CONSTRAINTS ON MODELS FOR THE HIGH- AND LOW-ENERGY X-RAY BACKGROUNDS

R. A. DALY
Joseph Henry Laboratories, Princeton University, Princeton, NJ 08544
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ABSTRACT

The high-energy X-ray background most likely results from unresolved sources, since the model in which the background results from a smooth hot intergalactic medium at semi-relativistic temperatures is ruled out by the COBE measurements. Models in which the high- and low-energy backgrounds are produced by sources are constrained by general arguments based on energetics. In addition, the properties of the sources which produce the low-energy background, and of those producing the high-energy background if the spectrum of these sources extrapolates to low energies, are constrained by the flux and isotropy limits of the Einstein deep survey. The application of the energy, flux, and isotropy constraints depend on the cosmological model, that is, on whether the universe is flat and matter dominated, open, or flat and dominated by a cosmological constant. The flux and isotropy arguments depend directly on the cosmological model, and the energy arguments are indirectly tied to the cosmological model through the Hubble constant. These constraints are discussed and applied to particular models to determine which models are viable sources of the high- and low-energy backgrounds in different cosmological models. Predictions of the models are discussed, as are the constraints which will result from the ROSAT deep survey.

Two of the three models considered have difficulty producing the high-energy X-ray background in a flat, matter-dominated universe. The massive black hole model is able to account for the observed characteristics of the high-energy X-ray background in a flat matter dominated universe, but the massive X-ray binary and supernovae models are inconsistent with the observations. In an open universe, or a flat universe dominated by a cosmological constant, both the massive black hole model and the supernovae model are consistent with the constraints and could produce the high-energy background.

The low-energy background could be produced by either massive black holes, or supernovae explosions in young galaxies, in any cosmological model. The massive X-ray binary model is only marginally acceptable as a source of the low-energy background and is unlikely to be the source of the high-energy background.

Subject headings: cosmology — X-rays: sources

1. INTRODUCTION

The X-ray background naturally divides into two energy regimes, that from 3 to about 200 keV, which is referred to as the high-energy background, and that from about 1 to 3 keV, which is referred to as the low-energy background. The origins of these backgrounds remain unknown.

The smooth hot intergalactic medium model discussed by Cowie & Kobetich (1972), Field (1972), Field & Perrenod (1977), Guilbert & Fabian (1986), Barcons (1987), and others for the high-energy X-ray background is no longer a viable candidate since the hot gas would produce an observable distortion in the Wien region of the microwave background radiation (Guilbert & Fabian 1986; Taylor & Wright 1989; Lahov, Loeb, & McKee 1990; Mather et al. 1990). Therefore, it is most likely that the backgrounds result from the integrated emission of unresolved sources, which is not meant to imply that the sources cannot be large and extended. Such models are discussed, for example, by Bookbinder et al. (1980), Leiter & Boldt (1982), Boldt & Leiter (1987), Daly (1987), Schwartz & Tucker (1988), Grindlay & Luke (1990), and Zdziarski (1988).

Models in which thermal bremsstrahlung emission from clumped hot gas produces the high-energy X-ray background appear to be ruled out (Rogers & Field 1990), which increases the likelihood that the source of the high-energy background is either emission from distant galaxies, or emission associated with active galactic nuclei. The difficulties that these models have in explaining the energy density of the X-ray background are discussed by Cowie (1990). The arguments presented by Rogers & Field (1990) are likely to constrain models in which the X-ray background is produced by thermal bremsstrahlung emission associated with the formation of structures in the universe (e.g., Daly & Turner 1988; Cen et al. 1990; Loeb & Ostriker 1991).

It is interesting to note that local sources contribute only a small fraction of the flux of the high- and low-energy backgrounds (e.g., De Zotti et al. 1989; Persic et al. 1989), and that no source with a spectrum remarkably similar to that of the high-energy X-ray background has been detected. This may indicate that the X-ray emission phase which produces the high- and/or low-energy X-ray backgrounds is long past, or that there have not been sufficient high-energy observations to detect the low-redshift X-ray sources that are similar to the sources which produce the high-energy background. This later possibility is suggested by the brightness fluctuations observed at high energies (Boldt 1989), and by the recent observations of Morisawa et al. (1990), as predicted by the model of Schwartz & Tucker (1988).

Assuming that the backgrounds result from discrete sources, the properties of these sources are constrained. Arguments based on energetics, discussed in § 2.1, constrain the properties of any source population which is to produce the high- or low-energy X-ray background. The source characteristics are also constrained by the flux limit of the Einstein deep survey if the sources are to comprise the low-energy background, or if
the sources comprising the high-energy background have spectra that extrapolate to low energies, as discussed in § 2.2. Likewise, the properties of the sources of the low-energy background are constrained by the isotropy of the background at low energies, as are those which comprise the high-energy background if they contribute some fraction of the flux in the low-energy band, as discussed in § 2.3.

The constraints imposed by the energy, flux, and isotropy arguments are cosmology dependent. The most severe constraints result in a flat matter-dominated cosmological model; the constraints are less severe in an open universe, and even less severe in a spatially flat universe dominated by a cosmological constant. The flux (§ 2.2) and isotropy (§ 2.3) constraints depend upon the cosmological model because the comoving coordinate distance to the source depends on the cosmological model. For a source at a given redshift the comoving coordinate distance is smallest in a flat matter-dominated universe, and becomes progressively larger in an open universe, and in a flat universe with a significant cosmological constant. Hence, it is easier to detect a source at a given redshift with a given luminosity in a flat matter-dominated universe than in an open universe, or one which is spatially flat but dominated by a cosmological constant. Therefore, the flux and isotropy constraints that are difficult to satisfy in an Einstein–de Sitter universe are more easily satisfied in an open model, or a flat model in which the energy density of the universe is dominated by a cosmological constant.

The energy arguments (§ 2.1) depend on the cosmological model through the value of the Hubble constant $H_0 = 100h$ km s$^{-1}$ Mpc$^{-1}$. The bound on the current age of the universe of $\gtrsim 12.5 \times 10^9$ yr implies that $h \leq 0.5$ in a flat matter-dominated universe, $h \leq 0.75$ in an open model, and $h \leq 0.95$ in a flat model with a significant cosmological constant (Peebles 1984); the last value has been obtained assuming that the current ratio of the mean density of matter relative to the critical value is $\Omega_0 = 0.1$, hence the energy density of the cosmological constant relative to the critical value at the current epoch is $\Omega_0 = 1 - \Omega_0$ (note that as $\Omega_0$ decreases $h$ may increase). It is much easier to account for the energy density of the X-ray background in models with large values of $h$ than it is in models with small values of $h$, as discussed in §§ 3.2.1, 3.2.5, and 4.

Three specific models are considered in light of the energy, flux, and isotropy constraints: the massive X-ray binary model (§ 3.1), active galactic nuclei–massive black hole models (§ 3.2), and the supernovae model (§ 3.3). The outcome of the comparison of the models with the observations in different cosmological models are summarized in Table 3.

The models fall into two broad categories: those in which the sources of the high-energy background have spectra that extrapolate to low energies, and those in which the sources of the high-energy background have spectra that turn over at $\sim 3$ keV (moving from high to low energies). In the first case one population comprises the majority of both the high- and low-energy backgrounds, and in the second case there are two distinct populations of sources, one of which produces the high-energy background, and one of which produces the low-energy background. The results of the ROSAT deep survey can be used to discriminate between these two possibilities, as discussed in §§ 3.2.4 and 4; since the models can reliably be predicted to fall into one of these categories, the results of the ROSAT deep survey should significantly constrain models for the high-energy background as well as the low-energy background.

2. CONSTRAINTS

2.1. Energy Constraints

The total energy density in the 2–200 keV X-ray background is about $10^{-16}$ ergs cm$^{-3}$ (e.g., Daly & Turner 1988), and that of the 1–3 keV background is, at most, about $2 \times 10^{-17}$ ergs cm$^{-3}$ (Wu et al. 1991). For simplicity, the total energy density $\rho_T$ in the background is parameterized by $\eta$: $\rho_T = \eta 10^{-16}$ ergs cm$^{-3}$, with $\eta \approx 0.1$–0.2 for the 1–3 keV background, and $\eta \approx 1$ for the 1–200 keV background. Sources with a comoving density $n_0$ each with a characteristic luminosity $L_\nu$ emitting X-rays for a time $t_\nu$ at a characteristic redshift $z_\nu$ must satisfy $\rho_T = n_0 L_\nu t_\nu (1 + z_\nu)^{-1}$ if they are to produce the high- or low-energy X-ray background. Each source emits a total energy $E_\nu = L_\nu t_\nu$ in X-rays. If rest mass is converted into X-rays with an efficiency $\epsilon$, then the total mass of each source involved with the X-ray emission is $M_{\text{rm}} = E_\nu \epsilon / c^2$, $c$ being the speed of light.

The comoving number density of the sources that produce the X-ray background may be compared to the comoving number density of galaxies at the current epoch. The current comoving number density of galaxies is obtained by integrating over the luminosity function. The observed luminosity function is characterized by a Schechter function (Schechter 1976) with a slope $\alpha = -1.07$, $M_\nu = -19.68$, and $\phi * = 1.56 \times 10^{-2}$ Mpc$^{-3}$ for a Hubble constant of $H_0 = 100h$ km s$^{-1}$ Mpc$^{-1}$ (Elstathiou, Ellis, & Peterson 1988). Integration of the luminosity function indicates that the mean number density of galaxies at the current epoch with a luminosity brighter than $L_\nu$ is about $3 \times 10^{-2} h^3$ Mpc$^{-3}$; galaxies brighter than $0.1 L_\nu$ have a mean number density of about $3 \times 10^{-2} h^3$ Mpc$^{-3}$; and galaxies with a luminosity greater than $0.01 L_\nu$ have a mean number density at the current epoch of about $10^{-1} h^3$ Mpc$^{-3}$. For reference the Milky Way galaxy is an $L_\nu$ galaxy, and a typical dwarf galaxy, such as the Small Magellanic Cloud, has a luminosity of about $0.01 L_\nu$; the respective masses including dark matter are estimated to be about $10^{12}$ and $10^9 M_\odot$.

The comoving number density of sources $n_0$ producing the X-ray background is parameterized in terms of the current comoving number density of galaxies, so that the number densities can be easily compared. Hence, let $n_0 = 10^{-2} h^2$ Mpc$^{-3}$; for values of $\eta$ of about 0.3 the comoving number density of the sources comprising the X-ray background is comparable to that of all galaxies brighter than $L_\nu$ at the current epoch, for $\chi \approx 3$ it is comparable to that of all galaxies brighter than $0.1 L_\nu$, and for $\chi \approx 10$ it is comparable to that of all galaxies brighter than about $0.01 L_\nu$.

Combining these expressions the total energy radiated as X-rays from a source $E_\nu = \rho_T n_0 (1 + z_\nu)$ gives, with $\rho_T = \eta 10^{-16}$ ergs cm$^{-3}$,

$$E_\nu \approx L_\nu t_\nu \approx 3 \times 10^{50} \chi^{-1} \eta^{-1} h^{-3} (1 + z_\nu) \text{ergs}. \quad (1)$$

The rest mass which is converted into X-rays with an efficiency $\epsilon$ is

$$M_{\text{rm}} \approx 10^\chi \eta^{-1} \frac{\epsilon}{0.1} \left(\frac{h}{0.5}\right)^{-3} M_\odot. \quad (2)$$

The strategy is to consider a particular model and calculate the efficiency factor with which rest mass is converted into X-rays. This indicates a minimum mass for the galaxy via equation (2). Given the mass, only certain values of $\chi$ are allowed since the value of $\chi$ is related to the total mass; if the total mass is fairly large, $\gtrsim 10^{52} \chi^{-1} \eta M_\odot$, then $\chi$ must be fairly...
small, \( \chi \lesssim 0.3 \), when \( \eta \sim 1 \). Hence, given the efficiency factor, and \( \eta \) (that is, the fraction of the energy density of the total background that the sources comprise), a mass is indicated which depends on the comoving number density of the sources through the parameter \( \chi \), on the characteristic redshift of the emission \( z_\text{e} \), and on Hubble's constant \( h \). Considerations of this mass indicate an upper bound on the comoving number density of the sources, that is, on \( \chi \).

Another important parameter is the mass density \( \Omega_{m,z} \) involved with the production of the X-ray background relative to the critical density \( \rho_c \):

\[
\Omega_{m,z} = \frac{n_\gamma M_{\text{rm}}}{\rho_c} \approx 6 \times 10^{-7} \eta (1 + z_\text{e}) \left( \frac{\epsilon}{0.1} \right)^{-1}.
\]  (3)

The fraction of the baryonic mass of the universe that must be actively involved with the X-ray production, referred to as the active fraction \( \Delta_\epsilon \), is

\[
\Delta_\epsilon = \frac{\Omega_{m,z}}{\Omega_\text{b}} \approx 6 \times 10^{-7} \eta (1 + z_\text{e}) \left( \frac{\epsilon}{0.1} \right)^{-1}
\]  (4)

for \( \Omega_\text{b} h^2 \approx 0.1 \), where \( \Omega_\text{b} \) is the current mass density in baryons relative to the critical density of the universe. Here, \( \epsilon \) is the efficiency factor relevant for the material that is directly involved in the X-ray production. Note that this is quite generous since \( \Omega_\text{b} h^2 \) is likely to be 0.01 (Olive et al. 1990). In addition, often the relevant parameter is the fraction of baryons within galaxies that are directly involved with the X-ray emission. Since this is easily less than 1/10 of the mean mass density of baryons used in equation (4), the active fraction relevant for baryons in galaxies is a factor of at least 10 larger than that given by equation (4). Note also that the active fraction \( \Delta_\epsilon \) depends primarily on the efficiency with which rest mass energy is converted into X-rays, and on \( n_\gamma \), the energy density in the background under consideration. It is independent of the Hubble constant and of \( \chi \), which characterizes the comoving number density of the sources contributing to the high- or low-energy background.

### 2.2. Flux Constraints

If the time scale \( t_\text{s} \) over which the X-ray emission occurs can be estimated, the characteristic luminosity per source \( L_\text{x} \) results:

\[
L_\text{x} = E_\gamma / t_\text{s}.
\]

If the characteristic redshift of the emission can be inferred, then the flux per source can be estimated, and the number of sources per square degree can be calculated from the comoving number density of the sources and the time of emission per source. These are constrained by the *Einstein* deep survey if the sources comprise the low-energy X-ray background, or if the spectra of the sources which produce the high-energy background extrapolate smoothly to low energies.

A source at a redshift \( z \) with a luminosity \( L_\text{x} \) will produce a flux \( f_\gamma \), given by

\[
f_\gamma = \frac{L_\text{x}}{4\pi r^2 a_\text{d}^2 (1 + z)^2},
\]  (5)

where the comoving coordinate distance \( r \) depends on the redshift of the source, and on the cosmological model; expressions for \( f_\gamma \) in a flat matter-dominated model and in an open model are given in the Appendix.

Equation (5) describes the relationship between the total X-ray luminosity and the total flux. The flux limit imposed by the *Einstein* deep survey applies to the flux observed over the waveband from 1 to 3 keV, that is, on the flux density of the source integrated over the energy range from 1 to 3 keV. Therefore, the fraction of the luminosity emitted over the (1–3)(1 + z) keV band relative to the total X-ray luminosity must be determined, so that the estimated fluxes, obtained from equation (5), can be compared with the *Einstein* deep survey limits.

The observed spectrum of the high-energy X-ray background is well fitted by thermal bremsstrahlung emission from gas at a temperature of about 35 keV \((1 + z)\). Assume that, irrespective of the X-ray emission mechanism, the sources of the background have an intrinsic spectrum that can be described by thermal bremsstrahlung emission from hot gas at a temperature \( kT_0(1 + z) \). Then, for energies \( E \ll kT \) the ratio of the luminosity emitted over the energy range from \( E_1 \) to \( E_2 \) relative to the total X-ray luminosity is

\[
\beta = \frac{L(E_1, z)}{L(z)} \approx 1.4 \left( \frac{E_2}{kT} \right)^{0.6} \left( \frac{E_1}{kT} \right)^{0.4},
\]

since the emission coefficient for thermal bremsstrahlung varies as \((E/kT)^{-0.4}\) for energies \( \lesssim kT \).

Note that the ratio \( L(E_1, z)/L(z) \) is independent of redshift, since the energy \( E \) scales with redshift, and the temperature \( T \) is written \( T = T_0(1 + z) \), where \( T_0 \) is the temperature at a redshift of zero which parameterizes the spectrum of the source. Choosing the energies relevant for the *Einstein* deep survey, \( 1–3 \) keV, the fraction of the total X-ray flux observable is

\[
\beta \approx 0.15 \quad \text{for} \quad kT_0 \approx 35 \text{ keV}, \quad \text{and} \quad \beta \approx 0.4 \quad \text{for} \quad kT_0 \approx 7 \text{ keV}.
\]

A large temperature of about 35 keV is applicable to the sources of the high-energy background, and a lower temperature is applicable to the sources of the low-energy background.

Note that this does not assume that the emission mechanism is thermal bremsstrahlung; it assumes that the emission spectrum resembles thermal bremsstrahlung.

The 1–3 keV flux per source, \( \beta f_\gamma \), is constrained by the *Einstein* deep survey to be less than about \( \beta f_\gamma \leq 2.6 \times 10^{-14} \) ergs s\(^{-1}\) cm\(^{-2}\) (Giacconi et al. 1979; Griffiths et al. 1983; Boldt 1987).

### 2.3. Isotropy Constraints

The isotropy of the high-energy X-ray background is constrained by the fluctuations analysis of the *HEAO* data. Shafer (1983) considered the rms fluctuations of the intensity on \( 3^\circ \times 3^\circ \) areas. This constrains the number of sources per square degree to be greater than about 50, assuming that the sources are randomly distributed. The isotropy of the low-energy X-ray background is indicated by the fluctuations analysis of the *Einstein* deep fields (Hamilton & Helfand 1987). Each pixel is about 1 arcmin\(^2\) and each field is about 30\(^\circ\) × 30\(^\circ\). Hamilton & Helfand find that, over the energy band from 1 to 3 keV, at least 3000 sources deg\(^{-2}\) are required if the sources are randomly distributed; a larger surface density of sources is required if the sources are clustered.

Some fraction of the flux in the 1–3 keV band may be due to the low-energy tail of the high-energy background, in which case the fluctuations analysis done at low energies constrains the number of sources per square degree that produce the high-energy background. If the sources comprising the background are randomly distributed, the fluctuations \( \delta I / I = 1 / \sqrt{N} \), where \( \delta I \) is the rms deviation of the flux per unit area from the mean value \( I \) and \( N \) is the number of sources per unit area; if the sources are clustered, then the same number density of sources produces larger fluctuations. Hence, given the isotropy constraint, if the sources are clustered, an even larger surface density of sources than is estimated here is needed for a model to remain consistent with the data.
Suppose that the low-energy tail of the high-energy background comprises a fraction \( f \) of the total flux in the 1–3 keV band. Then these sources will produce fluctuations in the 1–3 keV band which are suppressed by the factor \( f \), so the minimum number of sources per square degree producing the high-energy background is \( N_b \approx f^2 N_t \), where \( N_t \) is the minimum number of sources per degree squared inferred from the fluctuations analysis of the low-energy background. Hamilton & Helfand (1987) find that \( N_t \geq 5000 \). Wu et al. (1991) find that the total flux in the 1–3 keV band is at most twice that expected by extrapolating the spectrum of the high-energy background. Therefore, any model to explain the high-energy background in which the spectrum extrapolates to low energies must have \( N_b \geq 1000 \) sources per square degree.

The number of sources per square degree depends primarily upon two factors given that the comoving number density of sources is fixed: the probability \( P \) that a source is emitting X-rays at any given time, and the volume element \( dV \), which depends upon the cosmological model, as described in the Appendix. If the sources are X-ray emitters for the entire time span from \( z_2 \) to \( z_1 \), then \( P = 1 \). However, if a source is an X-ray emitter for a fraction of the Hubble time at the redshift of the source, then the probability that at any given time the source is contributing to the background is \( P = t_s / t_d(z) \), where \( t_s \) is the typical time scale for which a given object is an X-ray source and \( t_d(z) \) is the Hubble time at the redshift of the source.

The surface density of sources per steradian is

\[
N = \frac{1}{4\pi} \int_{z_1}^{z_2} (n P) dV,
\]

where \( n \) is the number density of sources and \( dV \) is the volume element. The surface densities of sources in a flat matter-dominated universe, and in an open universe with deceleration parameter \( q_0 = 0 \), for \( P < 1 \) and for \( P = 1 \) are computed in the Appendix, § A2, and tabulated in Tables 1 and 2.

Each of the models for the origin of the high- and low-energy X-ray backgrounds is considered in four limiting cases: (1) \( t_s \leq t_d(z_2) \) in a flat matter-dominated universe; (2) in an open \((q_0 = 0)\) universe; (3) \( t_s \geq t_d(z_2) \) in a flat matter-dominated universe; and (4) in an open universe. The equations that give the surface density of sources in each of these four cases are presented in the Appendix. Note that since in an open universe all of the models survive the comparison to the observations, they will survive a comparison to the observations by a wider margin if the universe is spatially flat and dominated by a cosmological constant.

A comparison of the number of sources per square degree in a flat, matter-dominated universe with that in an open \((q_0 = 0)\) universe over a specific redshift interval \((z_1, z_2)\) illustrates why it is much easier for a particular model to satisfy the isotropy constraints in an open model (or in a flat, cosmological constant-dominated model) than in a flat matter-dominated model. In Table 1 the number of sources per square degree in a matter-dominated flat universe \((N_1)\) and in an open model \((N_4)\) when \( t_s \leq t_d(z_2) \) are compared. In Table 2 the number of sources per square degree in a flat matter-dominated universe \((N_1)\), and in an open universe \((N_4)\), when \( t_s \geq t_d(z_2) \) are compared. The factor by which the surface density of sources increases in going from a flat matter-dominated universe to an open \((q_0 = 0)\) universe, given that the comoving number density of sources and the redshift interval are fixed, is indicated in the last column of each table.

### Table 1

<table>
<thead>
<tr>
<th>( z_1 )</th>
<th>( z_2 )</th>
<th>( N_1(q_0 = 0.5) )</th>
<th>( N_4(q_0 = 0) )</th>
<th>Factor (approximate)</th>
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<td>( 9 \times 10^4 P(t_s/t_h) )</td>
<td>3</td>
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<tr>
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</tr>
<tr>
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<td>5.0...</td>
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<td>( 10^4 P(t_s/t_h) )</td>
<td>5</td>
</tr>
</tbody>
</table>

Notes: Surface density of sources (per square degree) when the characteristic time scale of the X-ray emission \( t_s \) exceeds the Hubble time \( t_d(z_2) \), so \( t_s \geq t_d(z_2) \); in this case \( t_d(z_1) = t_s + t_d(z_2) \). Surface density of sources for the redshift intervals indicated have been computed using the expressions given in the Appendix for a flat matter-dominated universe \((q_0 = 0.5)\) and an open universe \((q_0 = 0)\); the surface density of sources is proportional to the comoving number density of the sources, parameterized by \( P \). The factor by which the surface density increases in an open model relative to a flat matter-dominated model is indicated in the last column.

### Table 2

<table>
<thead>
<tr>
<th>( z_1 )</th>
<th>( z_2 )</th>
<th>( N_1(q_0 = 0.5) )</th>
<th>( N_4(q_0 = 0) )</th>
<th>Factor (approximate)</th>
</tr>
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<td>( 2 \times 10^4 \chi )</td>
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Notes: Surface density of sources (per square degree) when the characteristic time scale of the X-ray emission \( t_s \) exceeds the Hubble time \( t_d(z_2) \), so \( t_s \geq t_d(z_2) \); in this case \( t_d(z_1) = t_s + t_d(z_2) \). Surface density of sources for the redshift intervals indicated have been computed using the expressions given in the Appendix for a flat matter-dominated universe \((q_0 = 0.5)\) and an open universe \((q_0 = 0)\); the surface density of sources is proportional to the comoving number density of the sources, parameterized by \( \chi \). The factor by which the surface density increases in an open model relative to a flat matter-dominated model is indicated in the last column.

### 3. Models

#### 3.1. The Massive X-Ray Binary Model

A model in which X-ray emission from massive X-ray binaries produces the high-energy X-ray background has been discussed by several authors (Bookbinder et al. 1980; Griffiths 1989 and references therein; Griffiths & Padovani 1990). This model is motivated by the resemblance between the spectra of massive X-ray binary systems and that of the X-ray background. Additional motivation arises from the correlation between the infrared and X-ray properties of starburst galaxies, making massive X-ray binary systems an attractive possibility for the source of the high-energy X-ray background (Griffiths & Padovani 1990).

Massive X-ray binaries consist of a \( \sim 20 M_\odot \) primary star and an accreting secondary, with a mass between about 1 and 4 \( M_\odot \), separated by less than 1 AU, that is, less than about \( 10^{13} \) cm. The X-ray emission results during the process of mass transfer from the primary to the secondary (van Paradijs 1983; van den Heuvel 1983). In the Galaxy, about 20 massive X-ray binary systems with a temperature greater than about 15 keV have been identified, and these have typical luminosities of
10^{38} \text{ ergs s}^{-1}. The X-ray-emitting phase is expected to commence about 10^7 yr after the binary system forms and is expected to last for about 10^5–10^7 yr. The X-ray luminosity may be as large as 5 \times 10^{38} \text{ ergs s}^{-1} \text{ (the Eddington luminosity for a 4 } M_{\odot}\text{ object), indicating an efficiency per binary system of } \epsilon_{eb} \sim 10^{-3} \text{ if the system is an X-ray source for } 2 \times 10^6 \text{ yr, which is probably an overestimate and therefore is erring in the direction of being generous to the model.}

There are two additional factors which determine the total efficiency \epsilon with which rest mass is converted into X-rays: the efficiency with which 20 \ M_{\odot}\text{ stars form, } \epsilon_{s}\text{, and the efficiency with which these 20 } M_{\odot}\text{ stars form very close (separations less than 1 AU) binary systems, } \epsilon_b. \text{ The total efficiency is } \epsilon = \epsilon_{eb}\epsilon_{s}\epsilon_b.\n
Three energy constraints are applied to the massive X-ray binary model to see if these systems can produce the high- or low-energy X-ray backgrounds in an open or flat cosmological model: considerations of the mass per galaxy and the comoving number density of galaxies with this mass (§ 3.1.1), the total fraction of the baryons in the universe that must be processed through a massive X-ray binary system (§ 3.1.2), and the metals produced by the massive X-ray binary systems (§ 3.1.3).

3.1.1. Constraints from the Mass per Galaxy

The total mass of the galaxy associated with the X-ray emission if this emission is to be the source of the high-energy X-ray background may be obtained from equation (2):
\[
M_{\text{rm}} \sim 10^9 \epsilon_{20}^{-1} \epsilon_s^{-1} \left( \frac{\epsilon_{eb}}{10^{-3}} \right)^{-1} \eta^{-1} (1 + z_e) \left( \frac{h}{0.5} \right)^{-3} M_{\odot}. \tag{7a}
\]

Since the product of the inverse of the efficiency factors in this equation is expected to be \((\epsilon_{20} \epsilon_s)^{-1} \gg 10^3\), the total mass of the galaxy must be fairly large, hence \(\chi \approx 0.3\), indicating that we are considering galaxies with a comoving number density similar to that of \(L^*\) galaxies at the current epoch, or objects that may be the precursors to present-day \(L^*\) galaxies, since lower mass galaxies do not have the requisite total mass.

This may be seen more readily by considering a few examples. Consider an Einstein–de Sitter universe; then, \(h\) must be small, \(h \approx 0.5\), so that the current age of the universe is reasonable. If the sources are at a redshift \(z_e \approx 1\) then the total mass per galaxy that is or once was in a massive X-ray binary system is \(M_{\text{rm}} \gtrsim 2 \times 10^6 \eta^{-1} h^{-3/2} M_{\odot}\). If these are to produce the high-energy background, \(\eta \approx 1\). So, if \(\chi \approx 3\), indicating galaxies with a comoving number density comparable to all galaxies greater than 0.1L\* (observed at the current epoch), then \(M \approx 7 \times 10^8 \ M_{\odot}\). If each of these galaxies has a total mass of \(\sim 10^{11} M_{\odot}\), then about 1% of the mass of the galaxy (including dark matter) would have to be or have been in a massive X-ray binary system; this means that about 10% of the available, baryonic mass of the galaxy (i.e., excluding dark matter) would have to be or have been in a massive X-ray binary system.

The fraction of the mass per galaxy involved with the X-ray emission is minimized when the low-energy X-ray background in an open universe is considered. In this case \(\eta \approx 0.1\), and it is reasonable to consider larger values of \(h, h \approx 0.75\), which gives a current age of the universe of about 12.5 \times 10^9 (h/0.75)^{-1} \text{ yr (see § 1). In this case the mass in X-ray binary systems (per galaxy) is } M_{\text{rm}} \approx 6 \times 10^7 \chi^{-1} \ M_{\odot} \text{ for } z_e \approx 1. \text{ Let us consider the extreme values of } \chi: \ 0.3–10. \text{ For } \chi \approx 10 \text{ the comoving number density of galaxies which are X-ray sources is comparable to all galaxies at the current epoch brighter than 0.01L\*. The requisite mass per galaxy in a massive X-ray binary system is } \sim 6 \times 10^6 \ M_{\odot}. \text{ These galaxies are expected to have a total mass of about } 10^8 \ M_{\odot} \text{ (about 10% of which is in the form of dark matter), so that about 0.6% of the mass of the galaxy, or about 6% of the known baryonic component (i.e., excluding dark matter), would have to be in or have been in a massive X-ray binary system, which seems quite large. For } \chi = 0.3, \text{ the mass per galaxy associated directly with massive X-ray binary systems (see eq. [7a]) is } M_{\text{rm}} \sim 2 \times 10^6 \ M_{\odot}, \text{ for } z_e \approx 1. \text{ This corresponds to about 0.02% of the mass per } L^* \text{ galaxy, assuming that each galaxy is about } 10^{12} \ M_{\odot} \text{, or about 0.2% of the known baryonic component. Is this acceptable? It seems like a very high fraction of the mass of the galaxy, but the constraints from the active fraction (§ 3.1.2) and from the metallicity (§ 3.1.3) do not demonstratively rule out the model, hence this model is deemed marginally acceptable as a source of the low-energy background, indicated by "M-OK" in Table 3. In an Einstein–de Sitter universe, } h \approx 0.5, \text{ and these fractions of galactic masses are increased by a factor of 3; hence X-ray binaries are unlikely to be the source of the low-energy background in an Einstein–de Sitter universe, but the model is conservatively deemed marginally acceptable. "M-OK." When the high-energy background is under consideration, } \eta \approx 1, \text{ and the mass per galaxy involved with the X-ray production is a factor of 10 larger than given above; hence in an open universe about 2% of the baryonic component of each } L^* \text{ galaxy, or 60% of the baryonic component in each } 0.01L^* \text{ galaxy, must be in or have been in a massive X-ray binary system. If a closed universe is under consideration, } h \approx 0.5, \text{ and these fractions increase by a factor of 3. Hence, in order for massive X-ray binaries to contribute significantly to the high-energy X-ray background a substantial fraction of the galactic mass must have been in a massive X-ray binary system, which is problematic, as indicated by the P's in Table 3. Cowie (1990), using energy arguments, arrives at a similar conclusion. The fraction of the galactic mass in stars with masses greater than about } 20 \ M_{\odot} \text{ may be estimated from the initial mass function (IMF). Assuming an IMF that is } \propto M^{-3.2}, \text{ for } 1 \ M_{\odot} \leq M \leq 20 \ M_{\odot} \text{ yields the mass ratio } M(M_{\star} > 20 \ M_{\odot})/M(1–20 \ M_{\odot}) \sim 10^{-2}. \text{ This indicates a value for } \epsilon_{20} \sim 10^{-2} \text{ which is probably a generous overestimate since stars with masses less than } 1 \ M_{\odot} \text{ are not accounted for: } \epsilon_{20} \sim 10^{-2}. \text{ For } \epsilon_{20} \sim 10^{-2}, \text{ the requisite mass per galaxy is}
\[
M_{\text{rm}} \approx 2 \times 10^{11} \epsilon_{b}^{-1} \left( \frac{\epsilon_{20}}{10^{-2}} \right)^{-1} \left( \frac{\epsilon_{eb}}{10^{-3}} \right)^{-1} \left( \frac{\eta (1 + z_e)}{2 \left( \frac{h}{0.75} \right)^{-3}} \right) M_{\odot}. \tag{7b}
\]

Keeping the total mass per galaxy \lesssim 10^{12} \ M_{\odot} \text{ requires that at least one in five of all stars with masses greater than } 20 \ M_{\odot} \text{ be in a very close binary system, if these are to be the source of the high-energy background; note that this would require that all of the mass of the galaxy be in baryons. If the available baryonic mass per galaxy (i.e., excluding dark matter) is } \sim 2 \times 10^{11} \ M_{\odot}, \text{ then every star with a mass greater than } 20 \ M_{\odot} \text{ would have to be in a massive X-ray binary system. In order for the massive X-ray binaries to produce the low-energy background, } \eta \approx 0.1, \text{ about one in 10 stars with mass greater than about } 20 \ M_{\odot} \text{ must be in very close binary systems, assuming that there is } 2 \times 10^{11} \ M_{\odot} \text{ of baryonic material available to form stars with masses greater than } 1 \ M_{\odot}.\n
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TABLE 3

<table>
<thead>
<tr>
<th>Model</th>
<th>Energy Constraints</th>
<th>Flux Constraints</th>
<th>Isotropy Constraints</th>
</tr>
</thead>
<tbody>
<tr>
<td>High-Energy Background (η ≈ 1) in an Einstein-de Sitter Universe (q₀ = 0.5, Λ = 0)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MXB</td>
<td>P</td>
<td>OK</td>
<td>OK</td>
</tr>
<tr>
<td>MBH (δ &lt; 0.5)</td>
<td>OK</td>
<td>OK</td>
<td>OK</td>
</tr>
<tr>
<td>Supernovae</td>
<td>OK</td>
<td>M-OK (h = 0.5)</td>
<td>P</td>
</tr>
<tr>
<td>High-Energy Background (η ≈ 1) in an Open Universe (q₀ = 0, Λ = 0)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MXB</td>
<td>P</td>
<td>OK</td>
<td>OK</td>
</tr>
<tr>
<td>MBH (h = 0.75)</td>
<td>OK</td>
<td>OK</td>
<td>OK</td>
</tr>
<tr>
<td>Supernovae</td>
<td>OK</td>
<td>OK (h = 0.75)</td>
<td>OK</td>
</tr>
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<tr>
<td>Supernovae</td>
<td>OK</td>
<td>OK</td>
<td>OK</td>
</tr>
</tbody>
</table>

**Notes:** Assessment of the models. The massive X-ray binary model (MXB), the massive black hole model (MBH), and the supernovae model have been considered in light of energy constraints, flux constraints, and isotropy constraints. The first panel shows the results of the comparison if the high-energy background is to be produced in a flat matter-dominated universe. An "P" indicates a problem. "M-OK" means marginally acceptable, and "OK" means that there is no conflict between the model and the relevant observations. The second panel shows the results if the model is to produce the high-energy background in an open universe with deceleration parameter q₀ = 0. The third and fourth panels apply to the low-energy background in flat matter-dominated and open universes, respectively.

Alternatively, one could appeal to a different initial mass function for young galaxies. However, as discussed above, even if all of the mass of the galaxy were in 20 M☉ stars, equation (7a) indicates that a substantial fraction of the stars about 0.2% would have to be in very close binary systems for the binary X-ray emission to be sufficient to explain the energy density observed in the high-energy X-ray background. A problem with energetics for the massive X-ray binary model ("MXB") is indicated in Table 3.

3.1.2. Constraints on the Active Fraction

Note that the argument indicating a problem producing the high-energy X-ray background cannot be circumvented by appealing to different types of galaxies as hosts for the massive X-ray binaries. This may be seen by considering the active fraction Λ_a of the baryons in the universe which must have been processed through a massive X-ray binary system if these are to produce either the high- or low-energy background. A generous estimate of the efficiency factor for the material directly involved with the X-ray emission process is ε_a = ε_{ph} ≲ 10^{-3}; so for z ≤ 1, the active fraction is Λ_a ≳ 10^{-5} η (see eq. [4]). In order for massive X-ray binary systems to produce the high-energy X-ray background about 0.01% of the baryons in the universe must be processed through a massive X-ray binary system, or at least 0.1% of the baryons in galaxies (see the discussion following eq. [4]).

If massive X-ray binaries are to produce the low-energy background, then about 0.01% of the known baryonic matter in galaxies must have been processed through a massive, X-ray binary system, that is, 10^{-4} of the baryons in galaxies must have been in 20 M☉ stars in a close (separations less than about 10^{13} cm) binary system. For the initial mass function assumed in § 3.1.1 with a 1 M☉ lower mass cutoff, this means that about 1% of the 20 M☉ stars were in very close binary systems, and each binary system produced an X-ray luminosity of about 5 × 10^{38} ergs s^{-1} for about 2 × 10^{9} yr.

3.1.3. Metallicity Constraints

A generous estimate of the efficiency with which a given massive X-ray binary system radiates X-rays is ε_{ph} ≈ 10^{-3}, which implies Λ_a ≈ 10^{-8} η (see §§ 3.1 and 3.1.2). When the massive star undergoes a supernova explosion, a significant mass of metals, primarily oxygen, will be produced. Given that the progenitor to the supernova has a mass M, the mass of metals M_f produced by the explosion will be M_f/M = 0.4 M - 4.2 (Woosley & Weaver 1986). For a 20 M☉ progenitor, the ratio of the mass of metals to the initial mass of the star is M_f/M ≈ 0.2. The ratio of the mean mass density of metals produced by the supernova explosions from massive stars in massive X-ray binary systems to the total mean mass density of baryons in the universe is (M_f/M)Λ_a ≈ 2 × 10^{-5} η. And, since the mean mass density of available baryons in galaxies is less than 0.01 h^{-2} (see the discussion following eq. [4]), the ratio of the mass of metals (primarily oxygen) produced by the stars in massive X-ray binary systems to the total baryonic mass of the galaxy is about 2 × 10^{-4} η. The local cosmic ratio of the mass in oxygen to baryons is about 0.01. This means that about 2% of the oxygen is produced by the supernovae explosions of massive stars that were once massive X-ray binary systems, if these systems are to produce the high-energy X-ray background. This implies that more than one in 50 of all stars with a mass greater than about 20 M☉ were in a very close binary system which was a very bright X-ray source; note that this is indepen-
dent of the environment (i.e., type of galaxy) in which the binaries are located.

3.1.4. Flux Constraints

Although this model appears to be in conflict with the energy requirements detailed in § 2.1, it does not appear to be in conflict with either the flux constraints (§ 2.2) or the isotropy constraints (§ 2.3). This is because the time scale over which the X-ray emission from each galaxy occurs, \( t_x \), seems to be unconstrained and therefore can be taken to be large.

If the X-ray emission from each source commences at a redshift less than about 1.5 and continues to a redshift of about 0.5, then

\[
t_x \simeq t_0(z_2 = 1.5) - t_0(z_1 = 0.5) \simeq 0.3 t_0 \simeq 3 \times 10^9 (h/0.5)^{-1} \text{ yr}
\]

in an Einstein–de Sitter universe. The X-ray luminosity is \( L_x \simeq E_x / t_x \), with \( E_x \) from equation (1). The flux per source in an Einstein–de Sitter universe is obtained from equation (A1), and, for the high-energy background, the flux is multiplied by \( \beta \simeq 0.15 \) to obtain the 1–3 keV flux (see § 2.2). Combining these expressions, and assuming a canonical redshift of emission of \( z_x \simeq 1 \), the 1–3 keV flux per source is \( f_x(1–3 \text{ keV}) \approx 4 \times 10^{-15} \nu_x(z_x/0.3)^{-1} \text{ ergs s}^{-1} \text{ cm}^{-2} \). The high-energy background will be produced when \( \eta \simeq 1 \). Thus, the calculated flux is consistent with the upper bound of 2.6 \times 10^{-14} \text{ ergs s}^{-1} \text{ cm}^{-2} \) (see § 2.2) for \( \chi \simeq 0.05 \). There is no conflict if these sources are to produce the low-energy background, since, in this case, \( \eta \simeq 0.1 \), though \( \beta \) may be large, \( \beta \simeq 0.4 \) (see § 2.2); the computed fluxes are consistent with the observational limits for \( \chi \simeq 0.01 \). And in an open universe the flux per source decreases (other factors remaining fixed); hence, there is no conflict between the Einstein deep survey flux constraints and the properties of these sources for either \( \eta \simeq 1 \) or \( \eta \simeq 0.1 \) in an open universe (with \( q_0 = 0 \)) or in a flat universe dominated by a cosmological constant.

This argument is general and applies to any model in which the X-ray emission occurs over the extended period of time discussed. This means that in any model in which the X-ray emission occurs at a characteristic redshift \( z_x \simeq 1 \), with a time scale \( t_x \simeq 3 \times 10^9 \text{ yr} \), from sources with a comoving number density such that \( \chi \simeq 0.05 \), the sources will have a flux below the flux limit of the Einstein deep survey if the sources are to produce the high-energy X-ray background, or the low-energy background (for \( \chi \simeq 0.01 \)), in a flat or open universe. Stronger limits on \( \chi \) result from the isotropy constraints.

3.1.5. Isotropy Constraints

The number of sources per square degree in an Einstein–de Sitter universe when \( t_x \gtrsim t_0(z_x) \), \( N_x \), may be obtained from equations (A7) and (A8) with \( z_1 \simeq 0.5 \), and \( z_2 \simeq 1.5 \) (see Table 2). The surface density of sources is \( N_x \simeq 3 \times 10^3 \nu_0(z_x/0.3)^{-2} \). For \( \chi \gtrsim 0.1 \) this is consistent with the lower bound of about 1000 deg^{-2} (see § 2.3) if the sources are to comprise the high-energy background, since in the massive X-ray binary model the sources are expected to have spectra that extrapolate to low energies. In an open model the number of sources per square degree over this redshift interval increases by about a factor of 2.5 (see Table 2), hence the isotropy limits are satisfied for \( \chi \gtrsim 0.04 \) in an open model.

Again, this argument, and the one that follows, is general and is applicable to any model in which the X-ray emission occurs over an extended period of time.

If the sources are to comprise the low-energy background, then the requisite number of sources per square degree is \( N_x \simeq 5000 \text{ deg}^{-2} \). This is obtained in an Einstein–de Sitter universe when \( \chi \gtrsim 0.5 \), given that the sources are X-ray emitters over the redshift interval from \( z_1 \simeq 0.5 \) to \( z_2 \simeq 1.5 \). And in an open model the number of sources per square degree increases (other factors held fixed); this model is consistent with the isotropy constraints in an open universe for \( \chi \gtrsim 0.2 \).

3.2. Massive Black Hole Models

3.2.1. Energy Constraints

Black hole models as a source of the X-ray background have the positive feature that massive black holes are expected to have very large efficiency factors, \( \epsilon \sim 0.1 \). However, equation (2) shows that for \( w_x = (1 + z_x) \sim 3 \), the remnant black hole must have a mass of

\[ M_{bh} \sim 10^7 \eta h^{-3}(\chi/0.3)^{-1}(w_x/e(0.1))^{-1} M_\odot. \]

This means that \( \chi \lesssim 0.3 \) since it would be difficult to hide massive black holes in galaxies that do not contain a considerable total mass. Note that in an Einstein–de Sitter universe, \( h \lesssim 0.5 \) (see § 1), and the requisite mass of black hole increases by about an order of magnitude from the requisite mass when \( h \sim 1.0 \).

The constraints on the typical mass of a black hole residing in \( L^* \) galaxies are weak; well-determined upper bounds have been obtained for a few local galaxies (see, e.g., Kormendy and Edgara 1988). For example, a black hole at the center of the Andromeda galaxy must have a mass \( \lesssim 10^7 M_\odot \), and one at the center of the Milky Way must be \( \lesssim 10^6 M_\odot \). Hence, a black hole mass of about \( 10^7 M_\odot \) per \( L^* \) galaxy, allowing for a few exceptions, is consistent with the current data. However, a black hole mass of about \( 10^8 M_\odot \) per \( L^* \) galaxy is in conflict with observations of local \( L^* \) galaxies.

The black hole mass per \( L^* \) galaxy will be less than about \( 10^7 M_\odot \) for \( h \sim 1.0 > h \gtrsim 0.5 \) when \( \eta \sim 0.1 \) and \( w_x \sim 3 \) (see eq. [8]). Therefore, X-ray emission from massive black holes could produce the high-energy X-ray background in a flat or open cosmological model. This agreement is indicated in Table 3. As stressed by Cowie (1990), this would require that most of the energy from the black hole emerge as X-rays.

It is interesting to note that the black hole mass per \( L^* \) galaxy is about \( 10^7 M_\odot \) for \( h \sim 1.0 \), \( \eta \sim 1 \), and \( w_x \sim 3 \), hence X-ray emission from black holes could produce the high-energy background in a flat universe dominated by a cosmological constant (see the discussion in § 1). For \( h = 0.75 \) the black hole mass per galaxy is about \( 2 \times 10^7 M_\odot \), hence X-ray emission from massive black holes could produce the high-energy background in an open \((q_0 = 0)\) universe, as indicated in Table 3.

In order for X-rays from massive black holes to produce the high-energy X-ray background in a flat matter-dominated universe, \( \eta \sim 1 \) and \( h \sim 0.5 \), the requisite mass per black hole per \( L^* \) galaxy is \( \sim 10^6 M_\odot \) for \( w_x \sim 3 \), which is inconsistent with the constraints from nearby \( L^* \) galaxies. X-ray emission from massive black holes could produce the high-energy background if the emission is produced by some fraction \( \delta \) of \( L^* \) galaxies each with a very massive black hole since, in this case, the constraints on black hole masses from observations of local galaxies are not applicable. For example, the high-energy background could be produced if one in six of each galaxy brighter than \( L^* \) (i.e., \( \chi \sim 0.05 \)) has a black hole of mass \( 5 \times 10^6 M_\odot \). Considerations of the isotropy of the background result in
a lower bound on the comoving density of the sources, that is, on $\chi$, as discussed below. However, note that if the X-ray emission is absorbed at low energies (§ 3.2.4), then the constraints on $\chi$ from the isotropy of the low-energy background do not apply.

It is shown in § 3.2.2 that if the spectrum of the X-ray emission extends to low energies, then the isotropy constraints imply that $\chi \gtrsim 0.05$ (i.e., $\delta \gtrsim \frac{1}{2}$). Hence, the high-energy background ($\eta \approx 1$) could be produced by massive black holes in an Einstein–de Sitter universe (so $h \approx 0.5$) if the emission results from very massive black holes ($\sim 5 \times 10^8 M_\odot$) in a small fraction (about one in six) of all galaxies brighter than $L^*$. This model predicts that very massive black holes exist in some bright galaxies and hence will be constrained when estimates of or limits on black hole masses in a larger sample of bright galaxies is available. However, it should be noted that if the X-ray spectrum is absorbed at low energies, then $\chi$ could be very small, and the black hole mass per X-ray emitter very large. The fraction of galaxies brighter than $L^*$ that have very massive black holes must be less than one in two, since neither the Galaxy nor Andromeda has a very massive ($M \gtrsim 10^9 M_\odot$) black hole, hence $\delta < 0.5$. And if the X-ray spectrum is not absorbed at low energies, then the isotropy constraints from the Einstein deep survey imply that $\delta \gtrsim \frac{1}{2}$. Therefore, X-ray emission from massive black holes could produce the high-energy X-ray background in an Einstein–de Sitter universe for $\delta < 0.5$ (indicated by "OK" in Table 3).

3.2.2. Isotropy and Flux Constraints

If the X-ray spectrum of a given source is not absorbed at energies of about $1 (1 + z)$ to $3 (1 + z)$ keV, then the isotropy constraints imply that there must be at least 1000 sources per square degree if the high-energy background is to be produced, or at least 5000 sources per square degree if the low-energy background is to be produced (see § 2.3). The number of sources per square degree is larger when the X-ray-emitting phase is long (i.e., $t_x > t_d(z_2)$ in which case $t_x \approx t_d(z_1) - t_d(z_2)$), than when the X-ray-emitting phase is short (i.e., $t_x < t_d(z_2)$). The surface densities of sources when the X-ray-emitting phase per source is long are listed in Table 2. In a flat matter-dominated universe the surface density of sources $N_\delta$ is about $2 \times 10^8 \chi$ for $z_x \approx 2$, and $(z_1, z_2) \approx (1, 3)$. A typical redshift of $z_x \approx 2$ is chosen since this is where the optical quasar counts peak, and a glance at Table 2 indicates that as $z_x$ increases the surface density of sources does not increase significantly. Hence, $\chi$ must satisfy $\chi \gtrsim 0.05$ if the high-energy background is to be produced, and $\chi \gtrsim 0.3$ if the low-energy background is to be produced in a flat matter-dominated universe; these limits are decreased by a factor of 2 if $(z_1, z_2)$ → (1.5).

Note that this is a general argument that applies to the comoving number density of any population of sources that are to produce the high- or low-energy background in a flat matter-dominated universe. The bounds on $\chi$ may decrease by factors of 3–10 (depending on $z_1$ and $z_2$) if the universe is open, as indicated by Table 2.

Given $t_x, t_x \approx t_d(z_2)$, and $\chi$, the flux per source can be computed. In a flat matter-dominated universe, the flux per source is given by equation (A1), and the luminosity is $L_x \approx E_x/t_x$, where $E_x$ is given by equation (1): $f_x(1–3 \text{ keV}) \approx 3.6 \times 10^{-13} \beta \chi^{-1} \eta(1 + z_2)(1 + z_1) - 0.5 \eta(1 + z_2) - 0.5 \eta(1 + z_2) - 0.5 \eta(1 + z_2) - 0.5 \eta(1 + z_2) \text{ ergs} s^{-1} \text{ cm}^{-2}$; note that this is independent of $h$. If massive black holes are to produce the high-energy background ($\eta \approx 1$) in a flat matter-dominated universe, then $\chi \gtrsim 0.05$ (assuming no low-energy absorption of the X-ray spectrum in the vicinity of the massive black hole) and $f_x \lesssim 2.7 \times 10^{-14} \text{ ergs} s^{-1} \text{ cm}^{-2}$ for $\beta \approx 0.15$ (see § 2.2), $z_x \approx 2$, and $z_2 \approx 1$, which is marginally consistent with the flux limits from the Einstein deep survey. As $\chi$ increases $f_x$ decreases.

Hence, the energy, isotropy, and flux limits are satisfied by the massive black hole model if this model is to produce the high-energy background in an Einstein–de Sitter universe for $0.05 \lesssim \chi \lesssim 0.1$, $z_x \approx 2$, and $t_x \approx 4(h/0.5) - 1 \times 10^9 \text{ yr}$. The flux limits are satisfied by a wider margin if the sources are at higher redshifts, $z_x > 2$, although in this case the mass per black hole increases, and the length of the X-ray-emitting phase $t_x$ decreases. Since the constraints are satisfied in a flat matter-dominated universe, they will also be satisfied in an open universe, or a flat universe dominated by a cosmological constant, as indicated in Table 3.

As shown above, if the low-energy background is to be produced in a flat matter-dominated universe, the isotropy constraints require $\chi \gtrsim 0.3$. The 1–3 keV flux per source is $f_x(1–3 \text{ keV}) \approx 1.2 \times 10^{-14} \text{ ergs} s^{-1} \text{ cm}^{-2}$ for $\beta \approx 0.4$, about a factor of 2 below the limit set by the Einstein deep survey (see § 2.2). Since the model is consistent with the flux and isotropy limits in a flat matter-dominated universe, consistency will also be obtained in an open model ($\Omega_0 = 0$), or in a flat universe dominated by a cosmological constant.

Therefore, either the high- or low-energy background are to be produced by massive black holes in an Einstein–de Sitter universe, then they may lie just below the flux limit set by the Einstein deep survey. Alternatively, the redshift of X-ray emission could be larger than $z_x \approx 2$, or the universe could be open, or flat and dominated by a cosmological constant, or the spectrum of the sources may be absorbed at low energies.

3.2.3. The Boldt-Leiter Model

The production of the high-energy background by a specific emission mechanism associated with a brief evolutionary phase of active galactic nuclei is discussed by Leiter & Boldt (1982) and Boldt & Leiter (1987). The model involves emission from a small volume, which extends to about 100 Schwarzschild radii, surrounding the black hole. The temperature of the gas in this region is $\approx 120–150 \text{ keV}$, and the number density of electrons is $n \approx 10^2 \text{ cm}^{-3}$. One possible set of parameters is (Leiter & Boldt 1982): a black hole mass of $M_b \approx 10^8–10^9 M_\odot$, a comoving number density of sources $\chi \approx 8 \times 10^{-2}$ (corresponding to about one in four $L^*$ galaxies), a characteristic redshift of emission $z_x \approx 4$, and a period of emission $t_x \approx 10^8 \text{ yr}$. This model is able to reproduce the spectrum of the high-energy background.

The model is constrained by the Einstein deep survey if the low-energy tail of the high-energy spectrum may be obtained by extrapolating the high-energy spectrum to low energies. These constraints are now discussed. But note since the emission originates near the central compact object during an early evolutionary phase, it is not unreasonable to expect the spectrum to be absorbed at low energies. The column densities of material needed to account for such absorption are discussed below (§ 3.2.4). In the case that the spectra are absorbed at low energies, the original parameter choices (Leiter & Boldt 1982) are allowed, and the updated parameter choices (Boldt & Leiter 1987) are consistent with both the flux and isotropy limits of the Einstein deep survey if the spectra of the sources extrapolate to low energies.
If the high-energy spectrum extrapolates to low energies, the number of sources per square degree is constrained by the fluctuations analysis of the Einstein deep survey. The number of sources per square degree in an Einstein–de Sitter universe may be obtained from equation (A4) since $t_s < t_d(z_s \approx 4)$, or from Table 1 [see $N_1$ when $z_1 = 3$, and $z_2 = 5$ with $t_s/t_q \approx 8 \times 10^{-3}(h/0.5)$ and $\chi \approx 0.08$]. This implies about $150/h(0.5)$ sources per square degree, which is a factor of 10 below what is needed to avoid overproducing fluctuations at low energies; note that for $\chi \approx 0.3$ the number of sources per square degree increases to $N_1 \approx 500/h(0.5)$, and this number may be increased by increasing the period of emission $t_s$ as discussed by Bold & Leiter (1987). If the epoch of emission extends from about a redshift of 3–5, and the universe is open (with deceleration parameter $q_0 = 0$), then the number of sources per square degree increases by a factor of 5 (see Table 1). In this case the number of sources per square degree does not violate the low-energy limits, even for the parameter set given above with $\chi \approx 0.08$ and $t \approx 1.5 \times 10^9$ yr.

3.2.4. Low-Energy Absorption

It is also possible that the X-ray emission spectrum of these sources is absorbed at low energies, so that the observed spectrum (moving from high to low energies) turns over at an energy of about 2–3 keV. In massive black hole models the emitting region is near the central compact object, and this region may be embedded within a volume of gas, either hydrogen- or metal-rich material. Given an initial spectrum, such as thermal bremsstrahlung, the properties of the surrounding ambient material may be estimated by requiring that this material absorb the incident radiation at an energy of about $2(1 + z_s)$ keV in the rest frame of the source; for definiteness this energy is taken here to be about 5 keV. The idea here is to choose a rest frame energy of absorption so that the observed spectrum will turn over at an energy of about 2–3 keV, in which case the sources contribute to the high-energy background but not to the low-energy background. If the sources are at high redshift, say $z_s \approx 4$, then the energy at which the optical depth would be required to go to unity would be about 10 keV. But the estimates given below are rough approximations even if the requisite energy of $\sim 2(1 + z_s)$ keV is as high as 10 keV.

The absorption cross section for hydrogen-like atoms is $a \approx 2 \times 10^{-23} Z^2 E^{-3}(\text{keV})^2$, where $Z$ is the atomic number of the absorbers (Allen 1976). When the optical depth $\tau = \int n a d l$ goes to unity, the specific intensity is decreased to $e^{-\tau}$ of its initial value. This energy-dependent absorption of the incident radiation may cause the emergent spectrum to turn over at low energies. For a column density of hydrogen of $n_l = 6 \times 10^{24}$ cm$^{-2}$ which may be expressed as $n(\text{cm}^{-3})(\text{kpc}) = 2 \times 10^4$, the optical depth goes to unity at 5 keV. It is convenient to express the column density in terms of $\text{cm}^{-2}$ kpc so that if the size of the absorbing medium can be estimated, the requisite number density in that region can be readily seen. If there is a substantial abundance of metals, then absorption becomes significant at much lower column densities. For example, if there is some fraction $x_0$ of the solar abundance of oxygen (by mass), then $\tau \approx 1$ at 5 keV for a hydrogen density $n$ such that $n(\text{cm}^{-3})(\text{kpc}) \approx 500 x_0^{-1}$; if there is a fraction $x_0$ of the solar abundance of iron (by mass), then $n(\text{cm}^{-3})(\text{kpc}) \approx 10 x_0^{-1}$ causes $\tau \approx 1$ at 5 keV. These are not unrealistically large numbers; in any model in which the X-ray emission originates near a central compact object, especially during an early evolutionary stage, it is reasonable to expect a substantial column density of material enshrouding the central compact region. Then, the constraints on the number of sources per square degree and on the flux per source from the Einstein deep survey are not applicable since this survey was done at low energies, 0.5–3.5 keV.

If the spectrum is absorbed at low energies, then these sources are unlikely to be detected by the ROSAT deep survey, which is expected to go at least an order of magnitude fainter than the Einstein deep survey. If the ROSAT deep survey does not uncover a new population of sources, such as a population of X-ray-bright starburst galaxies, then it is likely that the high-energy background results from a population of sources with spectra that turn over at low energies, such as that discussed here. If the survey does reveal a new population of relatively flat spectrum sources, these may or may not have spectra that extrapolate to high energies, and, hence, may or may not turn out to be the sources of the high-energy background. The characteristics of the fluctuations may be used to determine whether they could be the sources of the high-energy background. For example, the sources of the high-energy background should produce relatively large fluctuations on large scales: $\delta I/(I/h) \approx 1$ on a $1^4 \times 1^4$ scale, as discussed in § 4. recall that $f$ is the fraction of the flux in the 1–3 keV band that is due to the low-energy tail of the sources of the high-energy background (see § 2.3). A cross-correlation of the HEAO and Einstein data was not possible because of the relatively small sky coverage of the Einstein deep survey.

3.2.5. Implications

The general energy arguments discussed above imply that if X-ray emission from massive black holes produces the high-energy background, then a fraction $\delta$ of all galaxies brighter than $L^* \approx 10^{45} \approx 10^9 h^{-3}$ in an Einstein–de Sitter universe the current age of the universe requires that $h \lesssim 0.5$, and therefore $M \gtrsim 10^{58} h^{-1} M_{\odot}$. Limits on the masses of black holes in bright galaxies will constrain both $\delta$ and the black hole mass per galaxy.

If the X-ray spectrum of the active galaxies extrapolates to low energies without significant absorption, then the isotropy constraints from the Einstein deep survey imply that $\delta \lesssim 0.5$ for a flat matter-dominated universe. This then implies that at least one in six galaxies brighter than $L^*$ contains a very massive black hole. Observations of nearby bright galaxies indicate that $\delta \lesssim 0.5$.

It is interesting to note that in both the massive black hole and supernova models for the high-energy X-ray background, it is easier to produce the background if $h$ is large, $h \sim 0.75–1.0$. This simply results from the fact that the comoving number density of galaxies is $\propto h^2$, and so is larger if $h$ is larger.

3.3. The Supernovae Model

3.3.1. Energy Arguments

The supernovae model was proposed by Bookbinder et al. (1980), who point out that the high-energy X-ray background could be a by-product of the supernovae explosions needed to produce the metals observed in the bulges of $L^*$ galaxies. The emission mechanism is thermal bremsstrahlung from the hot gas associated with the supernovae ejecta. A single temperature for the gas is expected if all of the supernovae shells overlap. In fact, one of the most attractive features of this model is the rough correspondence (within a factor of 2) of the expected temperature with that of the high-energy X-ray background, as will be discussed below. If the supernovae shells do
not all overlap, a distribution of temperatures is expected, and the emission will not be concentrated in the X-ray regime. Hence, a basic requirement in this model is that all of the supernova shells overlap. Since the characteristic temperature of the shock heated gas is expected to be \( \sim 30-40 \text{ keV} \), the hot gas will flow from the galaxy as a wind; in order that the postshock temperature remain high this outflow is mandatory, as will be described below.

Two efficiency factors enter in the estimate of the galactic mass involved in the X-ray production: the efficiency with which a given star converts rest mass energy into the kinetic energy of the supernova explosion which is subsequently converted into the thermal energy of the gas, denoted as \( \epsilon_1 \), and the fraction of the thermal energy of the gas which is radiated as X-rays, denoted as \( \epsilon_2 \).

A characteristic energy of about \( 10^{51} \) ergs is released as kinetic energy from a 10 \( M_\odot \) star that undergoes a supernova explosion, indicating an efficiency of conversion of rest mass energy into, ultimately, thermal energy of \( \epsilon_1 \approx 5 \times 10^{-5} \).

Consider the inner region of an \( L^* \) galaxy with copious supernova activity so that the supernovae shells overlap. At any particular time, let the mass of gas in the X-ray-emitting region that has been processed through supernovae be \( M \), which is \( M_t \), where the rate at which mass is input to the medium from supernovae explosions is \( M \tanh(t/t_w) \) for the time scale to flow from the X-ray-emitting region is \( t_w \). The thermal energy in the gas is \( E_{th} = \epsilon_1 M c^2 \), \( c \) being the speed of light. In addition, \( E_{th} = 1.5 NKT \), where \( N \) is the total number of particles, and \( N \geq M/m_p \), \( m_p \) being the proton rest mass; if a significant fraction of the mass of the star is ejected during the explosion, the number of particles can only be \( \geq M/m_p \) and will exceed \( M/m_p \) if there is ambient matter shock heated by the explosion which did not originate in an exploding star; matter that is shock heated and subsequently cools without flowing out of the galaxy would be such a source of cool or "unshocked" ambient matter. These equations imply

\[
kT \approx \frac{2}{3} m_p c^2 \approx 35 \text{ keV} \left( \frac{\epsilon_1}{5 \times 10^{-5}} \right).
\]

So, if most of the material which is shock heated by the supernovae originated in a star that exploded, then the characteristic temperature of the shock-heated gas is just right for the X-ray background if we allow the efficiency factor \( \epsilon_1 \) to be about a factor of 2 larger than \( 5 \times 10^{-5} \). The characteristic redshift of the event, \( z_{\text{sh}} \), is unknown. In order for this model to allow a sufficiently high temperature to explain the high-energy background via thermal bremsstrahlung emission, let \( \epsilon_1 \approx 5 \times 10^{-5} (1 + z_{\text{sh}}) \) (so \( \epsilon_1 \approx 10^{-4} \) for \( z_{\text{sh}} \approx 1 \)); this is equivalent to setting \( kT \approx 35(1 + z_{\text{sh}}) \) keV. Note that if the gas cools before flowing from the galaxy, the mass of gas which will be shock heated by subsequent supernova explosions increases, causing the temperature to decrease. Hence, the shock-heated gas must flow out of the galaxy as a wind if the postshock temperature is to remain high.

Note that the total efficiency factor \( \epsilon = \epsilon_1 \epsilon_2 \) is \( \lesssim 10^{-4} \). Equation (4) shows that, for \( \Omega_g \beta^2 \approx 0.1 \), this requires that at least 0.1% of the baryons in the universe be processed through supernovae in the central regions of \( L^* \) galaxies, assuming \( z_{\text{sh}} \approx 1 \), if this is to be the source of the high-energy background; note that for the more realistic choice \( \Omega_g \beta^2 \approx 0.01 \), at least 1% of the baryons in the universe would have to be processed through the central regions of \( L^* \) galaxies.

The ratio \( \epsilon_1 (5 \times 10^{-5} (1 + z_{\text{sh}}))^{-1} \) is denoted for convenience by \( \epsilon_1 \). The total mass associated with the X-ray emission is (see eq. [2])

\[
M_{X} \approx 2 \times 10^{10} \epsilon_1^{-5} \epsilon_2^{-1} \chi^{-1} \eta(0.5)^{-3} M_\odot.
\]

The fraction of the thermal energy of the gas which is radiated as X-rays is \( \epsilon_2 = E_{X}/E_{th} \), where \( E_{th} \) is the total thermal energy of the gas: \( E_{X} = E(1/t_w) \), where \( E(1/t_w) \) is the thermal energy of the gas (at any given time) in the region where the supernovae shells overlap, \( t_w \) is the period of the active phase, and \( t_w \) is the time scale for the galactic wind to flow from the active region. The cooling time for the gas due to thermal bremsstrahlung emission is \( t_c = E_{th}/L_{X} \), and the X-ray luminosity is \( L_{X} = E_{X}/t_c \). The time scale for the wind, comprised of the hot gas, to flow from the active region is \( t_w = \pi/c_r \); \( R \) being the characteristic radius of the active region, and \( c_r \) the sound speed of the gas in that region. Combining these expressions, \( \epsilon_2 = t_w/t_c \), results.

As noted above, the requirement that the supernovae shells overlap, and that the temperature of the shock heated gas remain high, translates into a constraint on the second efficiency factor: \( \epsilon_2 \gtrsim 1 \), since if \( t_w < t_c \), the cool gas would increase the mass of material to be shock heated by subsequent supernovae which would decrease the temperature to values which are too low to be relevant for the high-energy X-ray background. Following Bookbinder et al. (1980) the total mass processed through supernovae in the inner regions of \( L^* \) galaxies may be large \( (M \approx 10^{12} M_\odot) \), but the number density of the galaxies cannot exceed the current number density of \( L^* \) galaxies, so \( \chi \approx 0.3 \). Substituting this value for \( \chi \) into equation (10), and requiring that the total baryonic mass processed through supernovae be \( 10^{12} M_\odot \), leads to the constraint \( \epsilon_2 \gtrsim 7 \times 10^{-7} \eta(0.5)^{-3} (\chi/0.3)^{-1} (h/0.5)^{-3} = 8 \times 10^{-3} \eta h^{-3} \), for \( \chi \approx 0.3 \), and \( \epsilon_2 \approx 1 \) (so that the emission has the observed characteristic temperature). If the mass processed through supernovae in the inner regions of the galaxy is less than \( 10^{12} M_\odot \), then the constraints become considerably tighter, and the model would be ruled out.

The time scale for the gas to flow from the active region is \( t_w = R/c_r \). The sound speed is \( c_r = \gamma P/\rho = \gamma kT/(\epsilon_1) \), where \( \gamma \) is the ratio of specific heats, assumed to be 5/3, \( P \) and \( \rho \) are the gas pressure and density, and \( \epsilon_1 \) is the mean mass per particle; hence, \( c_r^2 = \epsilon_1 c^2 \). So, \( t_w \approx 4 \times 10^{9} R/(1 + z_{\text{sh}}) \approx 3 \times 10^{-3} t_M \), yr, where \( R \) is the characteristic radius of the active region in kpc, and \( \epsilon_1 \approx 0.6m_p \), appropriate for a plasma of primordial composition, has been assumed, where \( m_p \) is the proton rest mass.

The cooling time for the gas is \( t_c = E_{th}/L_{X} \). The thermal energy of the gas is \( E_{th} = 1.5 NK T \), and the X-ray luminosity is

\[
L_{X} = 1.4 \times 10^{-27} \chi^{-0.3} C_{\text{L}} n^2 Z g \sigma_V \text{ ergs s}^{-1}.
\]

The temperature is in degrees Kelvin, the mean number density of electrons \( n \) is in \( \text{cm}^{-3} \), the volume \( V \) is in \( \text{cm}^3 \), the clumping factor \( C_{\text{L}} = \langle n^2 \rangle/\langle n \rangle^2 \) must be close to unity since the gas is isothermal, and the large sound speed of the medium will lead to pressure equilibrium. The average guant factor \( g \) and mean atomic number are taken to be about 1.2 and 1.6, respectively. Note that if the mean atomic number \( Z \) is substantially larger than 1.6, the temperature would decrease (see eq. [9]), though this would have the effect of increasing \( \epsilon_2 \). Since \( t_w \) would increase and \( t_c \) would decrease. The cooling time for the gas is \( t_c \approx 3 \times 10^{9} n^{-1/2} \), yr, and \( \epsilon_1 \approx 1.4 \times 10^{-2} n R(1 + z_{\text{sh}})^{-1} \epsilon_1^{-1} \), where the mean number density of electron \( n \) is in units of \( \text{cm}^{-3} \) and the characteristic radius of

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the active region $R$ is in units of kpc. This implies a baryonic mass (see eq. [10]) associated with the X-ray production of

$$M_{\text{rm}} \approx 1.7 \times 10^{12} R^{-1} n^{-1} \chi^{-1} \eta(1 + z_s)(h/0.5)^{-3} \ L_\odot \ . \ (12)$$

Note that this equation is independent of $\epsilon_r$, since $\epsilon_r$ depends on $\epsilon_r$, and therefore the mass involved in the event is independent of the temperature to which the gas is heated (see eq. [9]), as long as the supernova shells overlap and the shock-heated material flows from the active region as a wind.

The constraint that the total mass processed through stars in the active region of each $L^*$ galaxy is less than $10^{12} M_\odot$ leads to a lower bound on $\epsilon_r \gtrsim 7 \times 10^{-2} \eta(\chi/0.3)^{-1} \epsilon_r^{-1}(h/0.5)^{-3}$ which, using the expression for $\epsilon_r$ given above, implies that

$$nR \gtrsim 5n(\chi/0.3)^{-1}(1 + z_s)(h/0.5)^{-3} \ . \ (13)$$

Note that taking an upper bound of $10^{12} M_\odot$ for the baryonic mass processed through the central region of each $L^*$ galaxy is generous; a lower number would considerably tighten the limits presented here, and the model would be ruled out.

The constraint that $\epsilon_r \gtrsim 1$ implies that

$$nR \lesssim 70(1 + z_s) \ L_\odot \ . \ (14)$$

These constraints will be applied in the next section. It is interesting to note that equation (4) and the expression for the efficiency imply that the active fraction is

$$\Delta_\alpha \approx \frac{0.09n(1 + z_s)}{nR} \ (15)$$

$$\approx 0.17(nR)^{-1} \ . \ (16)$$

for $z_s \approx 1$ and $\eta \approx 1$. Note that this fraction $\Delta_\alpha$ is likely to be a factor of 10 larger than quoted here since $\Omega_\Lambda h^2$ is more likely to be 0.01 than 0.1 (see the discussion following eq. [4]).

In the supernova model the X-ray emission mechanism is optically thin thermal bremsstrahlung radiation, so the spectrum of the high-energy background extrapolates to low energies. Hence, given the spectrum of the high-energy background, that of the low-energy background, $\lesssim 3$ keV, is fairly well determined. This means that important constraints on the model result from the Einstein deep survey. First the constraints resulting from the upper bound on the flux per source resulting from that survey are discussed, followed by constraints from the isotropy of the low-energy background.

### 3.3.2. Flux Constraints

If the total time $t_s$ of the active phase can be determined, then the X-ray luminosity of the active region $L_x$ is that estimated: $L_x = E_x/t_s$, with $E_x$ from equation (1). The number of cycles is defined to be $N_x = t/t_s$. In addition, $N_x = M/L_x$, where $M_r$ is the total mass processed through supernova explosions (see eq. [12]), which implies that $N_x \approx 2 \times 10^{4} n^{-1} h^{-4} \chi^{-1}(1 + z_s)(h/0.5)^{-3}$. Combining these expressions yields

$$t_s \approx 7.5 \times 10^{9} n^{-2} \chi^{-1} \eta^{-1}(1 + z_s) h^{-0.5}(h/0.5)^{-3} \ yr \ . \ (16)$$

So, the X-ray luminosity is $L_x = E_x/t_s \approx 10^{49} n^{-1} h^{-4} \chi^{-1}(1 + z_s) h^{-0.5}(h/0.5)^{-3} \ yr$. (17)

Combining the expression for $t_s$ (eq. [16]) with the constraint equation (13) results in an upper bound on the total time of the active phase

$$t_r \approx 3.5 \times 10^8 R^{-1} \eta^{-1}(\chi/0.3)(w_s/2)^{-1.5} \epsilon_r^{-0.5}(h/0.5)^{3} \ yr \ , \ (17)$$

where $w_s \equiv 1 + z_s$.

A lower bound on $t_s$ results from the 1–3 keV flux limit placed on an individual source by the Einstein deep survey: $f_x(1–3 \text{ keV}) \gtrsim 2.6 \times 10^{-14} \text{ ergs s}^{-1} \text{ cm}^{-2}$. Assuming that the sources are at a characteristic redshift of 1, $z_s \approx 1$, and a thermal bremsstrahlung emission spectrum with a temperature (at a redshift of zero) of about $kT_e \approx 35$ keV leads to an expected 1–3 keV flux (i.e., $\beta \approx 0.15$; see § 2.2). Requiring that this flux be below the limit imposed by the Einstein deep survey leads to the bound

$$n^2 R^2 \lesssim 72 \left( \frac{h}{0.5} \right)^{-2} \frac{1}{\epsilon_r} \ , \ (18)$$

for an Einstein–de Sitter universe (i.e., using eq. [A1]). In an open universe with $q_0 = 0$ equation (A2) is used to determine the flux; requiring that this be below the flux limit of the Einstein deep survey yields

$$n^2 R^2 \lesssim 120 \left( \frac{h}{0.5} \right)^{-2} \frac{1}{\epsilon_r} \ , \ (19)$$

where $\beta = 0.15$ appropriate for $kT_e \approx 35$ keV has been assumed. Note that these constraints are tighter than that given by equation (14) and therefore are used below. Substituting the limits on $n^2 R^2$ (obtained from eqs. [18] and [19]) into the expression for $t_s$ leads to the upper bound on the total time over which the emission occurs. In an Einstein–de Sitter universe, equation (18) substituted into equation (16) indicates that

$$t_s \approx 3 \times 10^4 \eta^{-1}(\chi/0.3)(h/0.5)^{-1} (h/0.5)^{-1} \ yr \ . \ (20)$$

The bound expressed by equation (17), applicable to either an open or closed universe, with $z_s \approx 1$, is

$$t_r \approx 3.5 \times 10^8 R^{-1} \eta^{-1}(\chi/0.3)(h/0.5)^{-1} \ yr \ . \ (21)$$

If these sources are to comprise the high-energy background $\eta \approx 1$ and $\epsilon_r \approx 1$. In this case, the bounds given by equations (20) and (22) are quite close, indicating that only a small range of parameters is allowed for the case that $\Omega = 1.0, \Lambda = 0$, since in an Einstein–de Sitter universe $h$ must be small, $h \approx 0.5$. Hence, the bounds indicated by equations (16) and (18) must be close to equality. Treating them as such, they may be solved for $R$ and $n$: $R \approx 0.7 n^{-1/2}(\chi/0.3)^{1/2}(h/0.5)^{-1} \text{ kpc}$ and $n \approx 10 \eta^{-1}(\chi/0.3)^{1/2}(h/0.5)^{-1} \text{ cm}^{-3}$ for $z_s \approx 1$. Note that $R$ is decreased and $n$ increased if $\chi$ is decreased. A value of $h = 0.5$ is preferable to keep the age of the universe $\approx 12.5 \times 10^9$ yr old. Therefore, in order for this to be a viable source of the high-energy X-ray background (i.e., $\eta \approx 1$ and $\epsilon_r \approx 1$) in an Einstein–de Sitter universe, a consistent set of parameters is $z_s \approx 1, h \approx 0.5, M \approx 10^{12} M_\odot, \chi \approx 0.3, t_s \approx 5 \times 10^9 \text{ yr}, n_\approx 15 \text{ cm}^{-3}, R \approx 0.7 \text{ kpc},$ and $kT_e \approx 70 \text{ keV};$ there is not much room for play in these parameters. This marginal consistency with the flux constraints is indicated by “M-OK” in Table 3.

Hence, a fairly firm prediction of this model is that, in an Einstein–de Sitter universe, these sources must lie just below the flux limit of the Einstein deep survey if they are to comprise...
the high-energy background. Therefore, these sources should be detected by the ROSAT deep survey, and they should have a 1–3 keV flux per source $\sim 10^{-14}$ ergs s$^{-1}$ cm$^{-2}$. Note, however, that this model is inconsistent with the isotropy constraints (see § 3.3.3).

In the open model ($d_0 = 0$) the bounds are not as tight since $h$ may be increased, $h \approx 0.75$. Equations (21) and (22) show that the model is not seriously constrained. Equation (21) reads $t_s \gtrsim 2 \times 10^9 h_0 (0.3)^{-1} (h/0.75)^{-1}$ yr, and equation (22) reads $t_s \leq 1.2 \times 10^9 R^{-1} \eta^{-0.5} (h_0/0.75)^2 (1/\sqrt{e_\gamma})$ yr. These equations are satisfied for $R \lesssim 6 \eta^{-2} (0.3)^2 (h/0.75)^2 (1/\sqrt{e_\gamma})$, where $R$ is in units of kpc.

As discussed in § 1 a spatially flat cosmological model in which the energy density is dominated by a cosmological constant allows large values of $h$, $h \approx 1$. Therefore, the supernovae model is not seriously constrained by the energy, flux, or isotropy arguments in such a cosmological model.

Equations (20), (21), and (22) with $\eta \approx 0.1$ do not provide tight constraints, indicating that these sources could easily produce the low-energy X-ray background in either an Einstein–de Sitter universe, an open model, or a spatially flat model with a significant cosmological constant.

### 3.3.3. Isotropy Constraints

In the supernovae model the sources are expected to be at fairly low redshift so that the redshifted temperature is high enough to be able to account for the high-energy background, hence $z_s \sim 1$. The sources are expected to be high-energy X-ray emitters for $\sim 5 \times 10^7$ yr in an Einstein–de Sitter universe (see eq. [20]). Therefore, $t_s < t_d(z_s)$, so $N_1$ is used to estimate the number of sources per square degree (see the Appendix and Table 1). For $\chi \lesssim 0.3$, $(z_1, z_2) = (0.5, 1.5)$, $t_s \approx 5 \times 10^8$ yr, $N_1 \approx 300$. So, the number of sources per square degree is about a factor of 3 or 4 below than that necessary to avoid overproducing fluctuations in the low-energy background. The value of $N_1$ could be increased to about 1000 deg$^{-2}$ if $\chi$ were increased to about 1, but then the observed total mass per galaxy, including dark matter, is only about $10^{11}$ $M_\odot$, as discussed in § 2.1, whereas the model requires $10^{12} M_\odot$ per galaxy. If the redshift interval is extended to $z_1, z_2 = 0.5, 2$, this number increases to $N_1 \approx 600$ deg$^{-2}$, which is still a factor of 2 below that indicated by the fluctuations analysis (see § 2.3). This problem with overproducing fluctuations in a matter-dominated flat universe when $\eta \approx 1$ is indicated by the “P” in Table 3.

In an open universe the number of sources per square degree increases by about a factor of 2 over those stated above (see Table 1). In addition, in an open universe it is reasonable to consider values of $h \gtrsim 0.5$, which is problematic in an Einstein–de Sitter universe. In this case the period of the active phase may be as long as $\sim 10^7$ yr (see eq. [22] with $h \approx 0.75$), which will increase the number of sources per square degree by another factor of $\sim 2$. So, the total number of sources per square degree is $\sim 1000$, consistent with the results of the fluctuations analysis (see § 2.3).

Therefore the supernovae model in a matter-dominated Einstein–de Sitter universe (deceleration parameter $q_0 = 0.5$ and $h = 0.5$) is ruled out as a source of the high-energy background based on the low-energy fluctuations analysis. However, if the universe is open, or spatially flat with a significant cosmological constant, then the supernovae model is still a viable possibility for the origin of the high-energy X-ray background.

The low-energy background could result from the emission mechanism discussed here. This is because, for low-energy background (i.e., $\eta \sim 0.1$) the time scale over which the X-ray emission occurs can be very long, $t_s \lesssim 3 \times 10^7 R^{-1} (0.3)^{0.5}$ yr for $z_s \sim 1$ and $h = 0.5$ (see eq. [22]). The luminosity per source $L_s \approx E_s/t_s$ (with $E_s$ from eq. [11]) is $L_s \gtrsim 1.5 \times 10^3 (0.3)^3 (h/0.5)^{1/2} R_s/\sqrt{e_\gamma}$ ergs s$^{-1}$. And the 1–3 keV flux is $f_s \gtrsim (0.15 - 0.4) \times 10^{-13} (h/0.5)^{1/2} (0.3)^{3/2} R_s/\sqrt{e_\gamma}$ ergs s$^{-1}$ cm$^{-2}$, which is well below the Einstein deep survey limit (see § 2.2); the first factor in the expression for $f_s$ is applicable if $kT_\gamma \gtrsim 35$ keV, and the second is applicable if $kT_\gamma \gtrsim 7$ keV (see § 2.2). The number of sources per square degree is a minimum in an Einstein–de Sitter universe with $t_s \approx t_d(z_s)$, that is, $N_1$ (see § 2.3). Table 2 indicates that the number of sources per square degree is $N_1 \approx 3 \times 10^7 (t_s/t_0)$, for $(z_1, z_2) = (0.5, 1.5)$. With $t_s/t_0 \approx 0.1$, $N_1$ exceeds 1000 for $\chi \gtrsim 0.3$, and $N_1$ is greater than 5000 for $\chi \gtrsim 2$, which is reasonable when $\eta \approx 0.1$ (see eq. [12]).

### 4. DISCUSSION

Energy, flux, and isotropy constraints are applied to models for the high- and low-energy X-ray backgrounds to determine which models are consistent with the observations in various cosmological models. The first constraint, that the sources produce the observed energy density in the background, is not directly related to whether the universe is open or flat, but is indirectly related through the current value of the Hubble constant (see § 1). In an open universe the current value of the Hubble constant may be $h \approx 0.75$ indicating a current age of the universe $\approx 12.5 \times 10^9 (h/0.75)^{-1}$ yr, whereas in a flat universe with deceleration parameter $q_0 = 0.5$, $h$ must be small so that the current age of the universe is about $12.5 \times 10^9 (h/0.5)^{-1}$ yr. The comoving density of sources is $\propto h^3$ and therefore can be increased if the universe is open, which eases the energy requirement per source (see eq. [1]). The flux and isotropy constraints are affected by the cosmological model because each depends directly on the comoving coordinate distance to the source and this depends on the cosmological model (see the Appendix).

The energy constraints are independent of the intrinsic spectrum of each source and hence can be applied directly to the sources of either the high- or low-energy backgrounds (see § 2.1). Note that it is possible that there are two distinct populations of sources: one which produces the high-energy background and another which produces the low-energy background (see § 1); alternatively, one population may produce both the high- and low-energy backgrounds. Models in which the spectrum of the sources extrapolates smoothly from high to low energies are able to produce a significant fraction of each of the backgrounds with one population of sources. Models in which the high-energy spectrum of each source turns over at low energies, about 3 keV at a redshift of zero, are able to produce the high-energy background, but not the low-energy background, in which case the flux and isotropy constraints from the Einstein deep survey are not applicable (see § 1).

In the massive X-ray binary model and the supernovae model, the intrinsic spectrum of each source (i.e., galaxy) extrapolates smoothly from high to low energies. Therefore, the flux and isotropy constraints from the Einstein deep survey that are directly applicable to the sources which comprise the 1–3 keV background also constrain the properties of the sources of the high-energy background after the constraints.
have been suitably modified (see §§ 2.2 and 2.3). However, in
the massive black hole model the spectrum of each source may,
but need not, be absorbed at low energies, energies of about 3
keV at a redshift of zero (see § 3.2.4). If the spectrum is
absorbed at low energies, then the isotropy and flux con-
straints from the Einstein deep survey are not applicable.

Table 3 shows the results of the comparison of the models
with the observations. The supernovae and massive X-ray
binary models have difficulty producing the high-energy back-
ground in an Einstein--de Sitter universe. The supernovae
model is inconsistent with the isotropy constraints from the
Einstein deep survey (see § 3.3.3). The massive X-ray binary
model has difficulty producing the energy density observed in
the background (see § 3.1.1). The massive black hole model is
consistent with the observations if a fraction $\delta$ of all galaxies
brighter than $L^* \Delta$ have black holes with a mass of about
$\delta^{-1} \times 10^8 M_\odot$ (since $h = 0.5$); observations of nearby galaxies
constrain this fraction to be $\delta < 0.5$. If the spectrum of the
sources does not turn over at low energies, then the isotropy
constraints imply that $\delta \gtrsim \frac{1}{3}$ (see §§ 3.2.2 and 3.2.5). If the spec-
trum of the sources does turn over at low energies, then a
very small fraction of all galaxies, $\delta \ll 1$, each with a very massive
black hole, could produce the high-energy background.

Constraints on models to produce the high-energy back-
ground are significantly eased if the universe is open (see panel
2 of Table 3), or if the universe is flat and dominated by a
cosmological constant. The isotropy constraints which are a
problem for the supernovae model in a flat matter-dominated
universe are just satisfied in an open model with $h = 0.75$ due
to both the change in the comoving coordinate distance to a
source and to the increase in the Hubble constant (see § 3.3.3).

The mass per black hole in the massive black hole model is
decreased to about $2 \delta^{-1} \times 10^7 M_\odot$ for $h = 0.75$. The massive
X-ray binary model still has difficulty with the energy argu-
ments (see § 3.1.1).

The energy density of the low-energy background is about
1/10 that of the high-energy background. Both the massive
black hole model and the supernovae model can comfortably
account for the properties of the low-energy background in
either an open or flat cosmological model. The massive X-ray
binary model is strained to produce the low-energy back-
ground in a flat or open model, but is marginally acceptable
(see § 3.1.1).

It is interesting that the Einstein deep survey constrains
models for the high-energy X-ray background if the spectra of
the sources that comprise that background extrapolate
smoothly to low energies. Similarly, interesting constraints on
models to produce the high- and low-energy X-ray back-
grounds will result from the ROSAT deep survey. The ROSAT
deep survey is expected to go at least an order of magnitude
faunter than the Einstein deep survey. This survey should dis-
cover numerous quasars and perhaps starburst galaxies, and
may observe the low-energy tail of the sources that produce the
high-energy background. As discussed in § 3.2.4, the character-
istics of the fluctuations on large scales may be used as an
indicator of whether the sources observed correspond to those
that comprise the high-energy background, that is, whether the
low-energy tail of the sources which comprise the high-energy
background have been detected. If the ROSAT survey has
sufficient deep sky coverage, a cross-correlation of the ROSAT
survey with that of HEAO 1 can be used to determine whether
the sources contributing to the low-energy background have
the same spatial distribution as those contributing to the high-
energy background.

It is also possible that ROSAT will be unable to detect the
sources of the high-energy background, which is likely to be
the case if two distinct populations produce the high- and
low-energy backgrounds. If these sources of the high-energy
background have spectra that are absorbed at low energies,
then they cannot be detected by ROSAT. This may be the case
if the X-ray emission is associated with a central, massive
compact object surrounded by absorbing material (see §§ 3.2
and 3.2.4).

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APPENDIX

A.1. FLUXES IN DIFFERENT COSMOLOGICAL MODELS

A source at a redshift $z$ with an intrinsic luminosity $L_x = dE/dt$
will produce a total flux $f_x = dE(dt/dA)^{-1}$ that depends on
the comoving coordinate distance to the source, and hence is different for different cosmological models (see § 2.2). For a matter-
dominated Einstein--de Sitter universe (deceleration parameter $q_0 = 0.5$, cosmological constant $\Lambda = 0$), the comoving coordinate
distance is $a_0 r = (2c/H_0)(1 - 1/\sqrt{1 + z})$, and for an open universe with deceleration parameter $q_0 = 0$, the comoving coordinate
distance is $a_0 r = (c/H_0)(1 + z)^{-2}(1 + 0.5z)$ (e.g., Sandage 1988). For a flat matter-dominated universe (i.e., deceleration parameter
$q_0 = 0.5$) equation (5) becomes

\[
 f_x \approx \frac{L_x h^2}{4.3 \times 10^{37}(1 + z)^2(1 - 1/\sqrt{1 + z})^2} \text{ cm}^{-2},
\]

whereas for an open universe (i.e., deceleration parameter $q_0 = 0$) equation (5) becomes

\[
 f_x \approx \frac{L_x h^2}{1.1 \times 10^{37} z^2(1 + 0.5z)^2} \text{ cm}^{-2}.
\]
A.2. SURFACE DENSITY OF SOURCES IN DIFFERENT COSMOLOGICAL MODELS

The surface density of sources, given that the sources have a particular comoving number density, depends upon the cosmological model and the probability that a source is contributing to the background. The probability that a source, which shines for a period of time that exceeds the Hubble time at the redshift of the source, is contributing to the background, is one. The probability $P$ that a source which shines for a time less than the Hubble time at the redshift of the source $t_H(z)$ is contributing to the background depends on the fraction of the Hubble time that the source is shining: $P = \frac{t_s}{t_H(z)}$ where $t_s$ is the time for which the source is shining. As noted above, $P < 1$ for $t_s < t_H(z)$ and $P = 1$ for $t_s \geq t_H(z)$.

The surface density of sources (per square degree) $N$ may be written

$$N = (4.1 \times 10^4)^{-1} \int_{z_1}^{z_2} (nP) dV,$$

where $dV$ is the volume element which depends on the cosmological model, and the numerical factor arises because there are $4.1 \times 10^4$ deg$^2$ in $4\pi$ sr. The number density of sources evolves as $n = n_0 (a/a_0)^{-3}$ where $a$ is the cosmic scale factor, a subscript $0$ refers to the current epoch, and for convenience $n_0$ is parameterized by $\chi$: $n_0 = 10^{-22} h^2 \chi$ Mpc$^{-3}$; Hubble’s constant $H_0 = 100 h$ km s$^{-1}$ Mpc$^{-1}$.

The surface density of sources is computed for four limiting cases. The surface density of sources in a matter-dominated Einstein–de Sitter universe ($\Lambda = 0$, $\Omega = 1$) when $t_s < t_H(z)$ (case 1) is denoted by $N_1$, and when $t_s > t_H(z)$ (case 2) is denoted by $N_2$. The number of sources per square degree in an open universe with deceleration parameter $q_0 = 0$ when $t_s < t_H(z)$ (case 3) is denoted by $N_3$, and that obtained when $t_s > t_H(z)$ (case 4) is denoted by $N_4$.

In a matter-dominated Einstein–de Sitter universe the volume element is

$$dV = 4\pi a^3 r^2 dr\sqrt{1 - kr^2} = 4\pi a^3 r^2 dr,$$

where the comoving coordinate distance $r$ is defined to be $dr = -dt/a$, so $a_0 r = 3t_0 [1 - (a/a_0)^{0.5}]$ since $a/a_0 = (t/t_0)^{2/3}$ when $q_0 = 0.5$, hence $t_H(z) = t_0 (a/a_0)^{1.5}$, where $(a_0/a) = (1 + z)$ and $t_0$ is the current age of the universe. Equation (A3) is easily integrated to obtain the surface density of sources (per square degree):

$$N_1 = 13.5 \left( \frac{4\pi}{4.1 \times 10^4} \right) n_0 \frac{t_s}{t_0} \frac{c}{H_0} I_1,$$

where

$$I_1 = (z_2 - z_1) - 4[(1 + z_2)^{0.5} - (1 + z_1)^{0.5}] + \ln \left( \frac{1 + z_2}{1 + z_1} \right).$$

The value of $I_1$ typically falls in the range $\sim 0.4$–0.7. For example, for $(z_1, z_2) = (1, 4)$, $I \approx 0.6$; for $(z_1, z_2) = (2, 4)$, $I \approx 0.5$ and for $(z_1, z_2) = (0, 4)$, $I \approx 0.7$. Equation (A4) reduces to

$$N_1 \approx 3.3 \times 10^5 \frac{t_s}{t_0} c I_1 \text{ deg}^{-2},$$

where the relation $t_0 = (3H_0)^{-1}$ has been used. Some values of $N_1$ are tabulated in Table 1.

The number of sources per square degree for case 2, $N_2$, may be obtained from equation (A3) with $P = 1$. Assuming a matter-dominated Einstein–de Sitter universe,

$$N_2 \approx 2.2 \times 10^5 \chi I_2 \text{ deg}^{-2},$$

where

$$I_2 = \left( 1 - \frac{1}{\sqrt{1 + z_2}} \right)^3 - \left( 1 - \frac{1}{\sqrt{1 + z_1}} \right)^3.$$

Some values of $N_2$ are tabulated in Table 2.

The number of sources per square degree $N_3$ in a universe with deceleration parameter $q_0 = 0$ (and $\Lambda = 0$) when $t_s < t_H(z)$ (case 3) may be obtained from equation (A3) with $n = n_0 (a_0/a)^3$, $P = \frac{t_s}{t_H(z)}$, $t_H(z) = (a_0/a)_0$, and $dV = 4\pi (1 + r^2)^{-1/2} a^3 r^2 dr$ since the curvature constant $k = -1$. The comoving coordinate distance $r$ is related to the redshift $(1 + z) = (a_0/a)$ via the equation $r = (1 + z)^{1/2}(1 + 0.5z)$, which may be written $r = 0.5(a_0^2 - a^2)/(a_0 a)$, $0.5(a_0/a - a/a_0)$; these relations imply that $(1 + z) = a_0/a = r + \sqrt{r^2 + 1}$. With $a_0 = (c/H_0)$, these expressions imply

$$N_3 = \frac{4\pi}{4.1 \times 10^4} \frac{t_s}{t_0} \left( \frac{c}{H_0} \right)^3 \left( \frac{r^3}{\sqrt{1 + r^2}} + r^2 \right) dr.$$  

The first term in the integral may be integrated by parts and has the solution $1/3\sqrt{1 + r^2(r^2 - 2)}$. The full solution of the integral is $(1/12)[((1 + z)^3 - 6(1 + z) - 3(1 + z)^{-1}]$. Hence, the number of sources per square degree is

$$N_3 \approx \frac{1}{12} \left( \frac{4\pi}{4.1 \times 10^4} \right) \frac{t_s}{t_0} \left( \frac{c}{H_0} \right)^3 n_0 I_3 \approx 6.9 \times 10^3 \chi \frac{t_s}{t_0} I_3 \text{ deg}^{-2},$$

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where
\[ I_3 = [(1 + z_2)^3 - 6(1 + z_2) - 3(1 + z_2)^{-1}] - [(1 + z_1)^3 - 6(1 + z_1) - 3(1 + z_1)^{-1}] . \]  
(A11)

Some values of \( N_4 \) are tabulated in Table 1.

The number of sources per square degree in a universe with deceleration parameter \( q_0 = 0 \) (and \( \Lambda = 0 \)) when \( t_x \geq t_m(z_2) \), \( N_4 \), may be obtained from equation (A3) with \( P = 1 \), \( n = n_0 (a_0/a)^3 \), and \( dV = 4\pi a^2 t^2 (1/\sqrt{1 + r^2}) \) dr:
\[ N_4 = \frac{2\pi}{4.1 \times 10^4 n_0 a_0^3} I_4 \simeq 4.1 \times 10^{4} \chi I_4 \text{ deg}^{-2} , \]  
(A12)

where
\[ I_4 = (r_x \sqrt{1 + r_x^2} - \sinh^{-1} r_x) - (r_1 \sqrt{1 + r_1^2} - \sinh^{-1} r_1) , \]  
(A13)

with \( r = z(1 + z)^{-1}(1 + 0.5z) \) and \( a_0 = c/H_0 \) (see, e.g., Sandage 1988). This may also be written
\[ I_4 = \left\{ \frac{w_z^2 - w_2^2}{4} - \cosh^{-1} \left[ 0.5(w_z + w_2^{-1}) \right] \right\} - \left\{ \frac{w_1^2 - w_1^{-2}}{4} - \cosh^{-1} \left[ 0.5(w_1 + w_1^{-1}) \right] \right\} , \]  
(A14)

where \( w = 1 + z \). When \( z \) is not too large, \( z \leq 10 \), \( I_4 \) is well approximated by
\[ I_4 \simeq 0.25(z_x^2 - z_2^2) . \]  
(A15)

Some values of \( N_4 \) are tabulated in Table 2.

REFERENCES
Boldt, E. 1987, Phys. Rept., 146, No. 4, 215
(Noordwijk: ESA), 797
Battrick (Noordwijk: ESA), 707
De Zotti, G., Danese, L., Franceschini, A., Persic, M., & Toffolatti, L. 1989, in
Two-Topics in X-ray Astronomy, ed. J. J. Hunt & B. Battrick (Noordwijk:
ESA), 737
(Dordrecht: Kluwer), 235
J. J. Hunt & B. Battrick (Noordwijk: ESA), 74
Cosmic Plasmas, ed. P. Gorenstein & M. Zombeck (Cambridge: Cambridge
University Press), 276
Letters B, 236, 454
Persic, M., De Zotti, G., Danese, L., Palumbo, G. G. C., Franceschini, A.,
University Press), 303
Lewin & E. P. J. van den Heuvel (Cambridge: Cambridge University Press),
189
Woosley, S. E., & Weaver, T. A. 1986, ARA&A, 24, 205