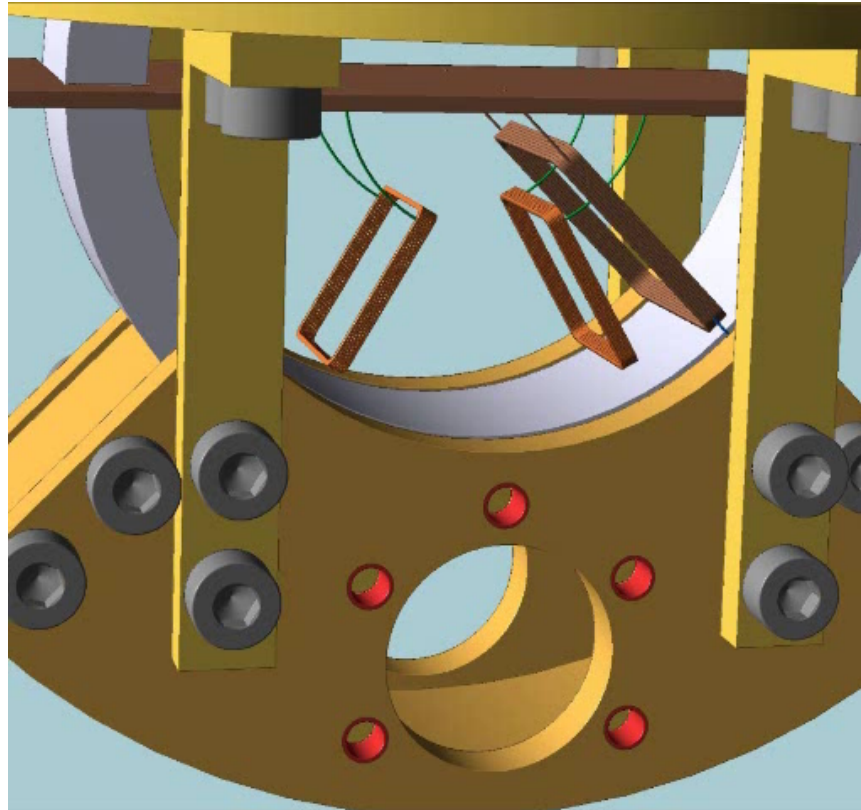
L can be calculated from attenuation of the electric current due to CVRs



Towed Grid that works at $T < 100\text{mK}$



Towed Grid



We monitor the free decay of the turbulence after towing the grid through the channel.

Zmeev, J. Low Temp. Phys (2014)

Need for new experiments

1. Generation (existing):

- vibrating objects (Lancaster, Osaka, Kharkov)
- non-steady rotation of a cryostat (Manchester, Helsinki)
- ion jet (Manchester)
- towed grid (Manchester) – talk by Zmeev at QFS

Generation (desired):

- steady flow
- simple flow (pipe, piston, classical vortex ring , or straight vortex (tornado))

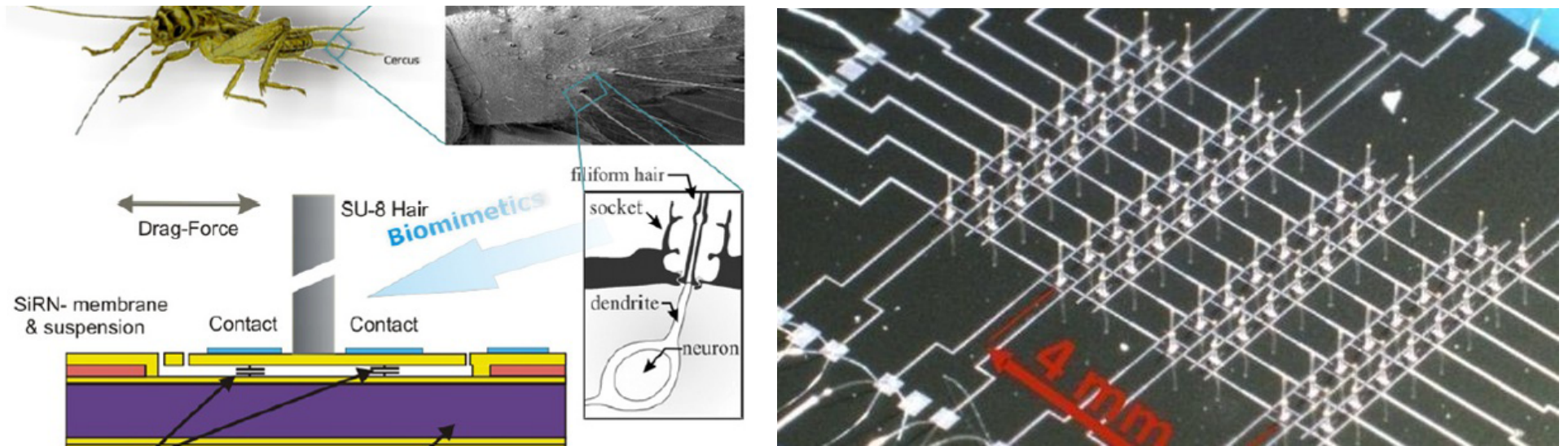
2. Detection of vortex lines (existing):

- Interaction with vibrating objects (Lancaster, Osaka)
- Scattering of quasiparticles in $^3\text{He-B}$ (Lancaster)
- NMR in $^3\text{He-B}$ (Helsinki)
- Scattering of charged vortex rings (Manchester) – talk by Golov at QFS
- Transport of trapped ions (Manchester) – talk by Walmsley at QFS

Detection (desired):

- Mapping velocity, vorticity, pressure fields
- Measuring drag on immersed objects and traction at solid walls
- Visualisation of individual vortices (seeded by fluorescent nanoparticles)

For inspiration:



“Towards a high-resolution flow camera using artificial hair sensor arrays for flow pattern observations”

Dagamseh et al., *Bioinspir Biomim.* **7**, 046009 (2012), Univ. of Twente