Symmetry Protected Topological Superfluids and Superconductors

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Topological Materials Science

トポロジーが紡ぐ物質科学のフロンティア

科研費 KAKENHI

OUTLINE

1. Topology and symmetry of ^{3}He: Overview
2. BW state: P_3 symmetry & SSB-induced TPT
3. ABM & planar states: Prototype of topological SCs
4. Challenges for theory & experiment

Review: **TM**, Y. Tsusumi, T. Kawakami, M. Sato, M. Ichioka, K. Machida, arXiv:1508.00787
TM, Y. Tsusumi, M. Sato, K. Machida, J. Phys.: Condens. Matter **27**, 113203 (2015)

Why ^{3}He

well-established knowledge on bulk

^{3}He as a “superclean” material

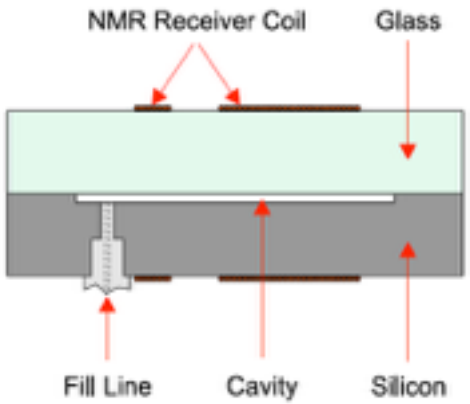
Huge symmetry group

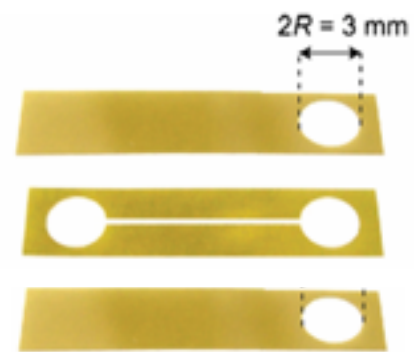
$$\mathrm{SO}(3)_S \times \mathrm{SO}_L \times \mathrm{U}(1)_\phi$$

⇒ a diversity of SSB

a variety of confinement geometries

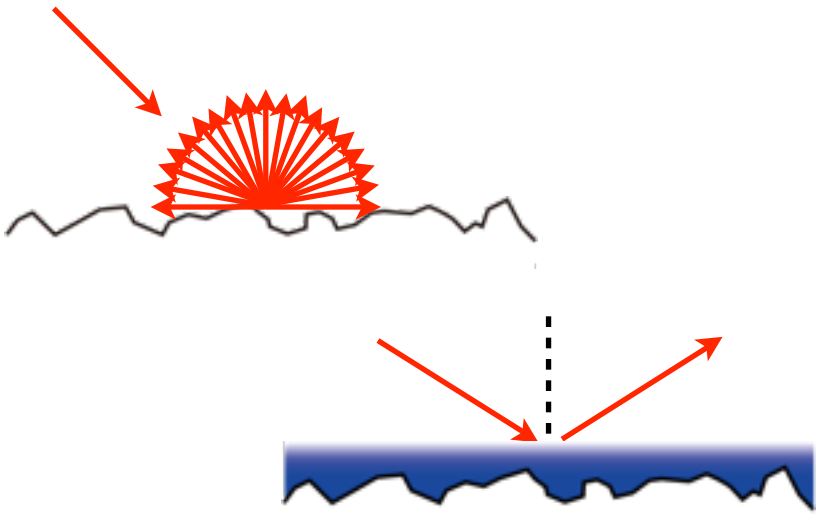
Bennett *et al.*, JLTP **158**, 163 (2010)





Yamashita *et al.*, JLTP **158**, 353 (2010)

Controllable surface conditions



Topology & Symmetry of ^{3}He

Symmetry group in normal state $G = \mathrm{SO}(3)_S \times \mathrm{SO}(3)_L \times \mathrm{U}(1) \times T$

spin rotation — orbital rotation — time-reversal symmetry

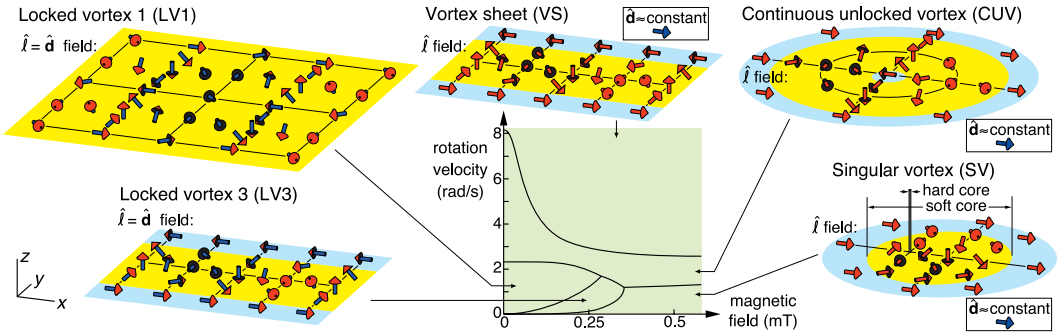
broken symmetry $R = G/H$

H **remaining symmetry**

Topological excitations: Cooper pairs

Huge order parameter manifold

==> Rich topology in *real* space



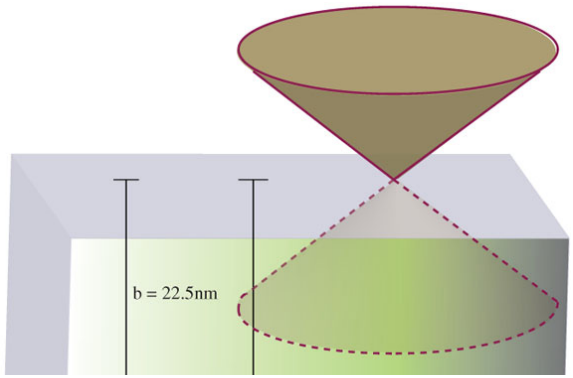
^{3}He: O. V. Lounasmaa and E. Thuneberg, PNAS **96**, 7760 (1999)

Topology of *fermions* (QPs)

Topology in *momentum* space

⇒ Majorana fermions in ^{3}He

Schnyder-Ryu-Furusaki-Ludwig, PRB '08
Qi-Hughes-Zhang, PRL '09; Volovik, JETP Lett '09,
Chung-Zhang, PRL '09; …



Symmetry Protected Topological Superfluid: Overview

Symmetry group in normal state $G = \mathrm{SO}(3)_S \times \mathrm{SO}(3)_L \times \mathrm{U}(1) \times T$

spin rotation | orbital rotation | time-reversal symmetry

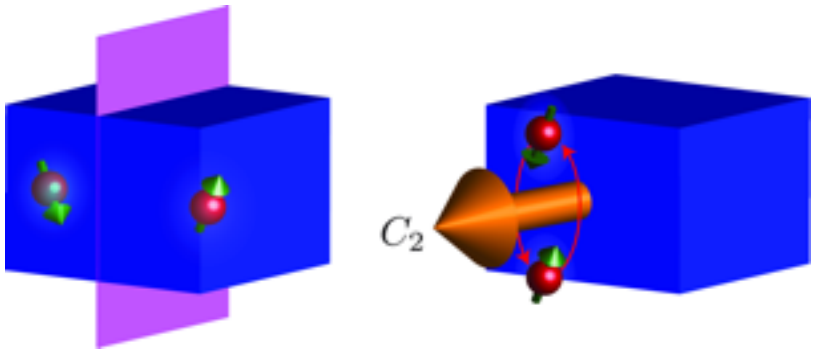
broken symmetry $R = G/H$

H

$R' = H/H'$ **broken symmetry**

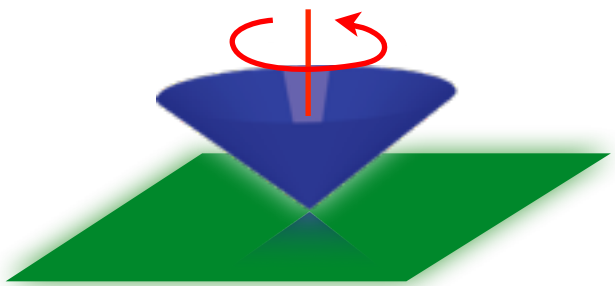
H'

P_2 P_3



mirror + TRS π-rotation + TRS

Salomaa and Volovik, PRB **31**, 203 (1985)



Symmetry-protected topological phase

Symmetry-broken non-topological phase

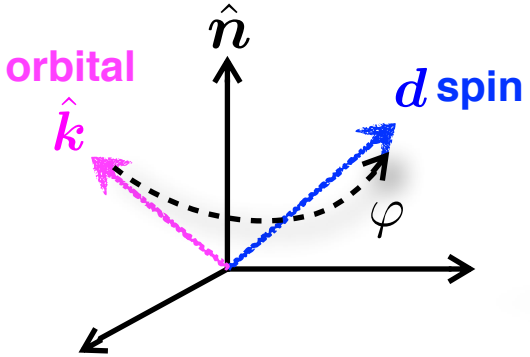
Spontaneous breaking of a discrete symmetry that protects topological invariant

SSB triggers off topological phase transition

Superfluid ^{3}He-B

BW state Balian and Werthamer, PR **131**, 1553 (1963)

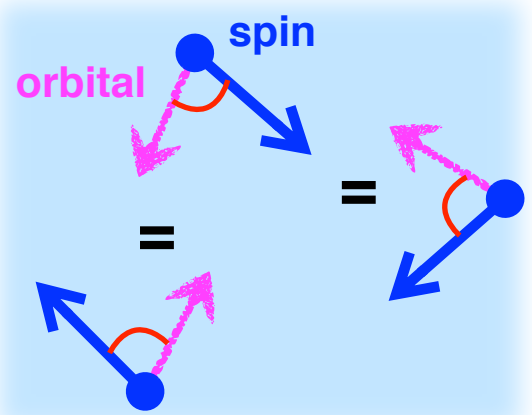
$$H = \mathrm{SO}(3)_{L+S} \times \mathcal{T} \times \mathcal{C}$$



OP: rotation matrix

$$R_{\mu i}(\hat{\boldsymbol{n}}, \varphi)$$

spin orbital

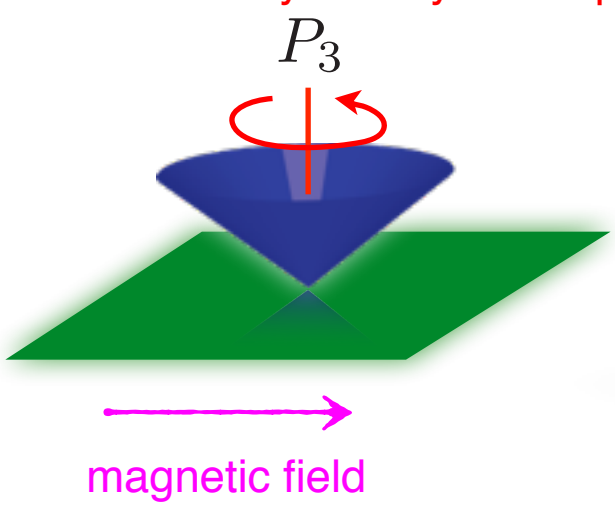


P_3 symmetry in the presence of a magnetic field

$$\mathcal{P}_3 \mathcal{H}(k_x, k_y, k_z) \mathcal{P}_3^{-1} = \mathcal{H}(k_x, k_y, -k_z) - \gamma H \hat{\ell}_z \sigma_z$$

symmetry breaking field ==> non-topological

P$_3$ symmetry ==> topological



The *P_3* symmetry is preserved when

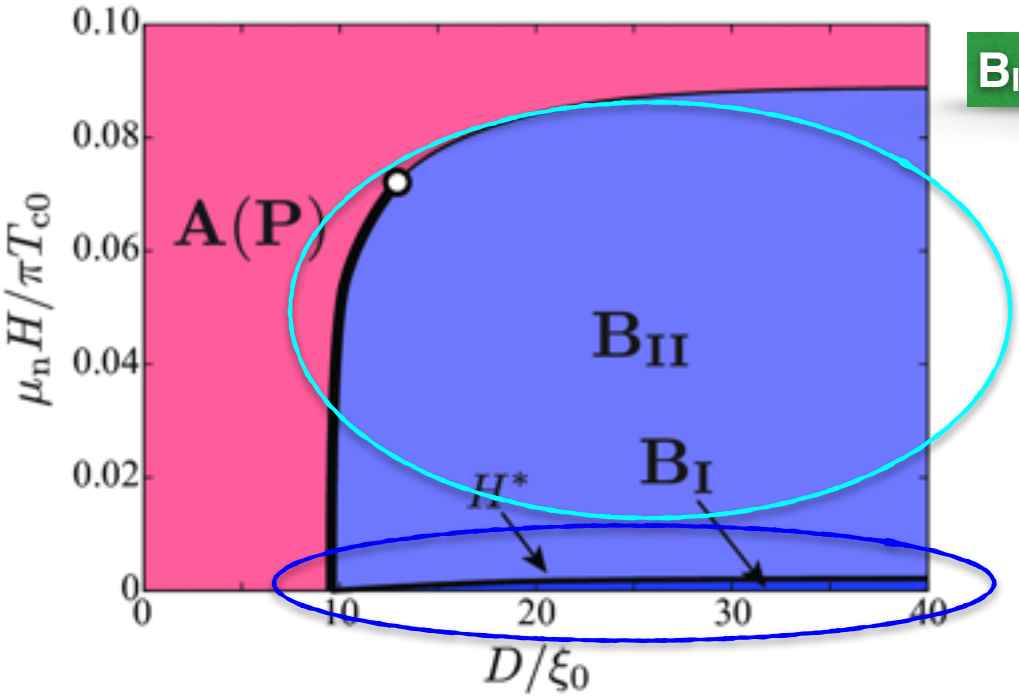
$$\hat{\ell}_z(\hat{\boldsymbol{n}}, \varphi) \equiv \hat{h}_\mu R_{\mu z}(\hat{\boldsymbol{n}}, \varphi) = 0$$

orientation of applied field

$SO(3)_{L-S}$

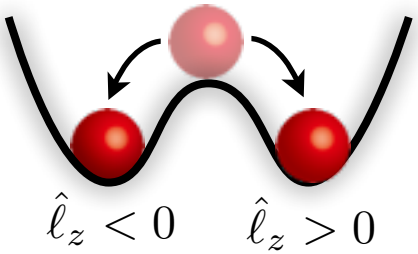
Topological Phase Diagram of ^{3}He-B

TM, M. Sato, and K. Machida, PRL **109**, 165301 (2012)

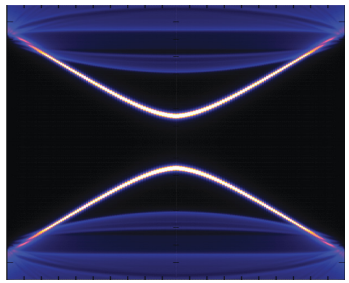


B_{II}: P_3-symmetry-broken non-topological phase

$$\hat{\ell}_z(\hat{\boldsymbol{n}}, \varphi) \neq 0$$

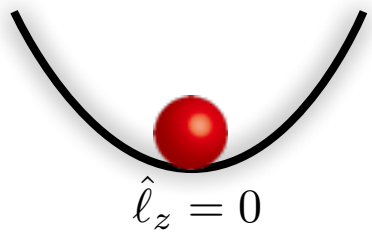


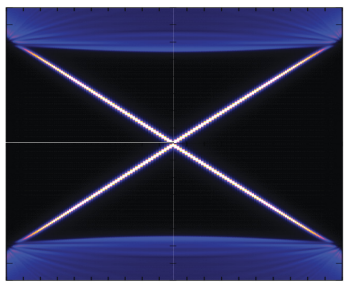
$$\mathcal{P}_3 : \hat{\ell}_z \mapsto -\hat{\ell}_z$$



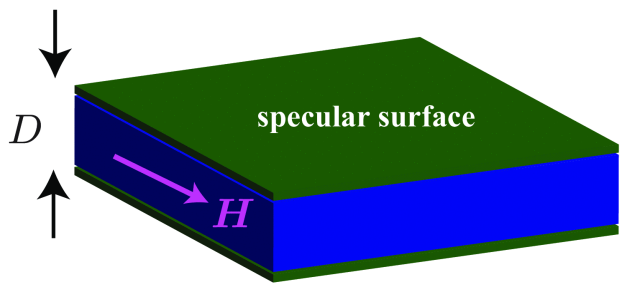
B_I: Symmetry protected topological phase

$$\hat{\ell}_z(\hat{\boldsymbol{n}}, \varphi) = 0$$





Spontaneous breaking of P_3 symmetry &
TPT w/o closing bulk gap



SSB-induced Mass Acquisition of Majorana Fermions

TM, M. Sato, and K. Machida, PRL **109**, 165301 (2012)

Surface Majorana fermions

$$\mathcal{L}_{\rm surf} = \frac{1}{2}\bar{\psi}_{\rm M}\gamma^{\mu}\partial_{\mu}\psi_{\rm M} + M\bar{\psi}_{\rm M}\psi_{\rm M}$$

effective mass associated w/ OP

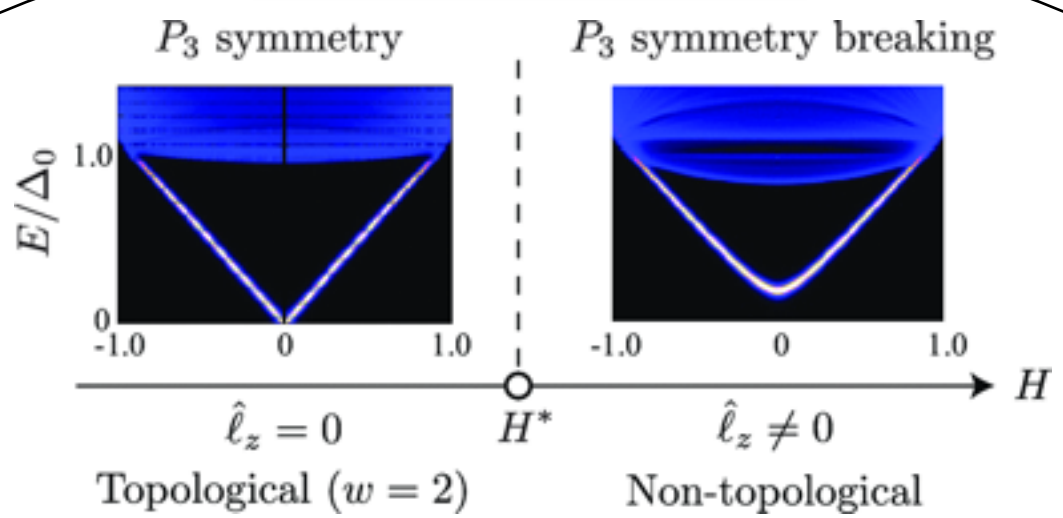
P_3 symmetry in the presence of a parallel field

$$M = \frac{\gamma H}{2}\hat{\ell}_z$$

$$\mathcal{P}_3\mathcal{H}(k_x,k_y,k_z)\mathcal{P}_3^{-1} = \mathcal{H}(k_x,k_y,-k_z) - \gamma H\hat{\ell}_z\sigma_z$$

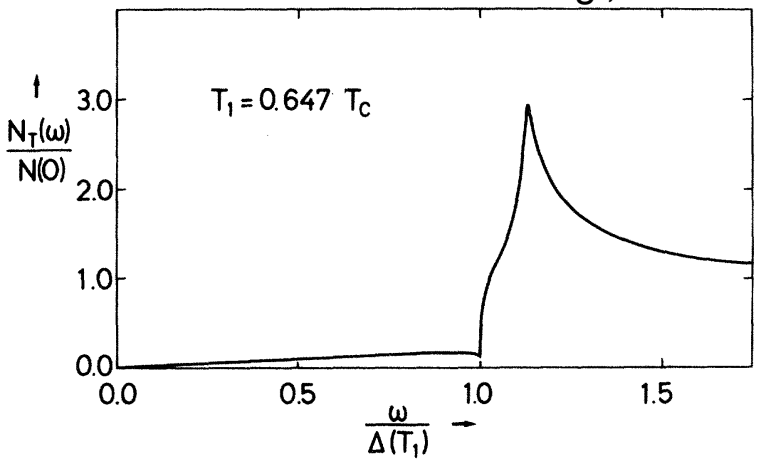
symmetry breaking field
"effective mass"

TPT w/o closing bulk gap



Majorana Fermions: A Special Kind of Andreev Bound States

Buchholtz and Zwicknagl, 1981



Majorara/Dirac fermion in SCs
= a spacial kind of **Andreev bound states**

Experimental observations of surface **Andreev bound states in ^{3}He-B**

- **Heat capacity measurement:**

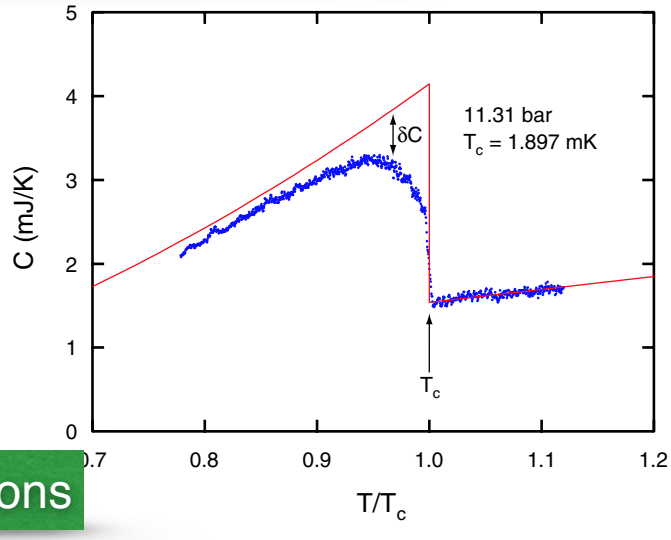
  Choi *et al.*, PRL (06); Y. Bunkov and R. Cazizulin, (15)

- **Anomalous attenuation of transverse wave:**

  J. P. Davis *et al.*, PRL (08)

- **Transverse acoustic impedance:**

  Murakawa *et al.*, PRL (09); JPSJ (11)



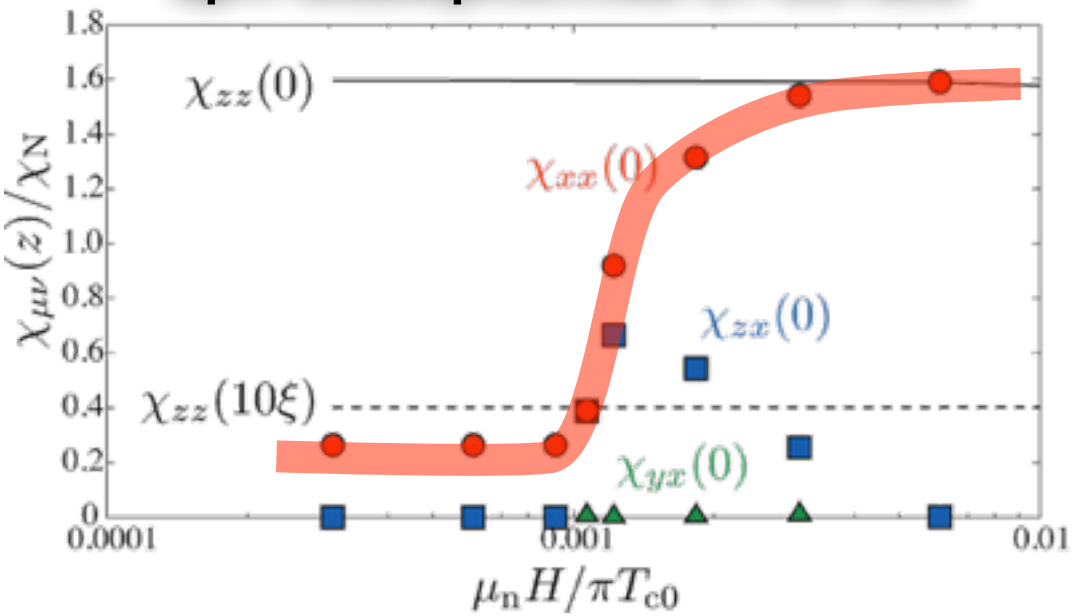
Direct observation of surface DOS w/ controllable surface conditions

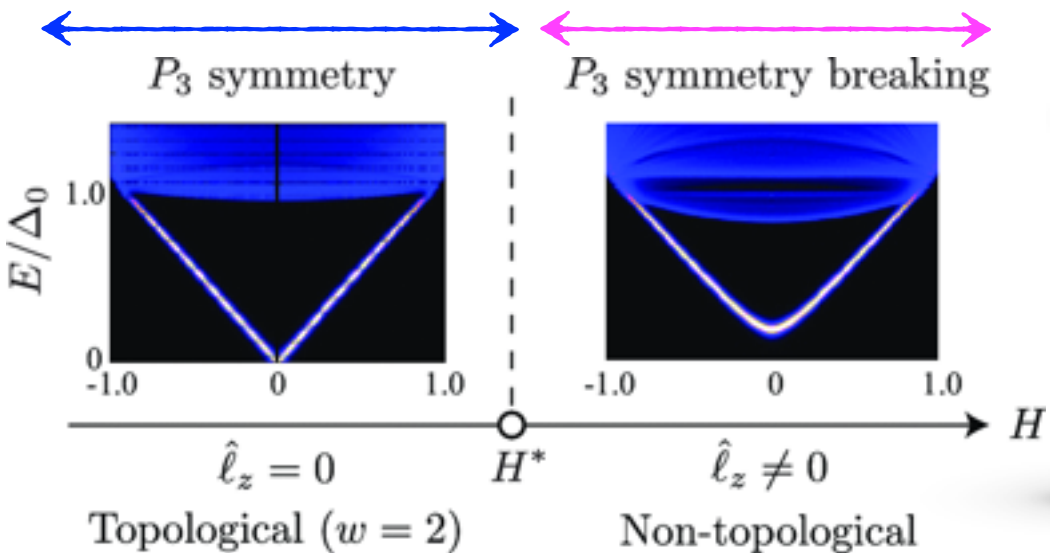
Hallmark of Majorana Fermions?

Majorana Ising spins: A hallmark of P_3 symmetric Majorana fermions

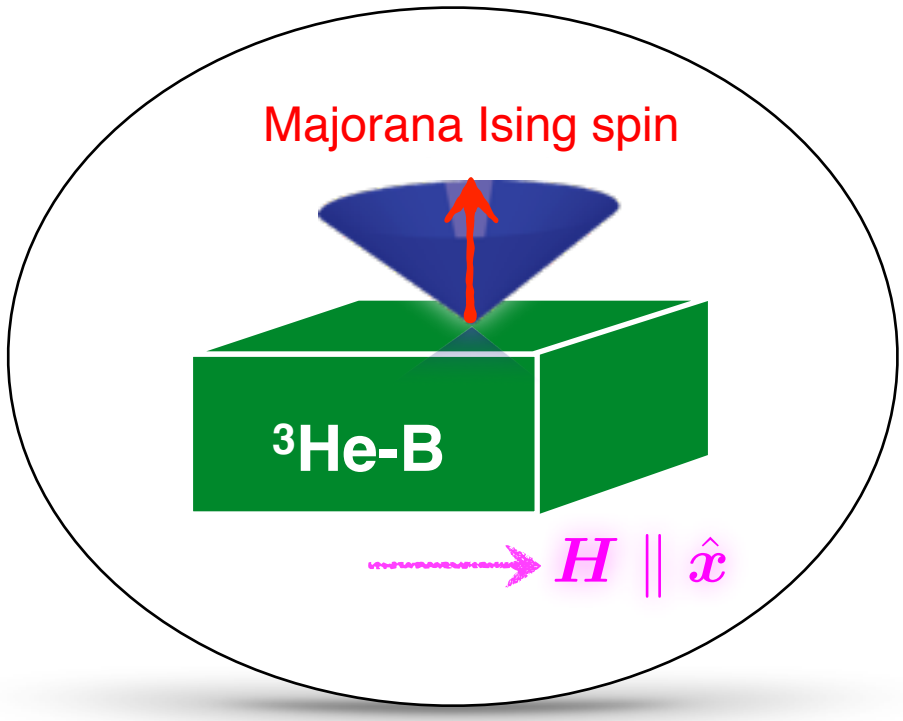
TM, M. Sato, and K. Machida, PRL **109**, 165301 (2012); **TM**, *et al.*, arXiv:1508.00787

Spin susceptibilities on surface





order-two magnetic point group sym



Chung-Zhang PRL (09)

B_I and B_{II} phases are detectable through the anomalous behavior of spin susceptibility

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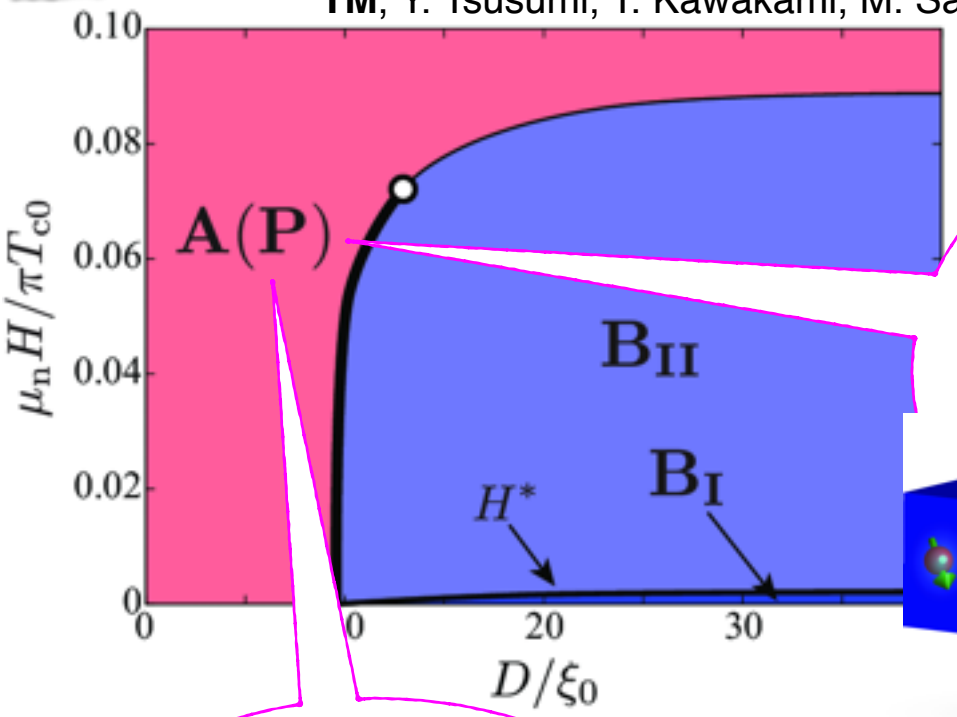
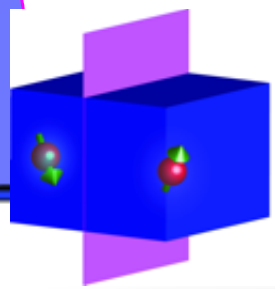
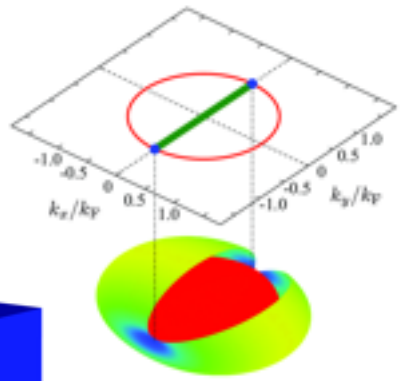
^{3}He as a New Paradigm of Topological Quantum Phenomena

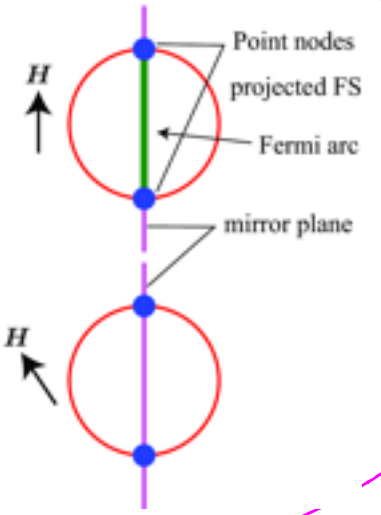
TM, Y. Tsusumi, T. Kawakami, M. Sato, M. Ichioka, K. Machida, arXiv:1508.00787





Planar state



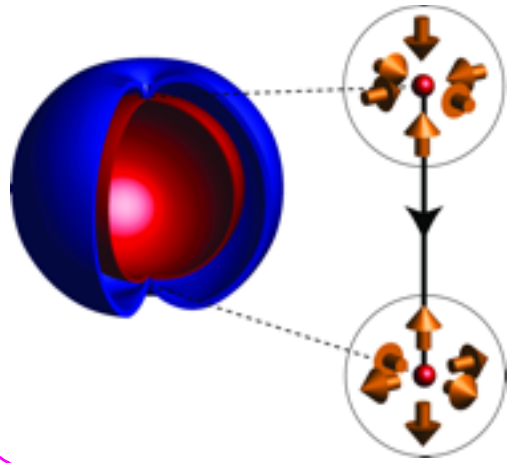


Point nodes protected by an extra symmetry

Y. Tsutsumi, M. Ishikawa, T. Kawakami, **TM** *et al.*, JPSJ (13)

Symmetry-protected surface Fermi arc
Anisotropic magnetic response

ABM: Weyl superfluid



Point nodes protected by *Chern # w/o extra symmetry*

G. E. Volovik, JETP Lett. **43**, 551 (1986)

Surface Fermi arc terminated to "**Weyl points**"
Weyl fermions & chiral anomaly

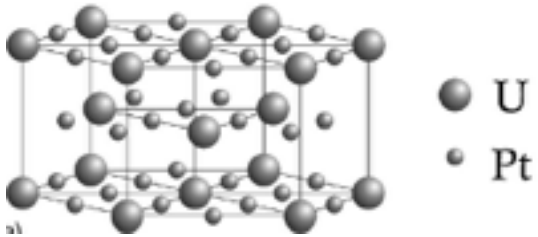
^{3}He as a New Paradigm of Topological Quantum Phenomena

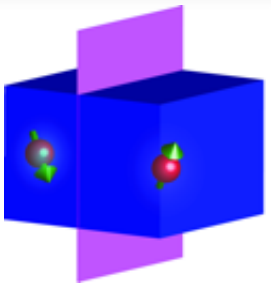
^{3}He

ABM as a Weyl superfluid

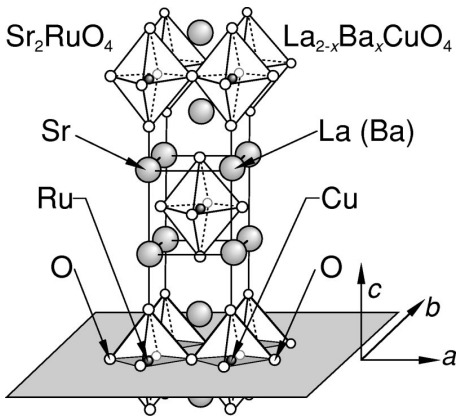
Planar: P_2-symmetry protected topo. SF

heavy fermion superconductor UPt_3





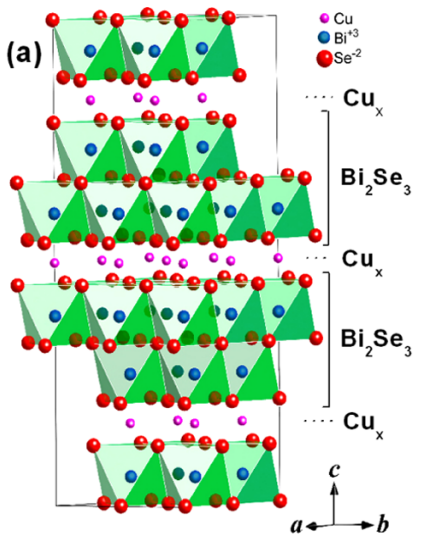
Chiral p-wave SCs(?): Sr_2RuO_4



Y. Maeno, *et al.*, JPSJ **81**, 011009 (2012)

Superconducting topological insulators: $Cu_xBi_2Se_3$, $Sr_xBi_2Se_3$

S. Sasaki and TM, Physica C **514**, 206 (2015)



Weyl SCs: URu_2Si_2, Noncentro SC, …

^{3}He as a New Paradigm of Topological Quantum Phenomena

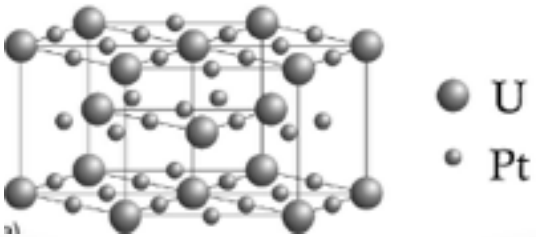
Y. Tsutsumi, M. Ishikawa, T. Kawakami, **TM** *et al.*, JPSJ **82**, 113707 (2013); **TM**, PRB **90**, 184056 (2014)

^{3}He

ABM as a Weyl superfluid

Planar: P_2-symmetry protected topo. SF

heavy fermion superconductor UPt_3



E_{2u} scenario

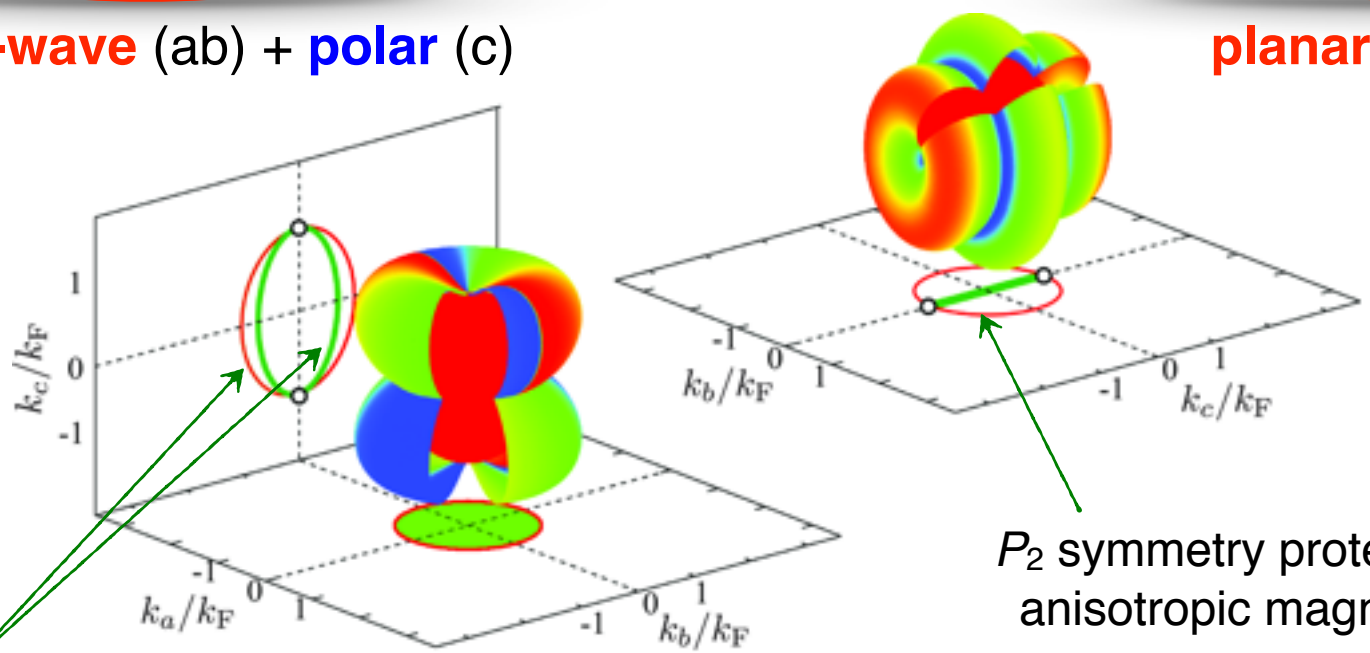
Sauls, Adv. Phys. **43**, 113 (1994)

chiral *d*-wave (ab) + **polar** (c)

E_{1u} scenario

Tsutsumi, *et al.*, JPSJ 2012

planar in *ab*-plane

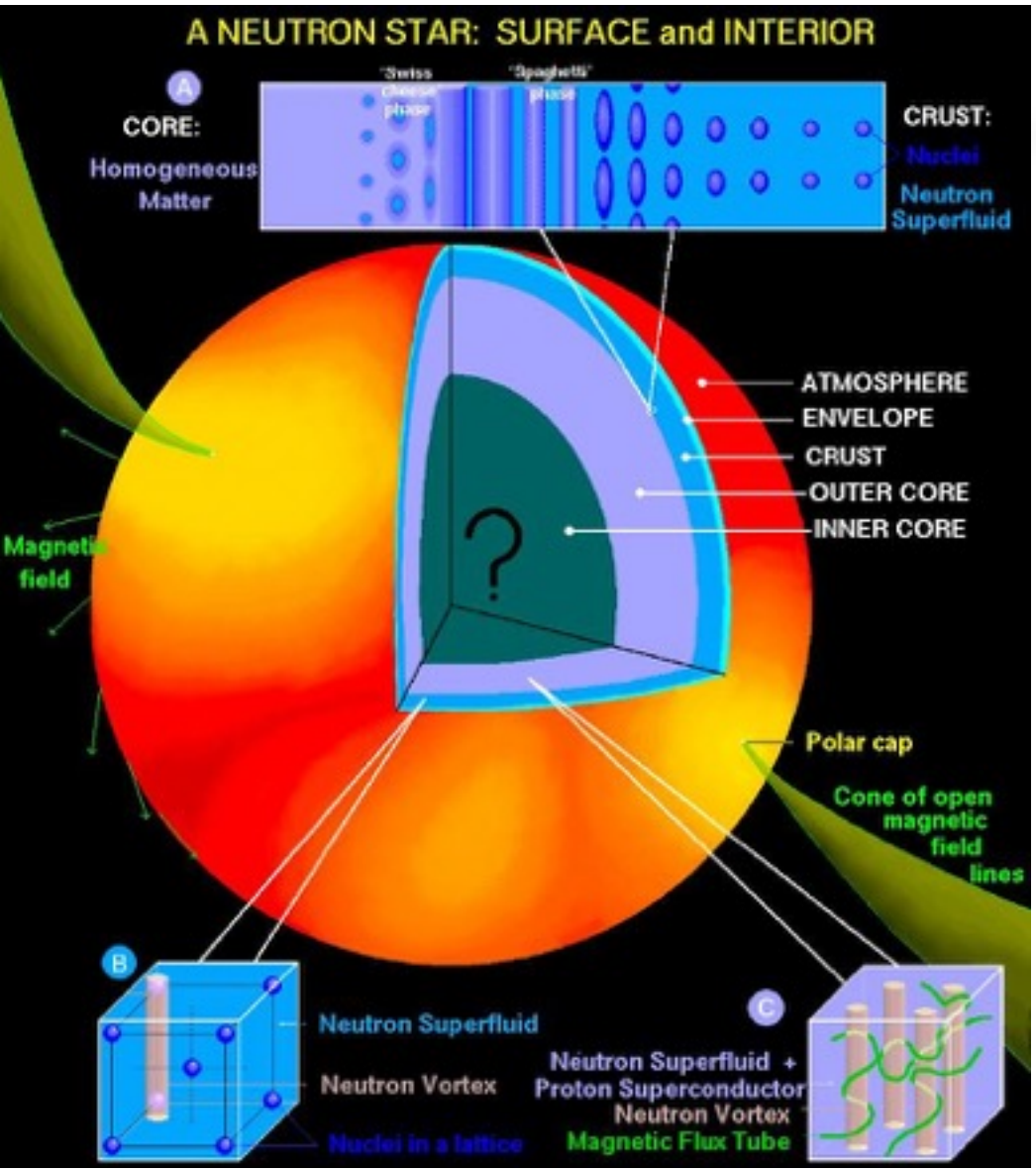


Fermi arc protected by Weyl points

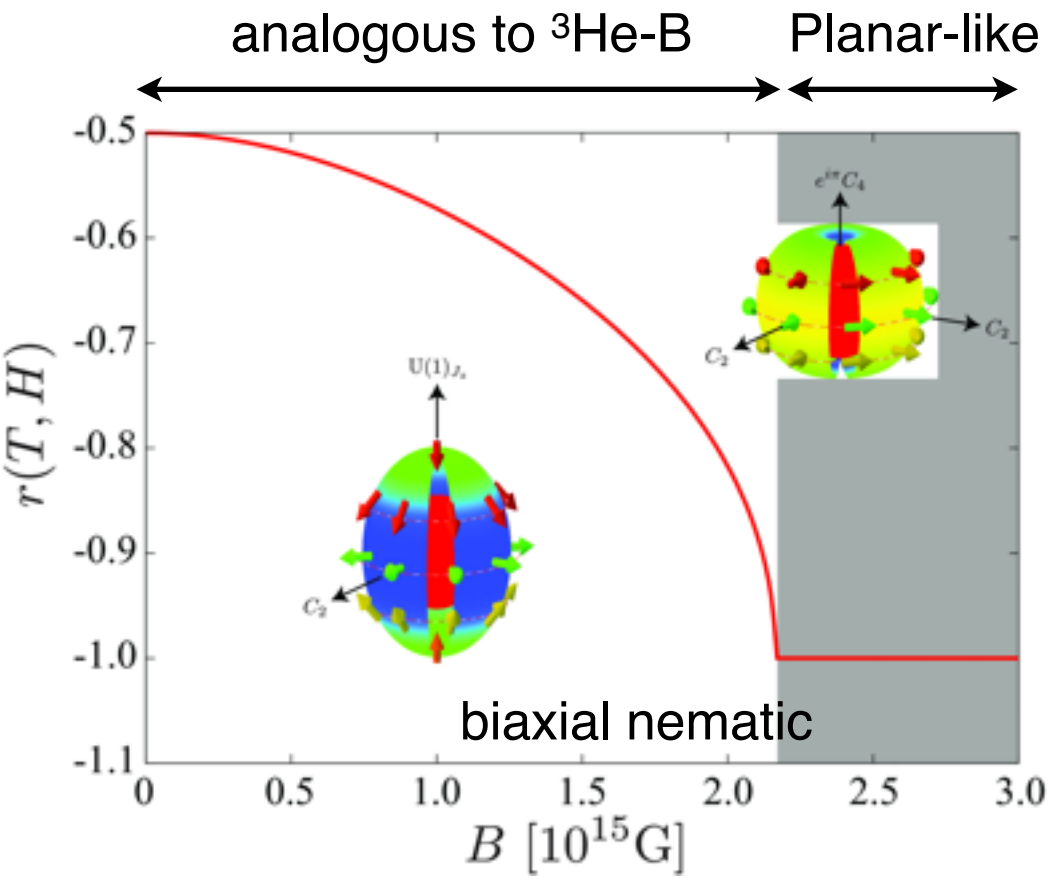
P_2 symmetry protected Fermi arc
anisotropic magnetic response

⇒ **tunneling spectroscopy**

^{3}He as a New Paradigm of Topological Quantum Phenomena



$^3P_2$ superfluid in neutron stars or more exotic phase (color SC)



new approach to a long standing issue on glitches and the origin of a huge magnetic field

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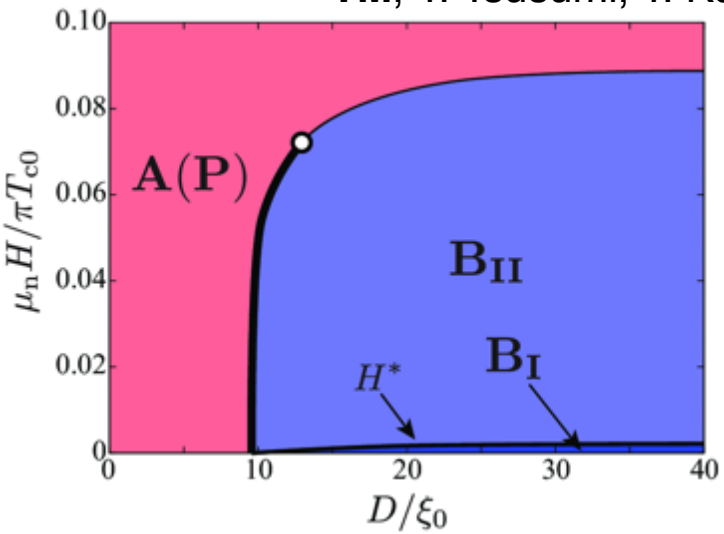
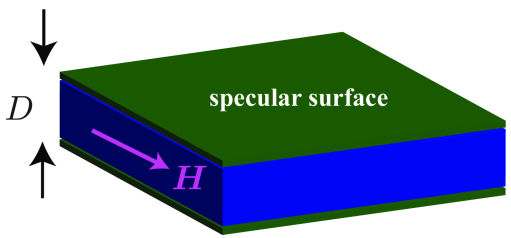
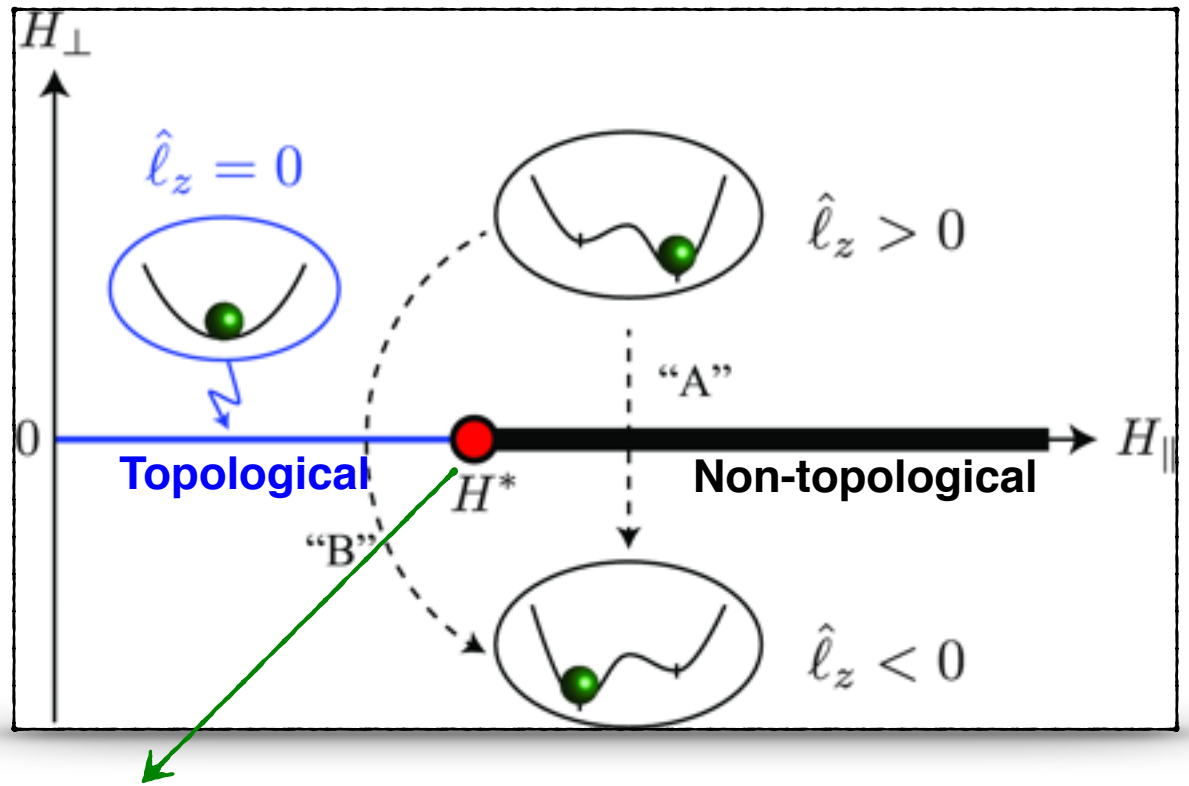
Challenges: "Topological" Quantum Critical Point

TM, Y. Tsusumi, T. Kawakami, M. Sato, M. Ichioka, K. Machida, arXiv:1508.00787

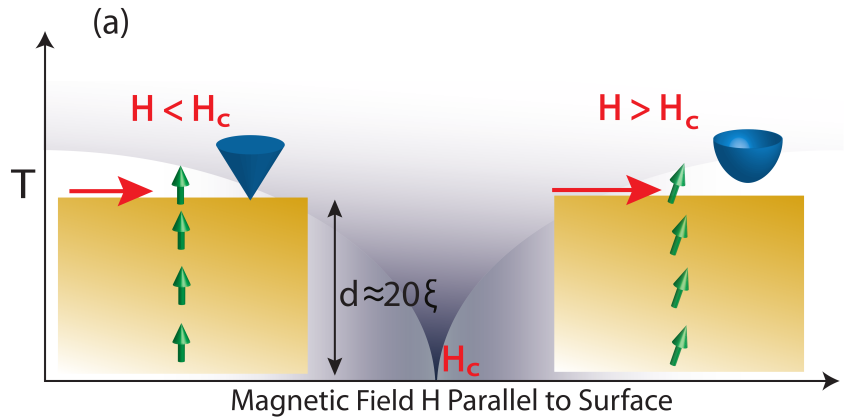


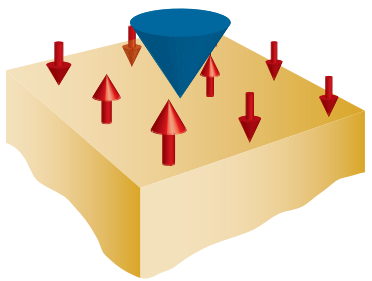




"Topological" quantum critical point?

Emergent supersymmetry: T. Grover, D. N. Sheng, and A. Vishwanath, Science **344**, 280 (2014)





Challenges: Detecting Majorana Fermions

Detecting Majorana fermions through transverse sound wave

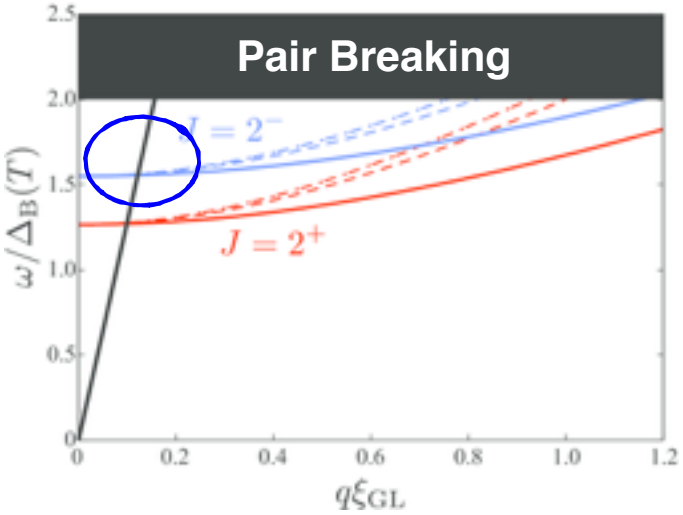
Why transverse?

longitudinal sound = density fluctuation ==> cannot be coupled to Majorana fermions

Transverse sound: overdamped in normal FL

Nambu-Goldstone mode $G/H_{\rm B} = {\rm SO}(3) \times {\rm U}(1)$

massless boson = "phase" & 3 spin modes



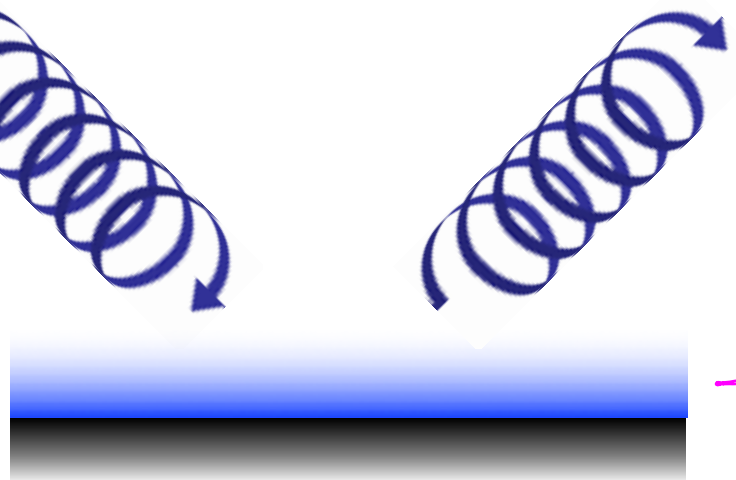
2. Massive bosonic mode:

Higgs modes

J=2 squashing modes
==> observed in sound wave attenuation

transverse sound: **squashing modes = restoring force**

Challenges: Detecting Majorana Fermions



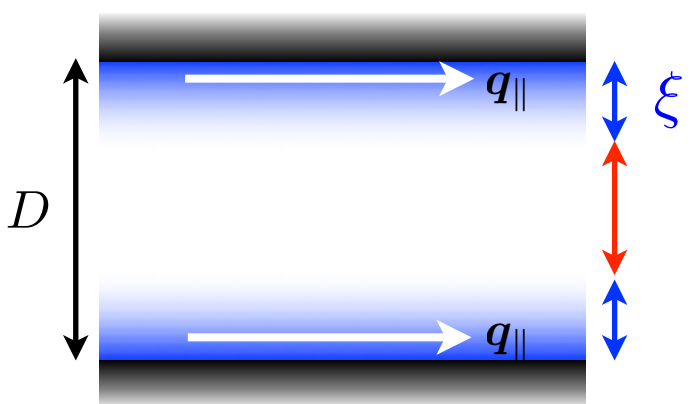
Bulk BW: empty of fermionic excitations

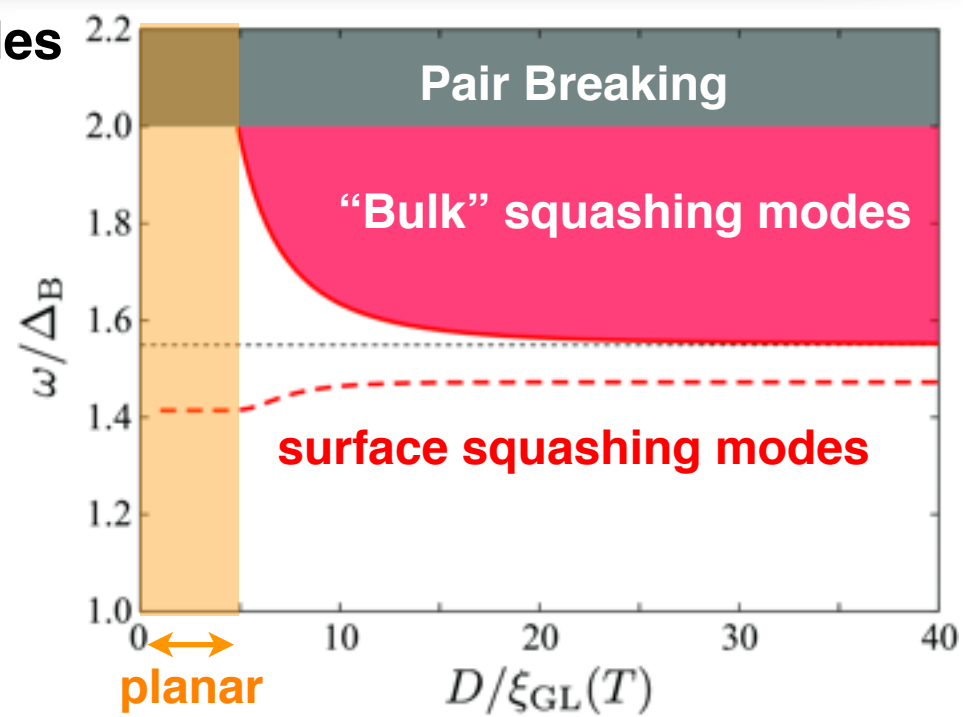
Majorana, low-lying bosonic modes,
& spin current

Coupling between symmetry-protected Majorana fermions and bosonic modes

New observation: "surface" squashing modes

TM and J. A. Sauls, in preparation





Challenges

Non-equilibrium transport in topological superfluid ^{3}He

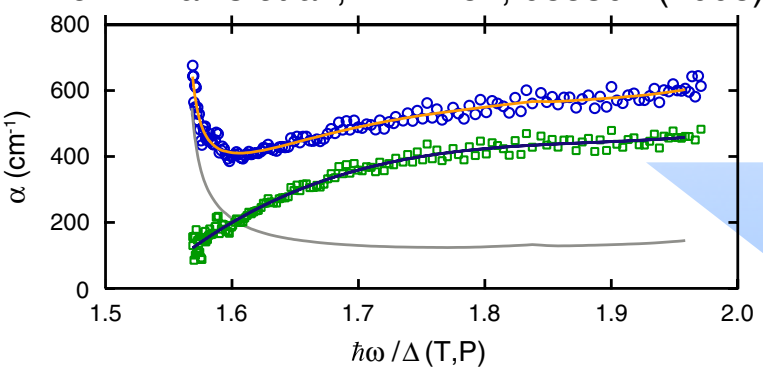
Exotic Quasiparticles — Majorana/Dirac fermions

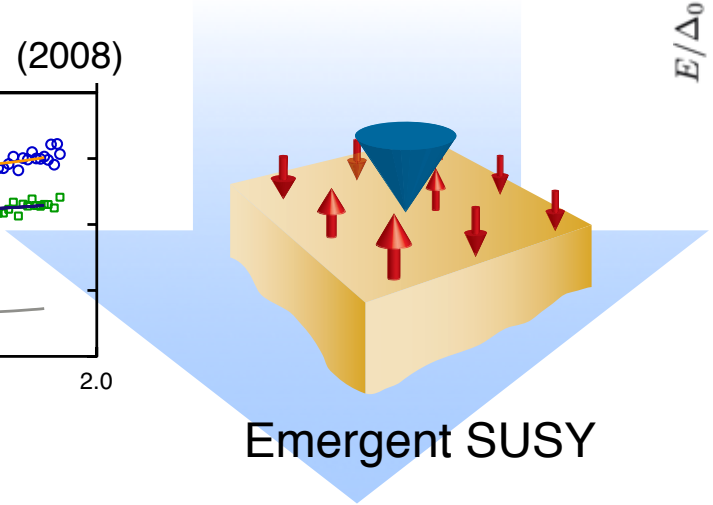
NG・Higgs modes — spin/mass current

Sound/Spin waves Transport

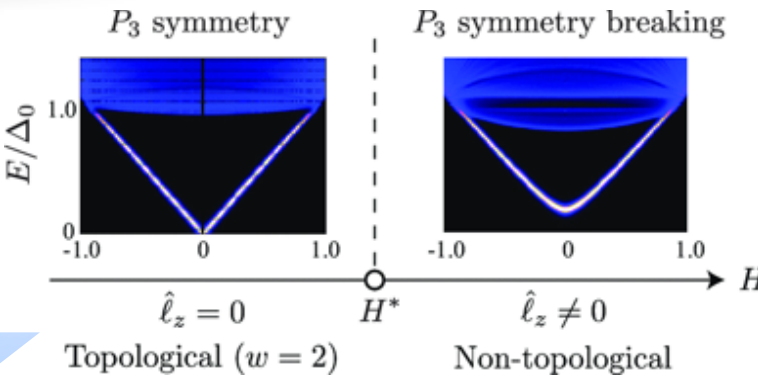
Anomalous attenuation of transverse SW

J. P. Davis *et al.*, PRL **101**, 085301 (2008)





Emergent SUSY



topo. phase transition

Non-equilibrium Transport in Unconventional & Topological Superconductors

Concluding Remarks

^{3}He is a treasure box of topological quantum phenomena

Uniqueness: ^{3}He-B under a magnetic field

SSB-induced TPT & mass acquisition of Majorana fermions

Commonality

ABM: Weyl, Planar (P_2) ==> Superconductors & neutron stars

Challenges: Detecting topological phenomena

coupling of Majorana to bosonic modes & sound/spin waves
==> high resolution spectroscopy for Majorana fermions