QUANTUM FLUIDS AND SOLIDS: SOME MYSTERIES AND CHALLENGES

A.J. Leggett

Dept. of Physics, University of Illinois at Urbana -Champaign

and

Institute for Quantum Computing, University of Waterloo, Ontario

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Outline

A) A couple of existing experimental anomalies (other than supersolid-like behavior in solid ^{4}He, liquid ^{3}He in aerogel, films...)

B) New regimes opened up by advances in cryogenics etc.

C) Probing the unknown: when existing theory may or may not be a guide

A) 1. Half-quantum vortices in superfluid ^{3}He-A: where are they ?

Reminder:

a) **"Ordinary" vortices** (^{4}He, BCS superconductors, ^{3}He-A and B...)

$\psi(\underline{r}) \sim f(|\underline{r}|)\, e^{(i\varphi)}$ φ - angle around vortex core

in neutral case (He) current j falls off as $\frac{1}{r} \rightarrow$ total $KE \propto \int j^2\, dr$

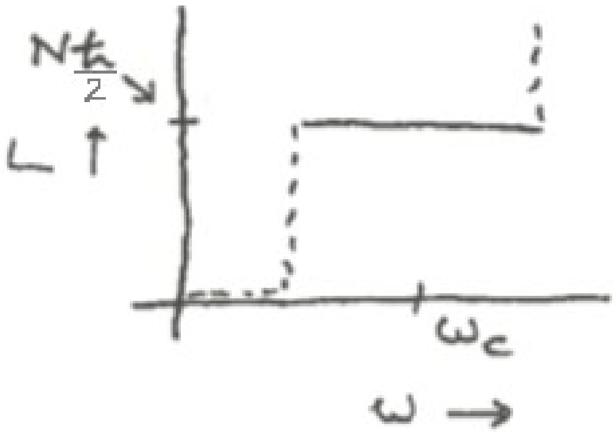
$\sim \int \frac{d^2 r}{r^2}$ $\rightarrow$ diverges (as ln R) R - sample size

in charged case, current screened by Ampère effect $\rightarrow$ falls off as $e^{(-\frac{r}{\lambda})}$

$\rightarrow$ total KE finite λ - London penetration depth

For neutral case under rotation, expect

(annular geometry, $\omega_c = \frac{\hbar}{2mR^2}$)



Half-quantum vortices in superfluid ^{3}He-A: where are they ?

b) **Half-quantum vortex** (HQV)

need "ESP triplet" Fermi superfluid (^{3}He-A, Sr_2RuO_4...)

then can have (at small r)

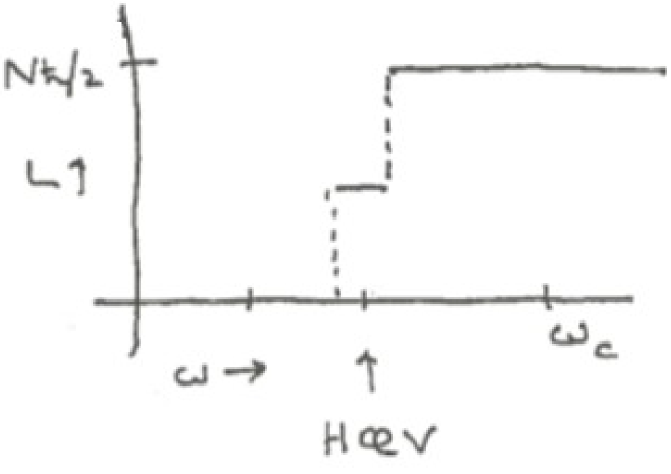
$$\psi_{\uparrow\uparrow}(\underline{r}) \sim f(|\underline{r}|)e^{(i\varphi)}, \qquad \psi_{\downarrow\downarrow}(\underline{r}) \sim const$$

in neutral case, current again falls off as $\frac{1}{r}$ → KE diverges as ln R, but $\rho_s \equiv \rho_{s\uparrow} = \frac{1}{2}\rho$ so KE of HQV $\frac{1}{2}$ that of ordinary vortex

in charged case, Ampère screening → for r >>λ, charge current → 0

but spin current finite $(\propto \frac{1}{r})$ → total KE diverges

For neutral case under rotation, expect



Upshot:

In a charged system (Sr_2RuO_4) HQV's are energetically disadvantaged vis-à-vis ordinary (Abrikosov) vortices, while in a neutral system (^{3}He-A) they are competitive with them and should occur.

Yet, experimentally: evidence for HQV's found* in Sr_2RuO_4, not found† (so far) in ^{3}He-A!

Possible explanations:

effect of dipole forces (x)

metastability

pathological narrowness of stability region

inadequacy of NMR detection technique

....

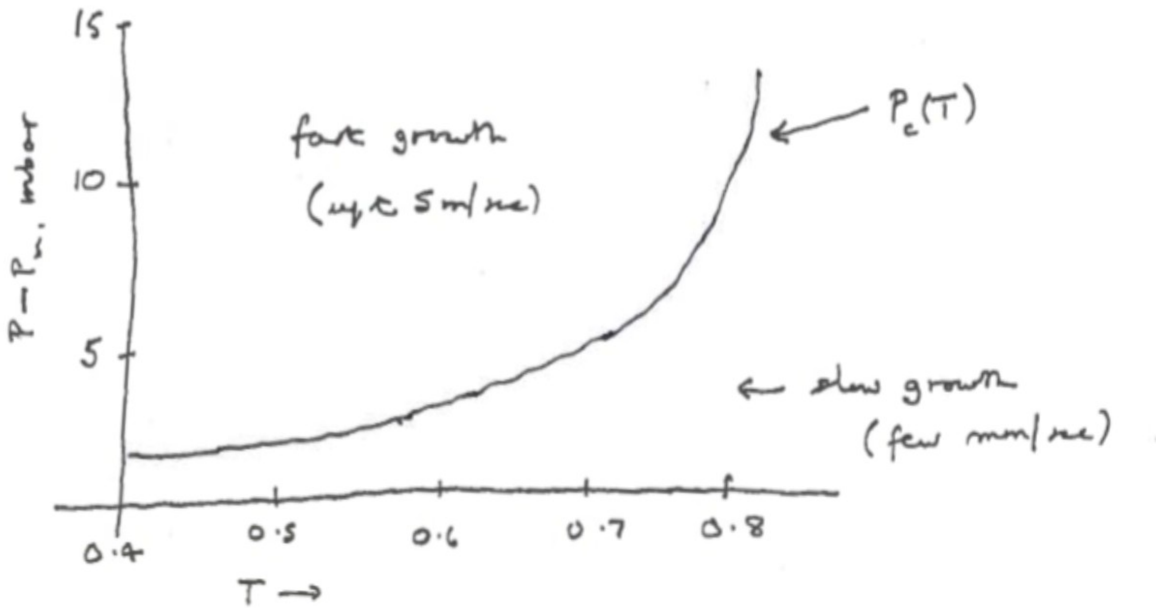
If none of these works....

* J.Jang et al,, Science 33, 186 (2011)

† M. Yamashita et al., PRL 101, 025302 (2008)

Y. Kimura et al., Proc. LT 27, 012006 (2014)

A) 2. “Explosive” growth of solid ^{4}He from the overpressurized superfluid liquid*



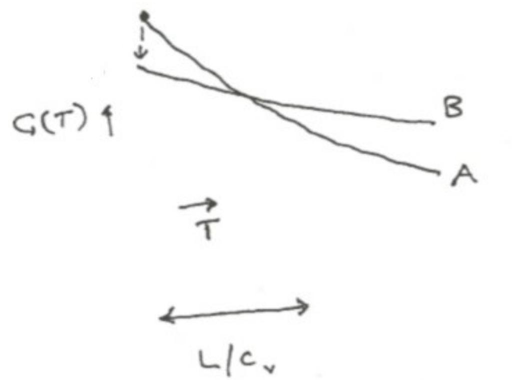
most analysis focuses on details of crystal facet roughenıng, etc., but:

could this simply be a “supercooling → hypercooling” transition ?

* V.L. Tsymbalenko, JLTP 138, 795 (2005)

V.L. Tsymbalenko, JETP 119, 700 (2014)

“Explosive” growth of solid ^{4}He from the overpressurized superfluid liquid



(a) “supercooling”: speed of transition limited by need to remove latent heat

(b)“hypercooling”: no need to get rid of latent heat →propagation at ~ speed of sound (“ice - IX”)

Predict: $$P_c(T) - P_m(T) = \frac{T(\Delta S)^2}{c_v|(\Delta V)|} \sim T\left|\left(\frac{dP_m}{dt}\right)\right|$$

general trend and order of magnitude right ...

any overpressurization experiments on superfluid ^{3}He ?

B) Novel regimes becoming available due to cryogenic advances etc.

1.Spin-wave-dominated regime in ultracold ^{3}He-B

Over most of current experimental range specific heat of B phase dominated by fermionic excitations. But for T$\rightarrow$0:

fermions: $c_v/k_B \simeq A_F(\frac{dn}{dE})(\Delta^{5/2}/T^{3/2})\exp(-\Delta/k_B T)$

bosons: $c_v/k_B \simeq A_B(\frac{k_B T}{\hbar\bar{c}})^3$

$A_F, A_B \sim 1$

$\bar{c} = \text{average velocity of boson}$

Since sound waves have $\bar{c} >> v_F$, but spin waves have $\bar{c} \sim \frac{1}{3} v_F$ at sufficiently low T, specific heat of ^{3}He-B dominated by spin waves.

Spin-wave-dominated regime in ultra ^{3}He-B

Ratio of bosonic/fermionic specific heat:

$$R(T) \equiv c_{vB}/c_{vF} = \frac{320\pi^3}{\sqrt{(2)}(\Delta/T_c)^{5/2}} \left(\frac{v_F}{\bar{c}_s}\right)^3 \left(\frac{k_B T_c}{p_F v_F}\right)^2 \left(\frac{T}{T_c}\right)^{7/2} \exp \Delta / k_B T$$

Constant factor ~ 2×10^{-3} (at svp)
R(T) is already ~ 1 at T/T_c =0.1 (at svp, at 20 bar >> 1!)

What would be "special" about spin-wave-dominated regime ?

Most obvious: since spin wave spectrum v. sensititve to magnetic field H,
thermodynamics will be strong f(H)
"normal" thermal conductivity K almost entirely due to spin waves:
"pseudo – Wiedemann-Franz" relation between K and D_s ? (D. Loss)

B2. “Bulk 2D state” of ^{3}He-A

General belief: (at least on diffusely scattering surface):

superfluidity in ^{3}He films requires $d \geq \xi(T)$

d – film thickness

T – coherence length (for T << T_c, ~ pair radius)

Is it interesting

(a) possibly by improving surface, to reach $d \leq \xi(T)$?

(b) with existing surfaces, by reducing T to reach $\xi < d \ll \hbar v_F / k_B T$?

Under condition (a), minimum gap is presumably “equatorial” one

→ thermodynamic properties similar to B phase

Under condition (b), minimum gap will be ~ $\hbar v_F / d \gg k_B T$

“Bulk 2D state” of ^{3}He-A

But in both cases normal - state Fermi sea contains lots of states with $k_z \sim k_F$

→ even if we accept “standard” wave function for p+ip state, namely

$$\psi_N \sim \left(\sum_k c_k a_k^\dagger a_{-k}^\dagger \right)^{N/2} |vac\rangle \qquad c_k \sim f(|\underline{k}|, k_z) \exp(i\varphi_k)$$

φ - angle of k in xy-plane

long -distance pair wave function in real space will **not** have the “MR” form

$$F(\underline{r}_i - \underline{r}_j) \propto \frac{1}{z_i - z_j}$$

C) When existing theory is challenged
1.Ultra – small samples of ^{3}He-A*

Longitudinal NMR in A phase described by effective Hamiltonian

$$\hat{H}\left(\hat{S}_z, \Delta\hat{\varphi}\right) = \left(\hat{S}_z^2/2\chi - \hat{S}_z h\right) - g_d \cos\left(\Delta\hat{\varphi}\right)$$

χ ≡ spin susceptibility

g_d ≡ dipole interaction

h ≡ external field

$\hat{S}_z$ ≡ z-comp. of total spin

$\hat{\Delta}\varphi$ ≡ phase difference of ↑↑ and ↓↓ Cooper pairs

$[\hat{S}_z, \Delta\hat{\varphi}] = -i$ canonically conjugate

$$|\Psi\rangle = \int_0^{2\pi} d(\Delta\varphi)\, c(\Delta\varphi) |\Delta\varphi\rangle$$

$$|\Delta\varphi\rangle \equiv \left(a_\uparrow^\dagger a_\uparrow^\dagger \exp(i\Delta\varphi/2) + a_\downarrow^\dagger a_\downarrow^\dagger \exp(-i\Delta\varphi/2)\right)^{N/2} |vac\rangle$$

* AJL, Synthetic Metals 141, 51 (2004)

Ultra – small samples of ^{3}He-A*

"Standard" theory of NMR based on semiclassical assumption

$$c(\Delta\varphi)=\delta(\Delta\varphi-\Delta\varphi_0)$$ (GP – like state)

Why does this work ?

$$\chi\sim N, g_D\sim N \rightarrow \langle(\delta S_z)^2\rangle\sim N, \langle\delta(\Delta\varphi)^2\rangle\sim 1/N$$

→ for N → ∞, $\Delta\varphi$ well-defined, S_z ill-defined

However, for small enough N situation is reversed !

Cross over occurs for $\lambda \equiv (g_D\chi) \sim 1$

i.e. V ~ 10^{-18} cm^3. (inclusions of liquid ^{3}He in solid ^{4}He ?)

The $ 64K question: for $\lambda \leqslant 1$, does a single sample of superfluid ^{3}He-A behave like the BEC gas in the Andrews et al. experiment ? i.e. in a given NMR run, does it "pick" a definite $\Delta\varphi$? If not, what does it do ?

2. Superfluid ^{3}He-A: angular momentum and surface currents

"p+ip" Fermi superfluid: Sr_2Ru0_4, ^{3}He-A ← much "cleaner" !

Infinite space:

standard ansatz: $\Psi_N \ (\hat{\Omega}^\dagger)^{N/2}|vac\rangle \ , \hat{\Omega}^\dagger = \sum_k c_k a_k^\dagger a_{-k}^\dagger$ $c_k \propto \exp(i\varphi_k)$ - polar angle of k (*)

Restricted (e.g. cylindrical) geometry**: $[\hat{L}_z, \hat{\Omega}^\dagger] = i\hbar\Omega^\dagger \rightarrow \langle \hat{L}_z \rangle = N\hbar/2$

most calculations based on quasiclassical Green's function methods, conclude that at least for reasonably smooth surface

$$\langle \hat{L}_z \rangle \sim N\hbar/2 \ , \ \langle \hat{J}_s \rangle \sim (N\hbar/2)/R$$ ←"large" ,

↑ - surface current

Problem†: in Sr_2Ru0_4 to date experiments fail to detect "large" surface current (+ set upper limit ~ 10^{-3} of expected value)

A radical solution: (*) is wrong

** Sauls, Phys. Rev. B 84, 214509 (2011)

Tsutsumi + Machida, Phys. Rev. B 85, 100506 (2012)

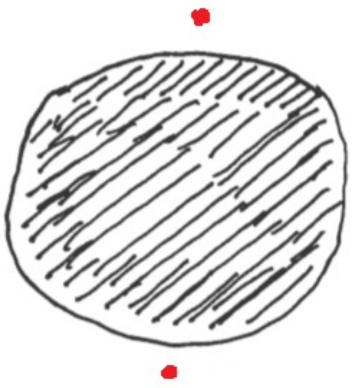
Nagai, JLTP 175, 44 (2014)

† C. Kallin, Reps. Prog. Phys. 75, 042501 (2012), section 7

Superfluid ^{3}He-A: angular momentum and surface currents

Consider: Cooper problem (2 extra electrons above F. Sea)

$$\psi_N \sim \sum_k c_k a_k^\dagger a_{-k}^\dagger |FS\rangle \qquad c_k \propto \exp(i\varphi_k)$$



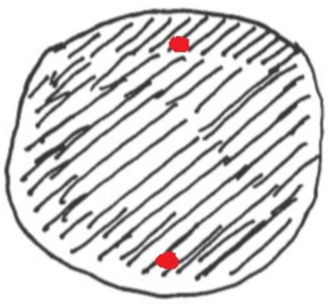
extra particle number: 2

extra angular momentum: $\hbar$

Superfluid ^{3}He-A: angular momentum and surface currents

But now consider "anti-Cooper" problem (2 holes below F. Surface)

$$\Psi_c \sim \sum_k d_k a_{-k} a_k |FS\rangle \qquad d_k \propto \exp(-i\varphi_k)$$



extra particle no: -2

extra angular momentum: $-\hbar$

What if GS is (schematically) a superposition of Cooper and anti-Cooper pairs ?

then (in infinite geometry) $\langle \hat{L}_z \rangle \leq (\Delta / E_F)^2 N\hbar/2 \ll N\hbar/2$

Can implement* for finite geometry: then predict

$$\langle J_s \rangle \sim (\Delta/E_F)^2 (N\hbar/2)/R << (N\hbar/2)/R$$

Can one test experimentally, in ^{3}He-A, not just for total angular momentum but for surface currents ?

*AJL + Yiruo Lin, unpublished

3. Majorana fermions and all that: are standard "mean field" methods adequate ?

Established picture: just as in a TI (topological insulator) one-electron states are quantum superpositions of different bands, e.g.

$$\Psi_{TI}=u_k a_s^\dagger+v_k a_p^\dagger$$

so in a TS (topological superconductor) one-fermion states are quantum superpositions of "particle" and "hole"

$$\hat{\Psi}_{TS}^\dagger=u(r)\hat{\psi}(r)^\dagger+v(r)\hat{\psi}(r) \qquad \left(\sim u_p a_p^\dagger+v_p a_{-p}\right) \qquad (*)$$

This is justified by appeal to ideas of "spontaneously broken U(1) symmetry" which unfortunately, IS WRONG!

Majorana fermions and all that: are standard "mean field" methods adequate ?

To conserve total particle number, need

$$\hat{\Psi}^\dagger_{TS} = u(r)\hat{\psi}^\dagger(r)+v(r)\hat{\psi}(r)\hat{C}^\dagger \text{ extra Cooper pair} \qquad (\dagger)$$

Does this make any difference ?

Probably not, if C. pairs are "boring" (no internal DOF's, etc)

So most existing applications unaffected

But maybe yes, if C. pairs are "interesting" ...

Example: Galilean invariance violated by (*), restored by (†)

One obvious consequence:

Majorana no longer self-conjugate !

Worse: when considering quantum-information operations such as braiding, need to consider the extra Cooper pair → may change results qualitatively

Major challenge to theory + experiment in next 20 years !