

Majorana fermions in He-3 Superfluid

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Grand Challenges in QFS
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Majorana Fermion concept (1)

(Wilczek, Nat Phys '09)



E Majorana (1906 - 1938?)

TEORIA SIMMETRICA DELL'ELETTRONE E DEL POSITRONE

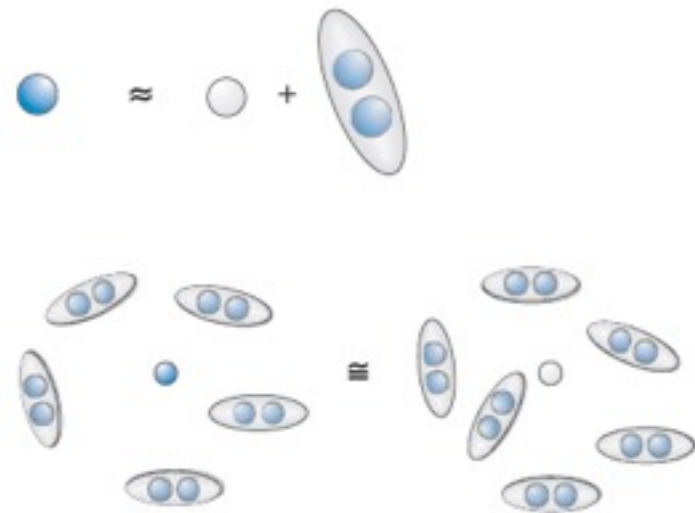
Nota di ETTORE MAJORANA

Il Nuovo Cimento **14** 171 (1937)

- Fermion identical to its own anti-particle
(a *real* fermion, $1/2$ of an electron)
- Proposed as a theory for elementary particle,
i.e. neutrino, but **NO** experimental confirmation

Majorana Fermion concept (2)

- Can *emerge* as quasi-particles of **superconductors**



(Wilczek, Nat Phys '09)

- ▶ **hole** can be considered as **anti-particle**:
real fermion \approx
particle-hole conjugation invariant
e.g. $\psi + \psi^\dagger$, $\psi - \psi^\dagger$

- Requires de-pairing **equal-spin** pairs

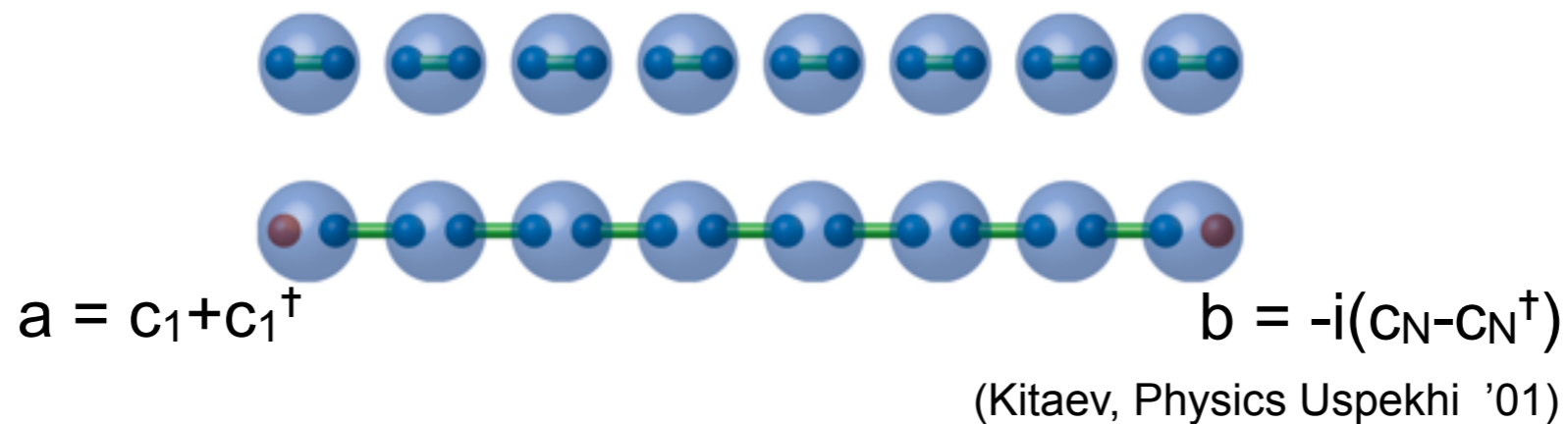
- Fermions (e.g. electron, He3 atom) **fractionalized** into localized $\psi + \psi^\dagger$ and $\psi - \psi^\dagger$ separated from each other

Outline

- Current status of Majorana fermions in $^3\text{He-B}$
- Grand Challenge: Qualitative detection of $^3\text{He-B}$ Majorana fermions
 - ▶ Surface ion nano-bubble
 - ▶ Possibility in thin slab
- Further challenge: interaction effect in Majorana surface

Majorana in condensed matter systems

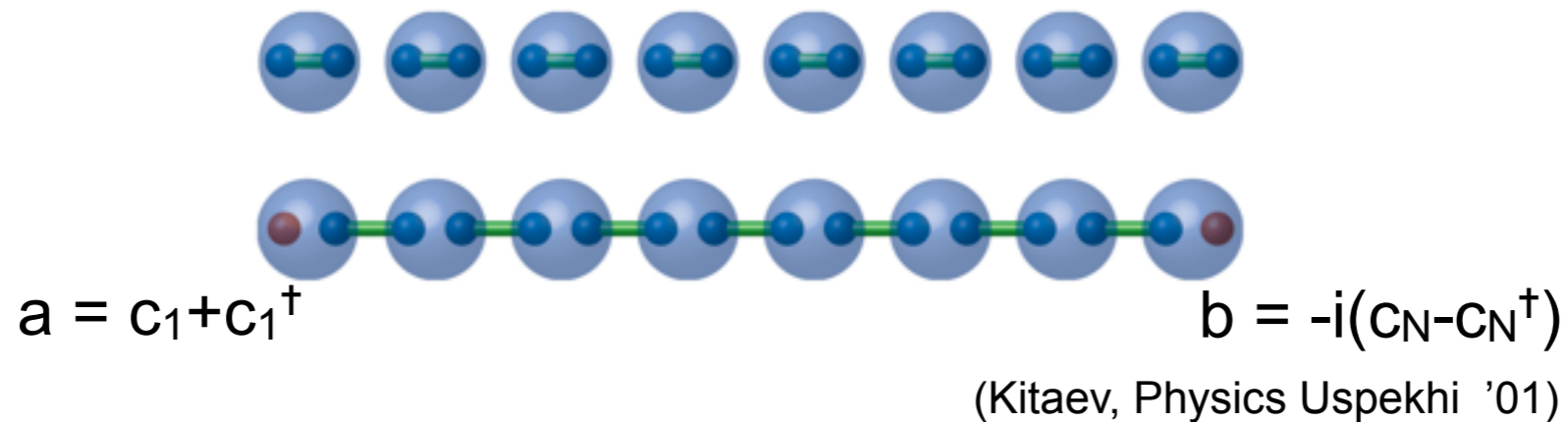
- Best publicized in 1D topological superconductor (TSC)



→ **Majorana zero modes** topologically protected by the **bulk energy gap**

Majorana in condensed matter systems

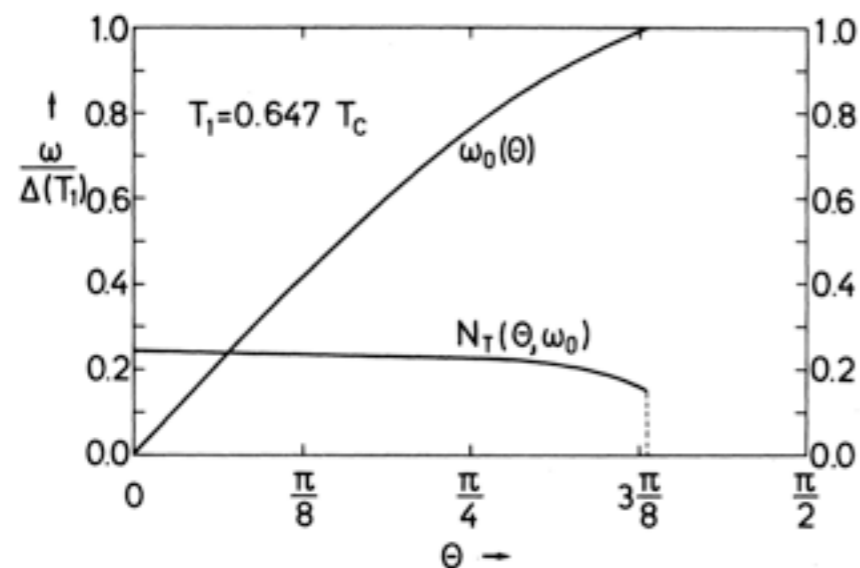
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→ Majorana zero modes topologically protected by the bulk energy gap

- 3He superfluid: **FIRST** condensed matter system identified!

(1) 3He-B surface state



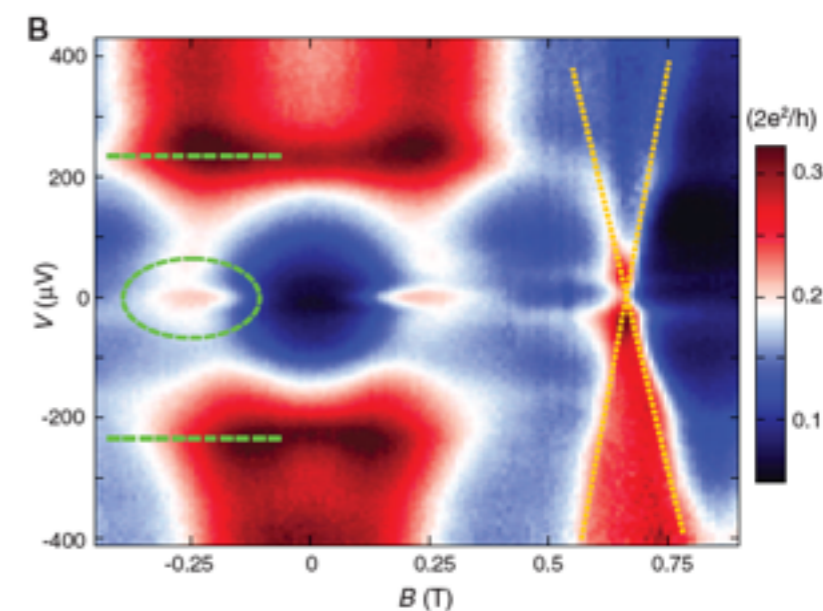
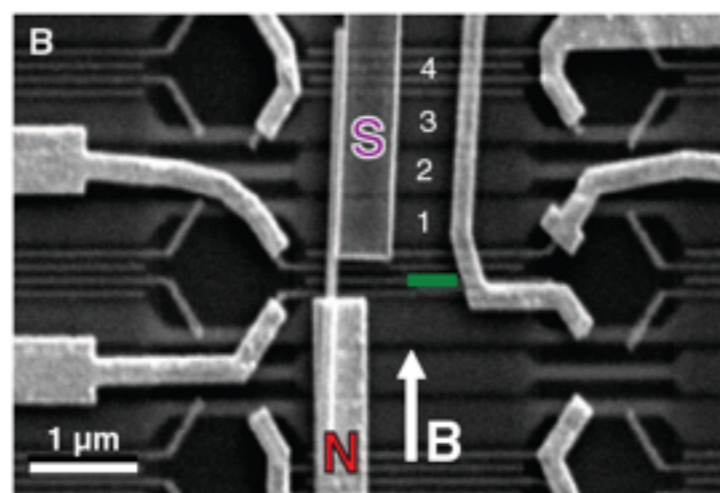
(Buchholtz and Zwicknagl, PRB 1981)

(2) 3He-B topology

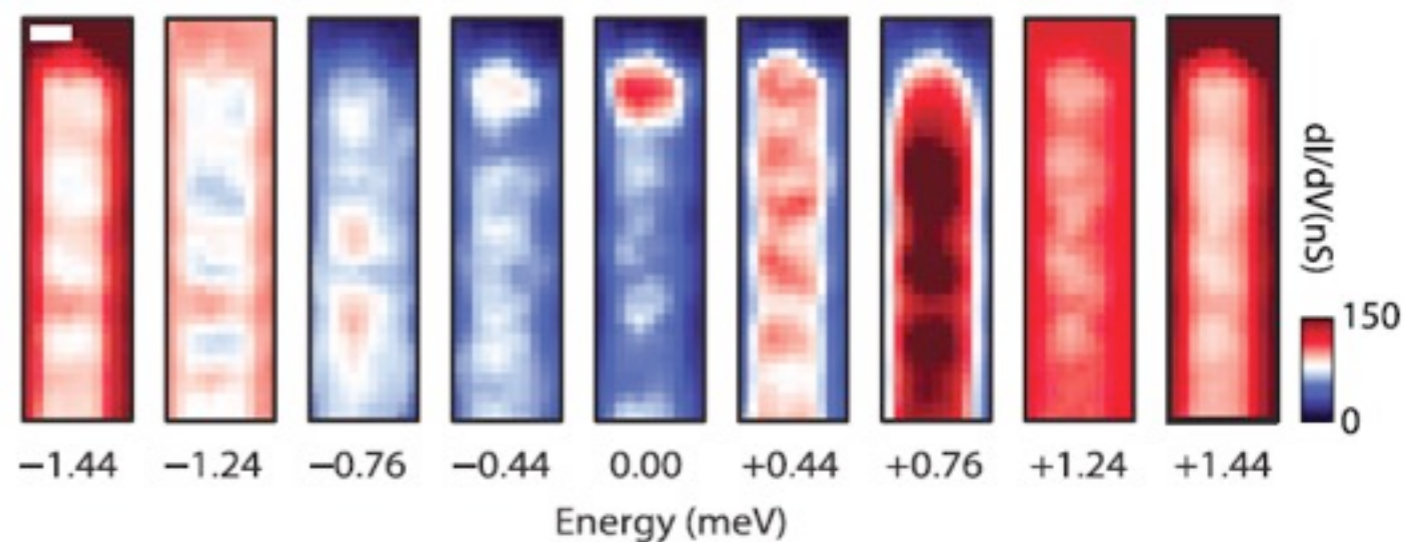
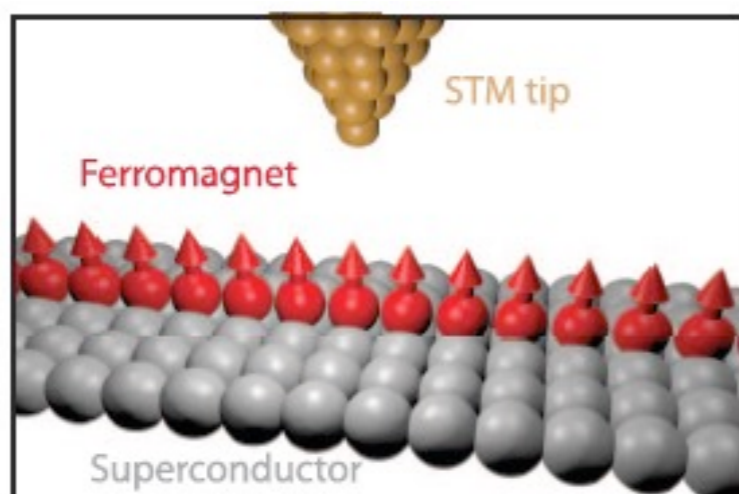
$$N = \frac{1}{48\pi^2} \int_{S^3 \text{ around instanton}} dS_\mu \epsilon^{\mu\nu\alpha\beta} \text{Tr}(U^\dagger \partial_\nu U)(U^\dagger \partial_\alpha U)(U^\dagger \partial_\beta U)$$

(Salomaa and Volovik, PRB 1988)

Recent solid state experiments:



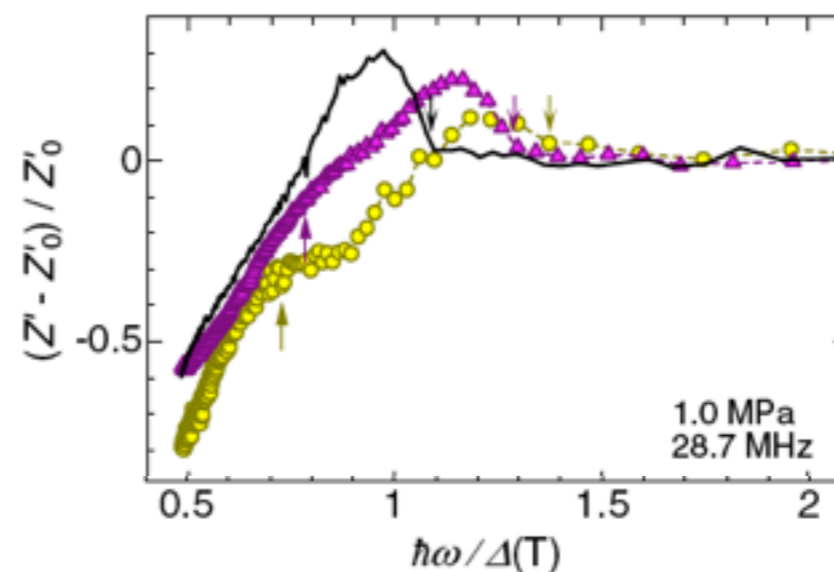
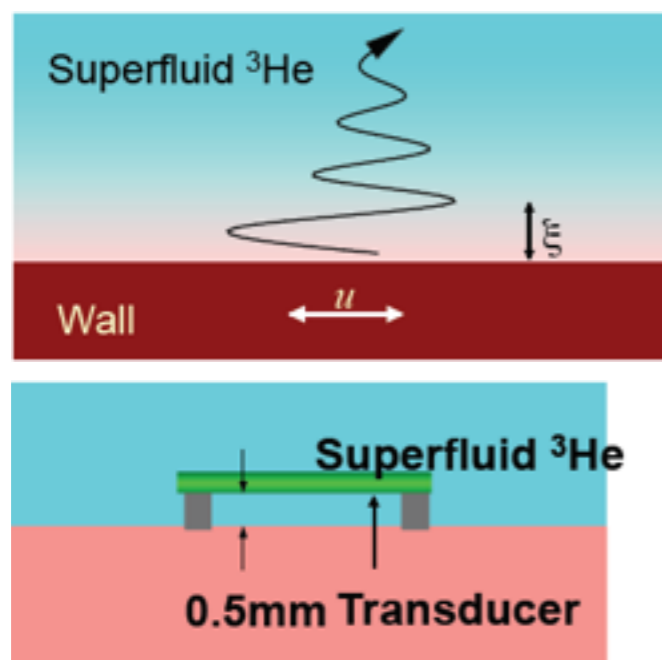
(Mourik, Kouwenhoven et al Science '12)



(Nadj-Perge, Yazdani et al Science '14)

Analogous earlier experiments 3He-B!

- Surface acoustic impedance below pair breaking frequency

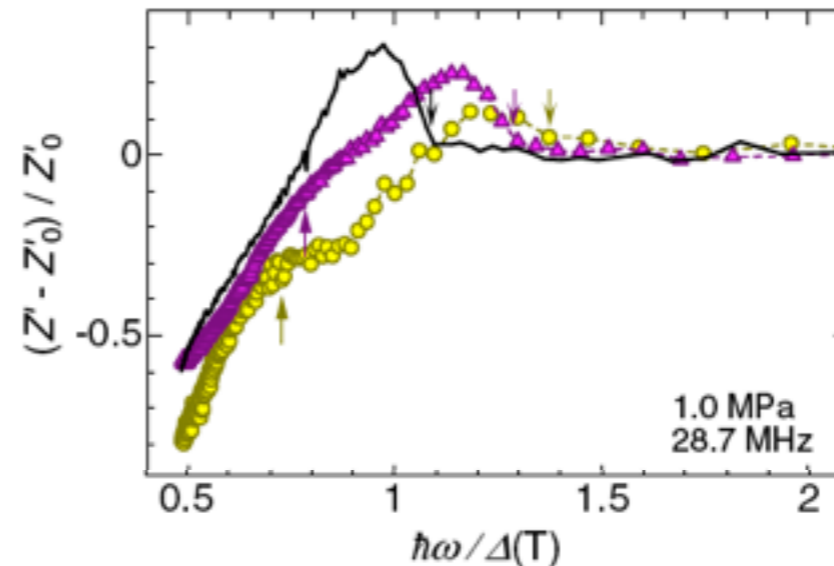
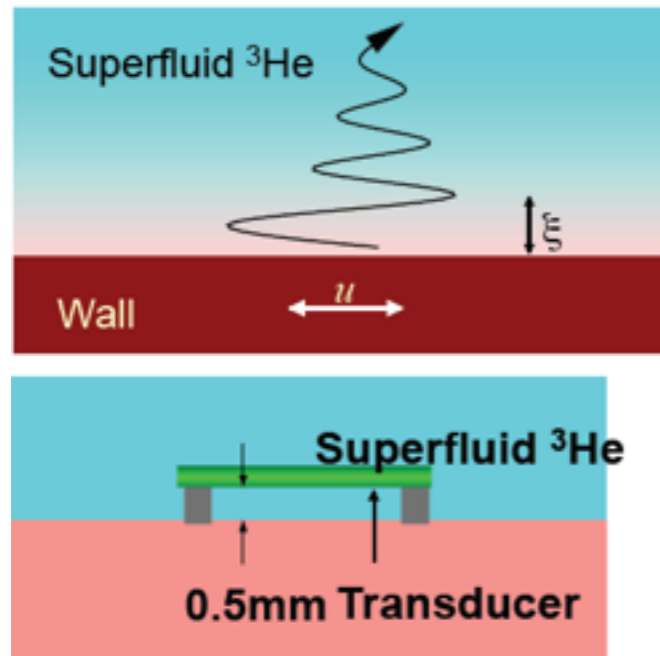


(Aoki, Nomura *et al.* PRL '05; Murakawa, Nomura *et al.* JPSJ '10)

⇒ indicates subgap quasiparticles at the surface
i.e. qualitatively equivalent to the zero bias anomaly

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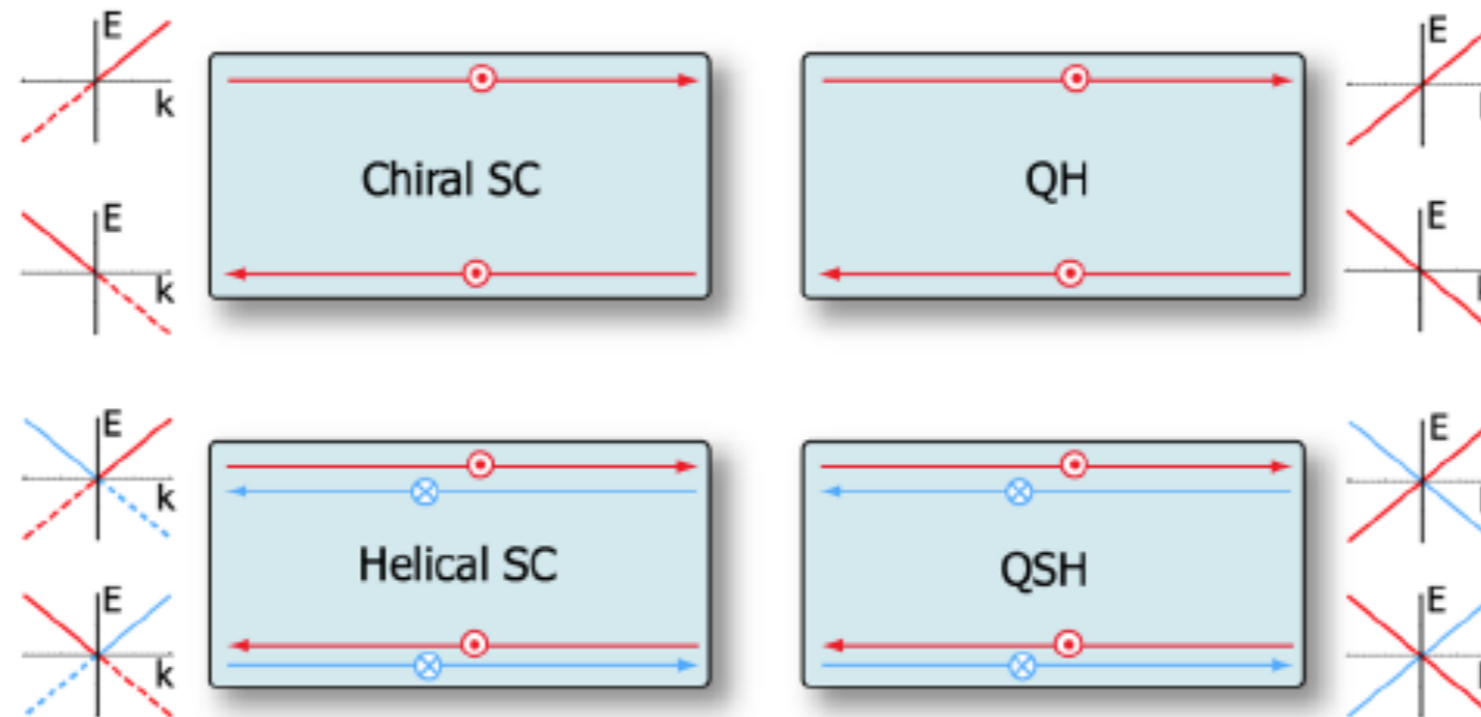
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- **No qualitative** (e.g. *more* than zero bias anomaly) Majorana detection in **either** ^3He superfluid or any solid state systems

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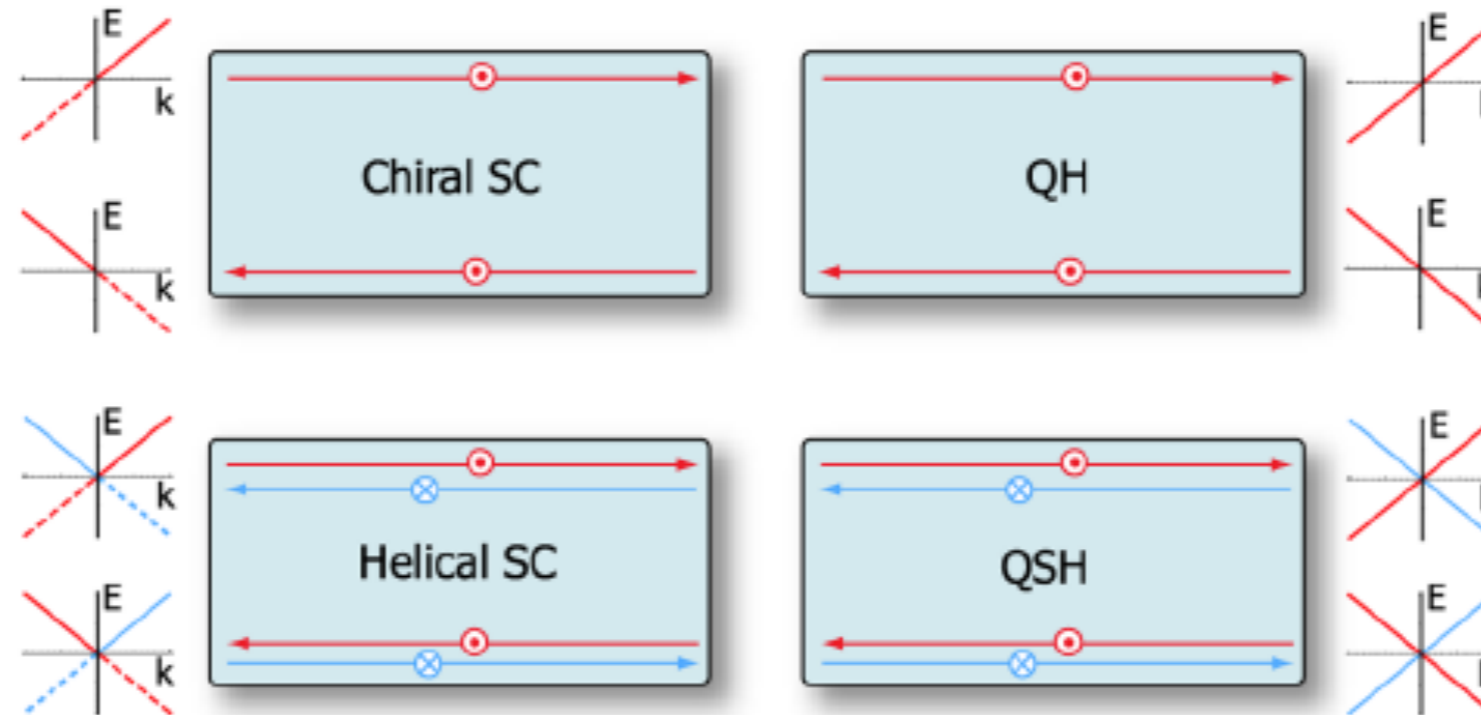
TI surface and TSC Majorana surface analogy



(Qi, Hughes, Raghu and Zhang PRL '09)

- The **Majorana edge state** same dispersion as the **TI edge state** for both chiral and time-reversal invariant (TRI) TSC
 - ▶ e.g. $p+ip$ (chiral) superconductor: $E = \nu k$ (like $\nu=1$ IQH)
 2D TRI TSC : $E = \sigma \nu k$ (like QSH)
 (requires **both** $|\uparrow\uparrow\rangle$ & $|\downarrow\downarrow\rangle$)

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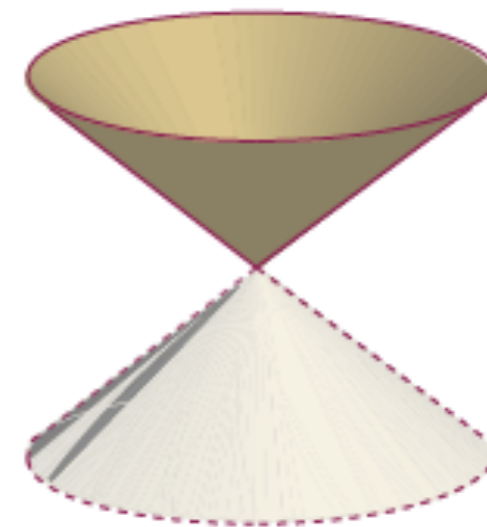
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 (requires **both** $|\uparrow\uparrow\rangle$ & $|\downarrow\downarrow\rangle$)
- Opposite surfaces of TI have **opposite spin-momentum locking**
 Opposite surfaces of TSC have $\psi_\sigma + \psi_\sigma^\dagger$ **Majorana** on one
 $\psi_\sigma - \psi_\sigma^\dagger$ **Majorana** on the other

Qualitative feature of 3He-B Majorana surface

(SBC, Zhang PRL '09)

$$\begin{bmatrix} \hat{\psi}_{\rightarrow}(\mathbf{r}) \\ \hat{\psi}_{\leftarrow}(\mathbf{r}) \\ \hat{\psi}_{\rightarrow}^{\dagger}(\mathbf{r}) \\ \hat{\psi}_{\leftarrow}^{\dagger}(\mathbf{r}) \end{bmatrix} = \sum_{\mathbf{k}} (\hat{\gamma}_{\mathbf{k}} e^{i\mathbf{k}_{\parallel} \cdot \mathbf{r}_{\parallel}} + \hat{\gamma}_{\mathbf{k}}^{\dagger} e^{-i\mathbf{k}_{\parallel} \cdot \mathbf{r}_{\parallel}}) \begin{bmatrix} \cos \frac{\phi_{\mathbf{k}} + \pi/2}{2} \\ \sin \frac{\phi_{\mathbf{k}} + \pi/2}{2} \\ \cos \frac{\phi_{\mathbf{k}} + \pi/2}{2} \\ \sin \frac{\phi_{\mathbf{k}} + \pi/2}{2} \end{bmatrix} \\
 \times u_{\mathbf{k}} \sin(k_{\perp} z) e^{\Delta z / \hbar v_F} + (\text{gapped modes})$$

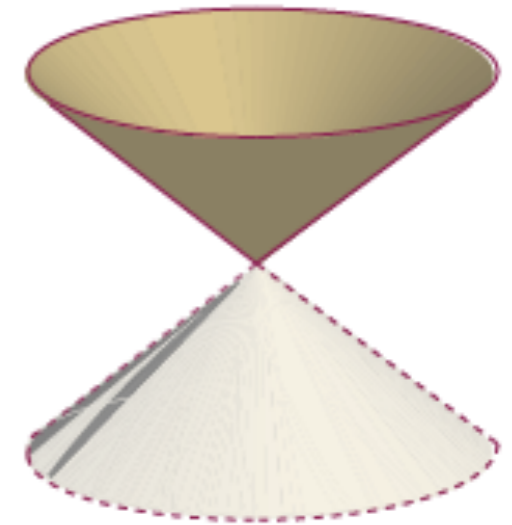


- Momentum eigenstates **NOT** particle number or spin eigenstates!
- Surface state of $\psi_{\sigma} + \psi_{\sigma}^{\dagger}$ for any in-plane spin quantization
 ← 3He-B spin-orbit rotation symmetry

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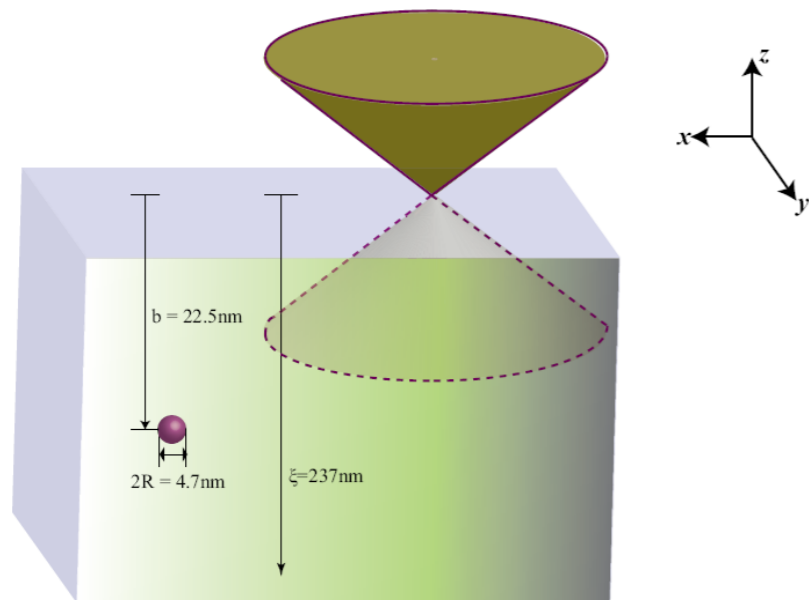
- Surface excitation **change**

$$I_z = i\psi_{\rightarrow}\psi_{\leftarrow}$$

Surface excitation **do not change**

$$\rho = \sum_{\sigma} \psi_{\sigma}^{\dagger} \psi_{\sigma} \quad I_x = \psi_{\rightarrow}^{\dagger} \psi_{\rightarrow} - \psi_{\leftarrow}^{\dagger} \psi_{\leftarrow} \quad I_y = \psi_{\rightarrow}^{\dagger} \psi_{\leftarrow} + \psi_{\leftarrow}^{\dagger} \psi_{\rightarrow}$$

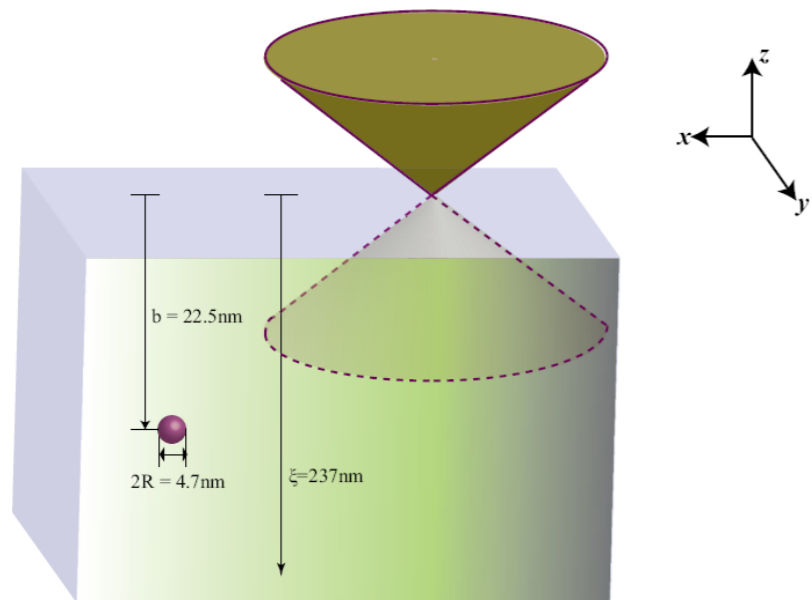
Toward qualitative Majorana detection



- Surface Majorana fermion detection through spin relaxation - shows **Ising** spin fluctuation of the surface state

(SBC, Zhang PRL '09)

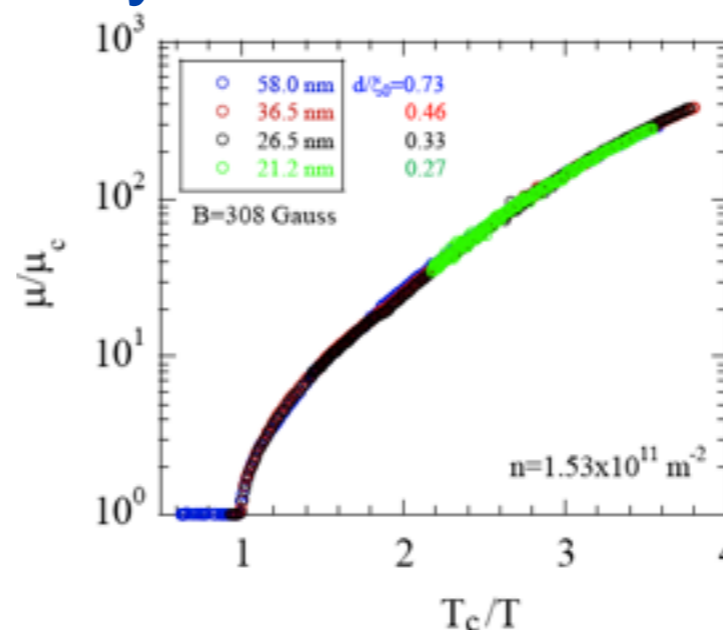
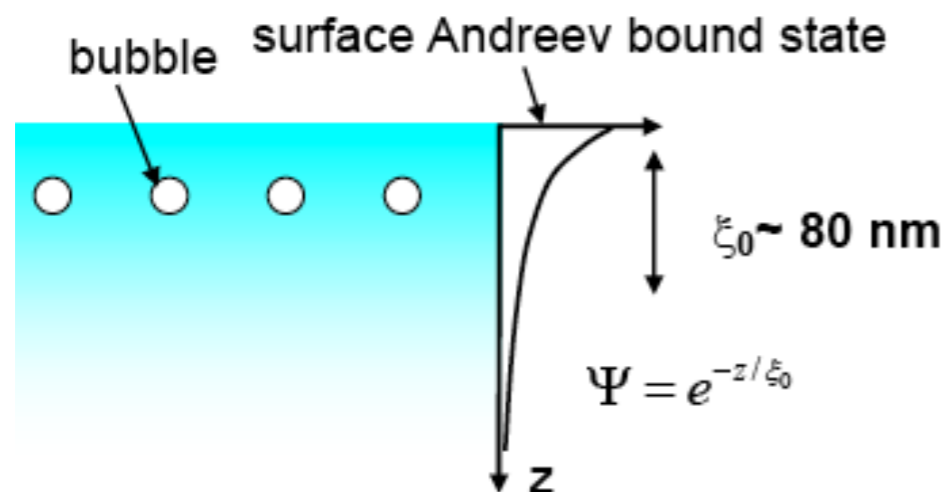
Toward qualitative Majorana detection



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(SBC, Zhang PRL '09)

- Ion bubble mobility experiment with parallel electric field \Rightarrow **no** sign of enhanced **density fluctuation** at the surface

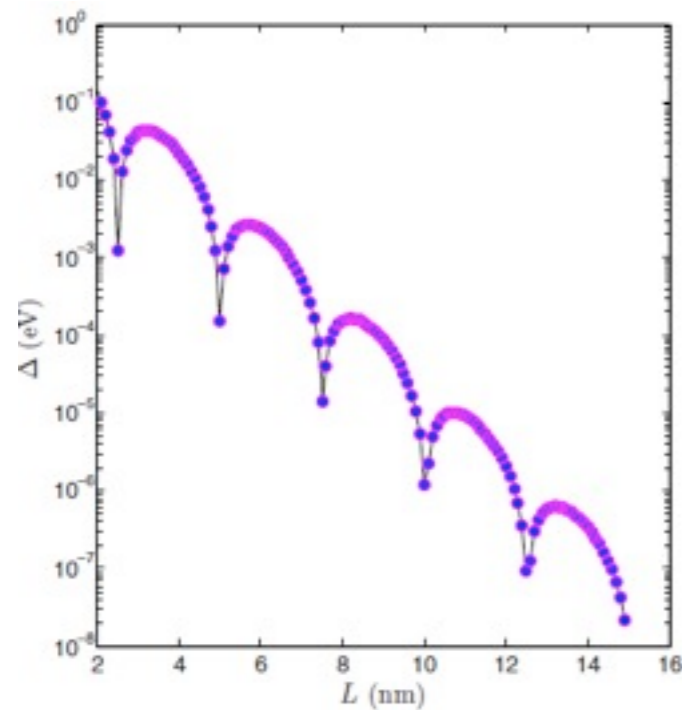


(Ikegami, SBC, Kono JSPJ '13)

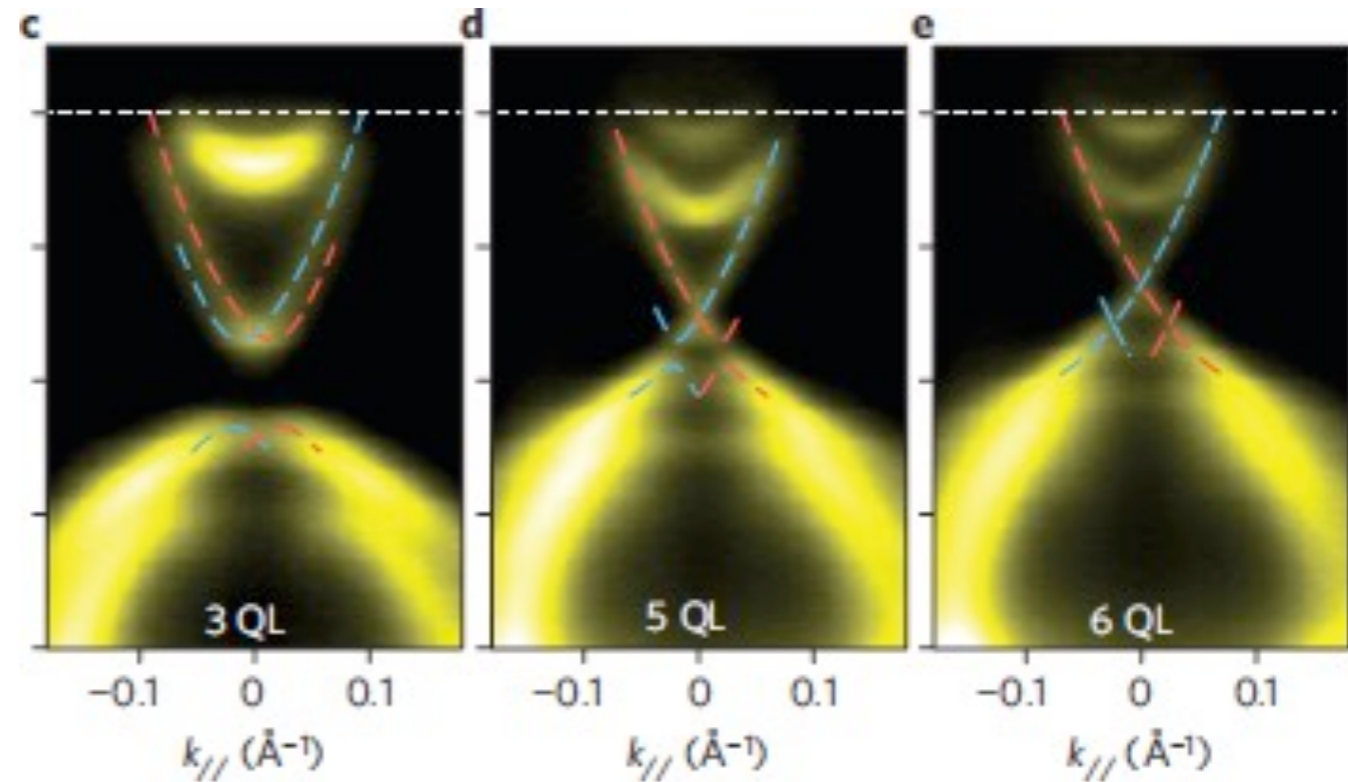
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Surface state in topological insulator film



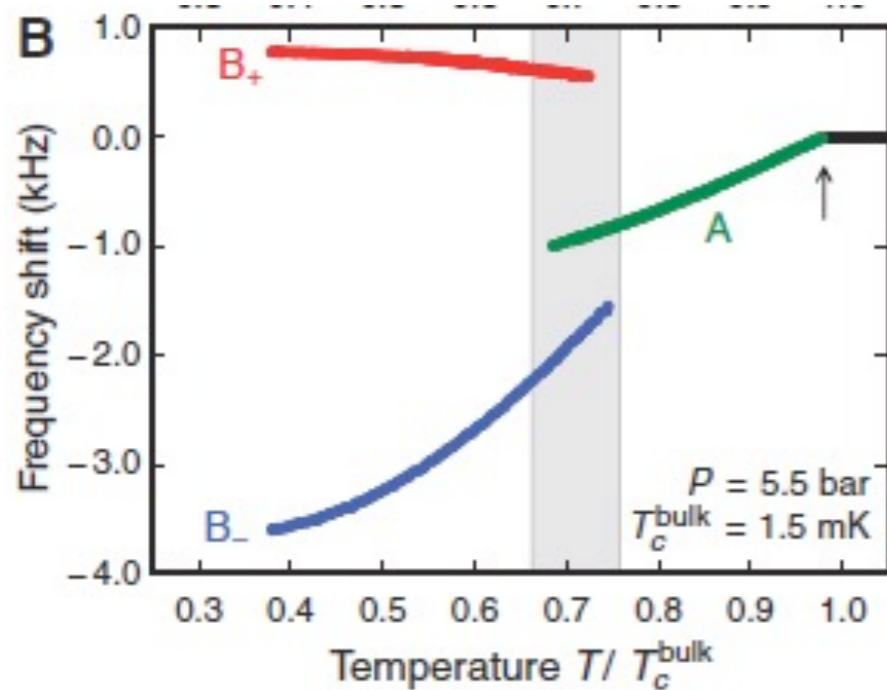
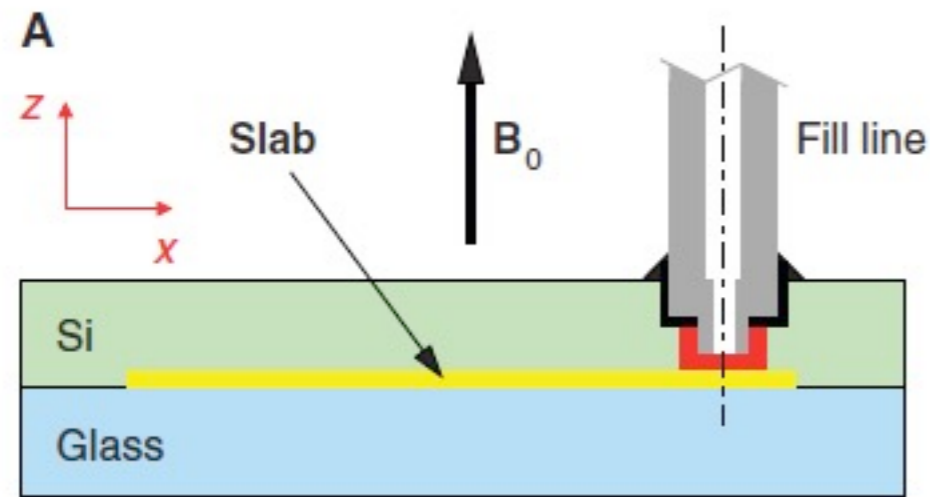
(Linder et al. PRB '09;
Liu et al PRB '09)



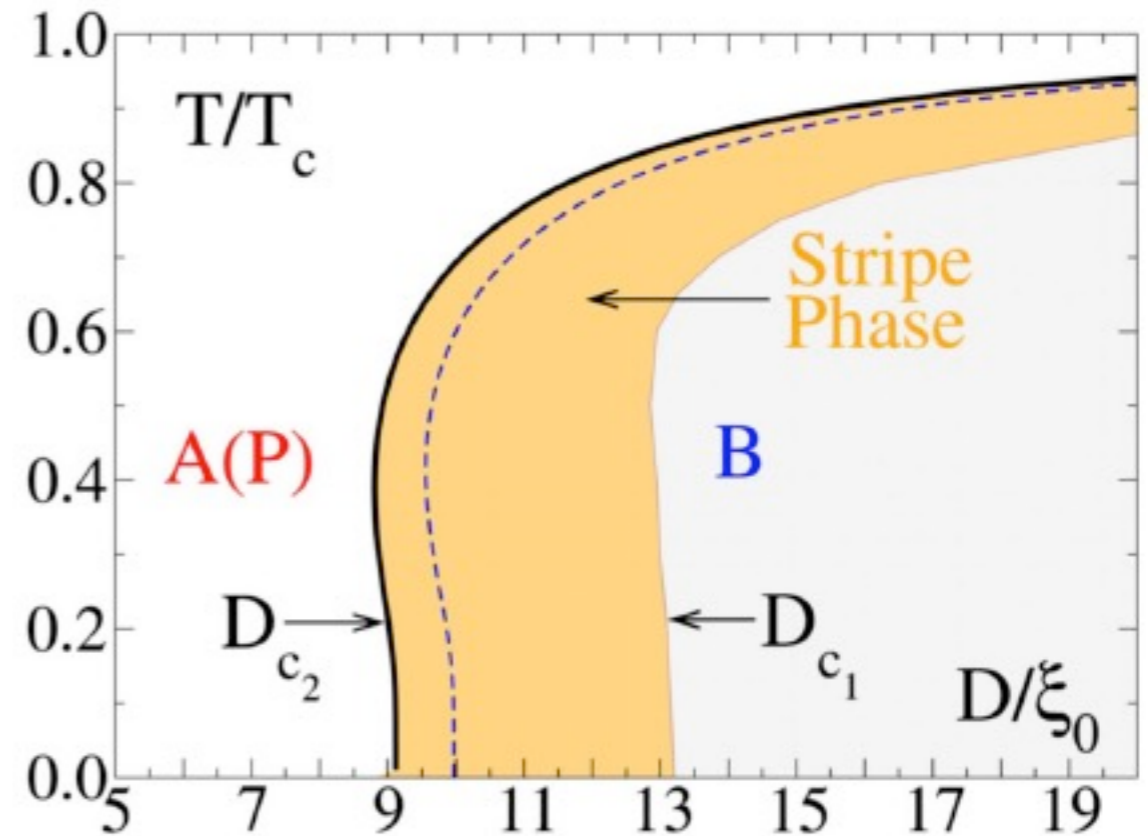
(Y Zhang, Xue et al. Nat Phys '10)

- Tunneling between two **TI surfaces** gaps out the surface state
 \Rightarrow momentum / spin locking gone for $k=0$
- Tunneling between two **Majorana surfaces** mean fusing back
 $\psi_\sigma + \psi_\sigma^\dagger$ and $\psi_\sigma - \psi_\sigma^\dagger$

Helium-3 in thin slab



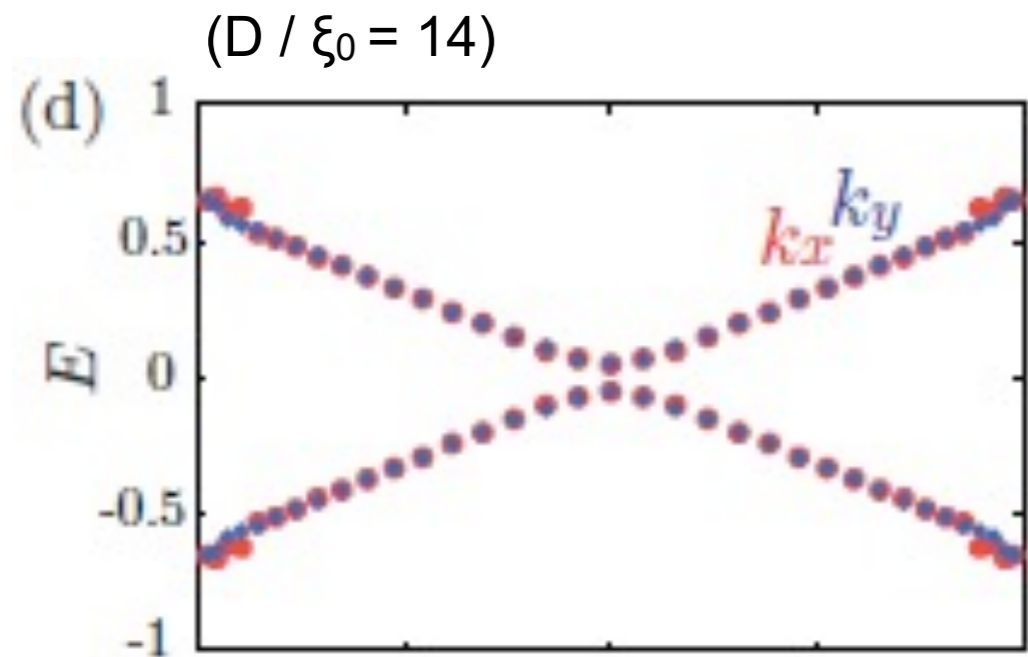
(Levitin, Saunders et al. Science '13)



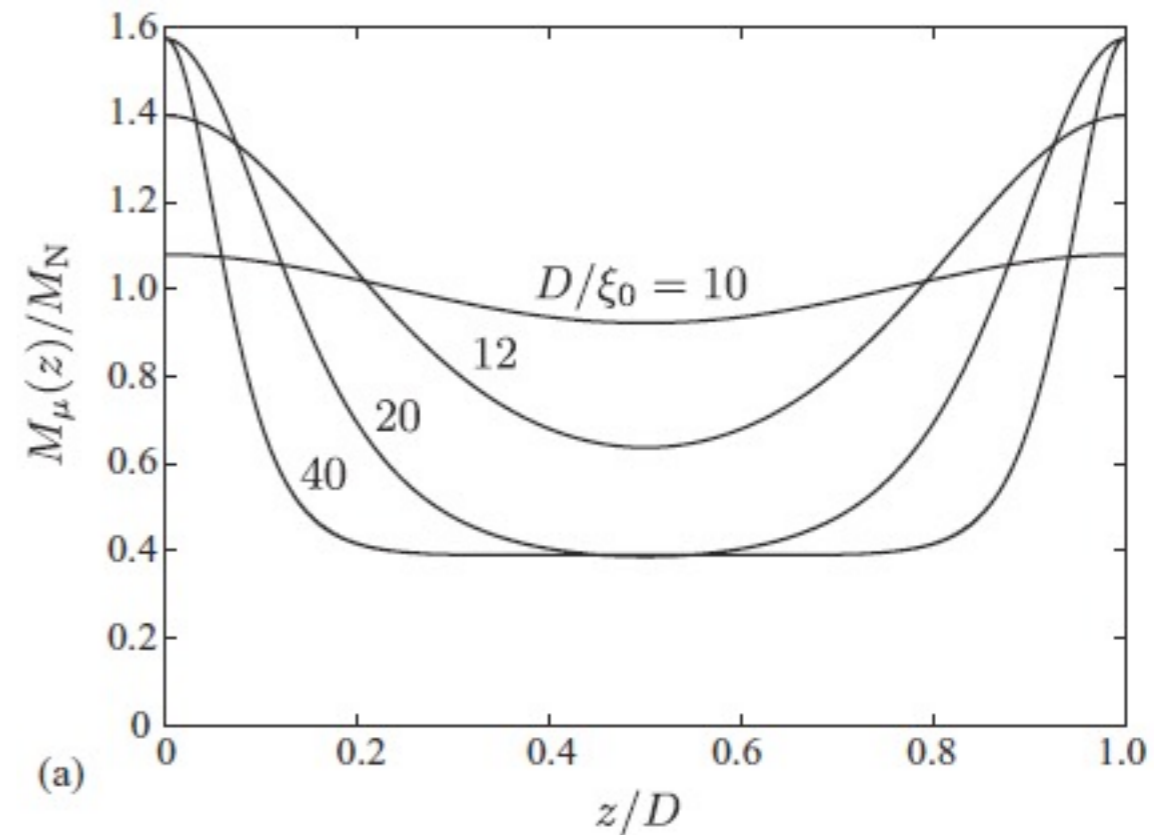
(Vorontsov, Sauls PRL '07)

- Experiments on slab **thin enough** ($D \approx 650\text{nm}$, $\xi_0 \approx 77\text{nm}$) to see transition between different superfluid states

Majorana detection in thin slab



(Tsutsumi et al PRB '10)



(Mizushima PRB '12)

- Significant tunneling between the $\psi_\sigma + \psi_\sigma^\dagger$ surface and the $\psi_\sigma - \psi_\sigma^\dagger$ surface within the B-phase
 - ▶ **Ising** spin fluctuation \longrightarrow **Isotropic** spin fluctuation
 - ▶ **Zero** density fluctuation \longrightarrow **Non-zero** density fluctuation

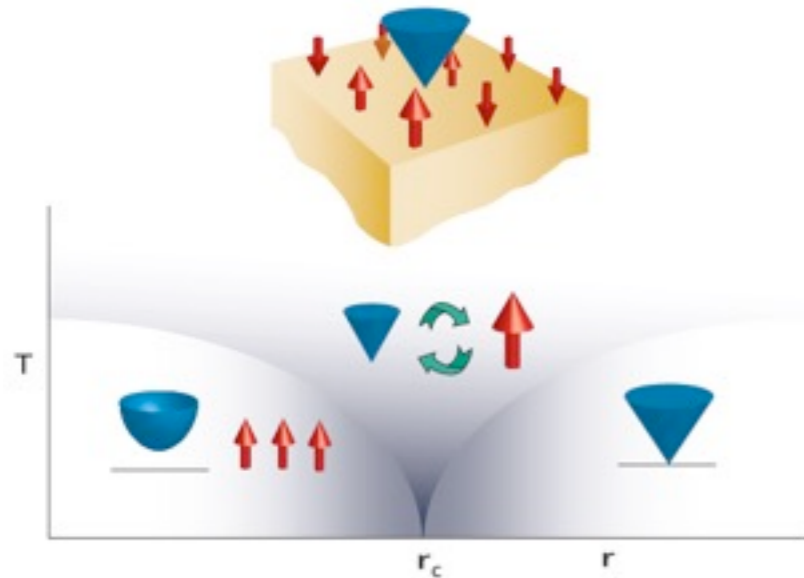
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Interaction effect on Majorana surface

- Interaction alters TSC classification: $\mathbb{Z} \rightarrow \mathbb{Z}_8$

(Fidkowski, Kitaev PRB '10; Yao, Ryu, PRB '13;
Qi, New J Phys '13; You, Xu '14)



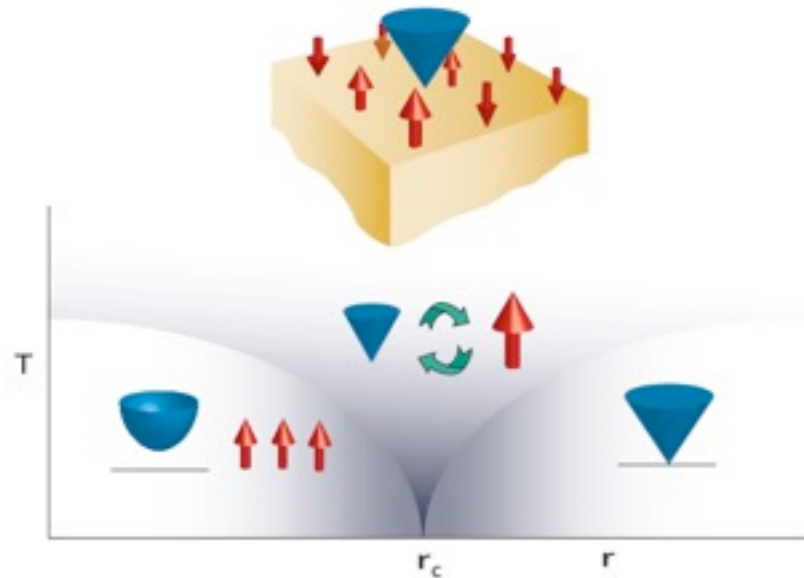
- Emergent **supersymmetry** in symmetry breaking transition of a single Majorana cone

(Grover, Vishwanath Science '14)

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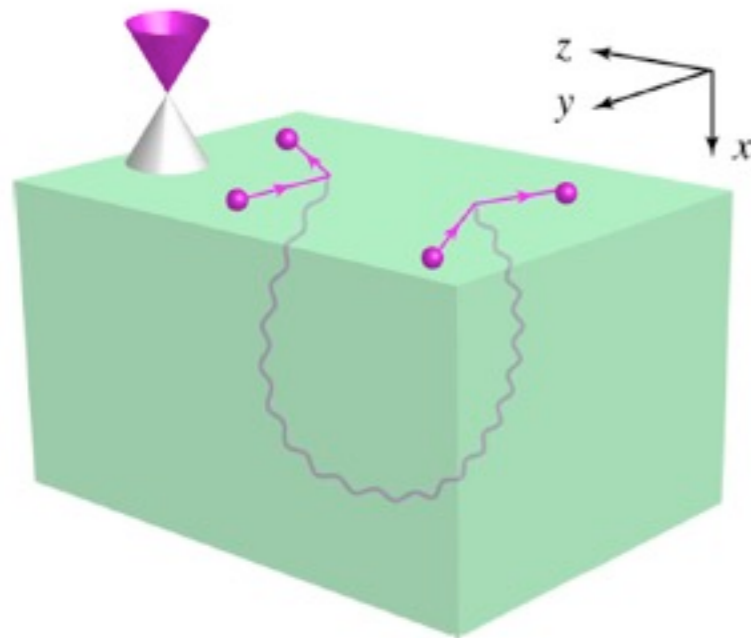
(Grover, Vishwanath Science '14)

- $^3\text{He-B}$ remains the only known system with the surface Majorana cone
 - ▶ **Non-interacting quasiparticles** insufficient for full physics as it ignores **bosonic modes**

Interaction on $^3\text{He-B}$ surface

- Surface spin current renormalization from Fermi liquid correction

(Wu, Sauls PRB '13)



- $J=1$ collective mode mediated interaction

(Park, **SBC**, Maciejko PRB '15)

- Ongoing study on possible non-equilibrium interaction effect
 \Rightarrow detection of **many-body** response of Majorana surface

(Taylo, Kallin, PRB '15;
SBC, Park, Maciejko *in progress*)

Conclusion

- There is **as much** experimental evidence for **Majorana fermions** in ^3He superfluid as any solid state systems.
- There are both experimental proposals and **technique** available for **qualitatively** detecting ^3He superfluid Majorana fermions.
- ^3He superfluid is a good venue for studying interaction effect on Majorana fermions.