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SPECTRAL DISTORTIONS OF THE MICROWAVE BACKGROUND RADIATION RESULTING FROM THE DAMPING OF PRESSURE WAVES

R. A. DALY
Joseph Henry Laboratories, Princeton University, Princeton, NJ 08544
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ABSTRACT

Initial inhomogeneities of the energy density of the primeval radiation field which enter the horizon prior to recombination lead to the formation of pressure waves as they enter the horizon. On scales less than about $5 \times 10^{16}\ M_{\odot}$ these waves will be damped prior to recombination by the processes of nonlinear dissipation and photon diffusion. The damping of these waves leaves its signature in spectral distortions of the microwave background radiation. The type of distortion depends upon the epoch of the damping. The magnitude of the distortion may be related to the amplitude of the waves which have been damped during a particular epoch. The amplitude of the perturbations of the energy density of the radiation field on a given scale may be related to those of the mass distribution on the same scale, in a manner which depends on whether the perturbations are of the isocurvature or adiabatic type. The limit on the amplitude of the initial perturbations of the mass distribution on the damping scale is used in conjunction with that on a much larger scale: the amplitude of the initial perturbations of the mass distribution needed to account for the observed variance of the number counts of galaxies at the current epoch, which depends on cosmological parameters. Combining these, an upper bound to the index of the initial power spectrum of the mass distribution is obtained. This bound depends on cosmological parameters, and on whether the initial perturbations are of the isocurvature or adiabatic type.

The current and projected limits of the magnitude of the Compton y-parameter and of a chemical potential are used to constrain the index of the initial power spectrum of the mass distribution for several cosmological parameter sets for isocurvature and adiabatic perturbations.

Subject headings: cosmic background radiation — cosmology — early universe — wave motions

1. INTRODUCTION

The existence of galaxies and clusters of galaxies may be explained if density perturbations are present in the early universe, since these will grow gravitationally to form the currently observed structures. Constraints on the type and amplitude of perturbations present at recombination result from the observed isotropy of the microwave background radiation, from the epoch of the formation of structures with a given mass (assuming that the perturbations that lead to the structures are present at recombination), and from deviations of the spectrum of the radiation from that of a blackbody, which is discussed here.

Perturbations of the mass density are accomplished by perturbations of the energy density of the radiation field, and hence of the radiation pressure. When these perturbations enter the horizon, pressure gradients cause pressure waves to be established. The oscillation of these pressure waves (i.e., acoustic waves) is affected by the processes of nonlinear dissipation, which damps waves with a particular amplitude, and photon diffusion, which damps waves with a particular scale. The damping of the waves produces spectral distortions of the microwave background radiation. The type of distortion depends on the epoch of the damping, and the magnitude of the distortion depends on the amplitude of the waves which are damped. The damping of the waves smooths the perturbations of the energy density of the radiation field, causing the radiation field to be homogeneous on scales less than the damping scale.

On scales less than the damping scale at recombination ($\sim 5 \times 10^{16}\ M_{\odot}$), observed anisotropies of the microwave background radiation, or limits thereon, do not contain direct information about the initial perturbations of the energy density of the radiation field (and hence of the matter) precisely because the damping of these perturbations has erased the inhomogeneities: the signature of the initial perturbations of the radiation field is contained in a spectral distortion.

The distribution of dark matter remains unaffected by the damping of the pressure waves (if the dark matter is uncharged). After the universe becomes matter-dominated, matter perturbations begin to grow gravitationally, and scattering between photons and moving electrons, and the Sachs-Wolfe effect, produce inhomogeneities in the microwave background radiation. The resulting anisotropies, present at recombination, are only observable on fairly large scales compared with galactic and galaxy cluster scales because of the large thickness of the last scattering surface.

The damping of pressure waves that could produce observable distortions of the microwave background radiation have comoving wavelengths corresponding to mass scales of roughly 1 to $10^6\ M_{\odot}$, scales much less than galaxy or galaxy cluster scales. Hence, limits on the amplitude of the initial perturbations of the mass density determined from limits on the magnitude of spectral distortions apply to scales much less than galaxy scales, whereas limits resulting from microwave background anisotropy observations apply to scales much greater than galaxy scales.

The baryons are affected by the damping processes because the photons and electrons are tightly coupled through Thomson scattering; charged dark matter should be similarly affected. The damping of adiabatic perturbations washes out

the perturbations in the baryon distribution, and the damping of isocurvature perturbations leaves perturbations in the baryon distribution which are identical in character to the initial perturbations of the baryons (Peebles 1983). This means that if galaxy and cluster formation result from the gravitational growth of initial density perturbations, then either the perturbations are of the isocurvature variety, or there is a significant component ($\Omega \gtrsim 0.1$) of cold nonbaryonic dark matter; isocurvature perturbations allow but do not require the existence of nonbaryonic dark matter, while adiabatic perturbations require the existence of dark matter unless galaxies and clusters of galaxies form as the result of some process other than the gravitational growth of initial mass inhomogeneities.

There is a substantial body of literature discussing spectral distortions resulting from processes occurring prior to recombination; see, for example, Chan & Jones (1975a, b), Danese & De Zotti (1978), Ilarionov & Sunyaev (1975a, b), Peebles & Yu (1970), Silk (1968, 1974), Sunyaev & Zel'dovich (1970a, b), and Zel'dovich & Sunyaev (1969). Coles & Barrow (1990) discuss work similar to that presented here.

It is important to note that the damping of pressure waves does not, strictly speaking, constitute an energy input to the radiation field. It is a redistribution of the energy of the radiation field present initially, as discussed in §§ 2.2 and 2.3. Therefore, the familiar expressions (e.g., Sunyaev & Zel'dovich 1980) relating an energy input to the radiation field with the magnitude of the resulting distortion cannot be directly applied. The alterations of these expressions are derived in § 2.

The epochs during which the damping of pressure waves produces a chemical potential and a Compton distortion are discussed in § 2. The spectral properties of the radiation field at the beginning and end of each of these epochs, and the relation between the magnitude of the distortion and the amplitude of the waves whose damping led to the distortion, are addressed. The damping mechanisms, nonlinear dissipation, and photon diffusion are considered in § 3. It is straightforward to determine when the operative damping mechanism will be nonlinear dissipation and when it will be photon diffusion (see § 3.3). The observational limits on the magnitude of spectral distortions imply limits on the initial amplitude of pressure waves on the damping scale, which imply limits on the amplitude of mass perturbations on this scale, which is combined with the estimated amplitude of mass perturbations on a much larger scale to constrain the index of the initial power spectrum of the mass distribution, detailed in § 4. Isocurvature and adiabatic perturbations are considered, as are various choices of cosmological parameters. Photon production, which may be significant in the isocurvature model, is discussed (see § 4.3.2). Two effects which may weaken the constraints are discussed in § 4.5. The results are summarized in § 5.

Many aspects of the problem must be discussed before they can be used to obtain limits on the index of the initial power spectrum of the mass distribution. To guide the reader, a brief prologue is now presented.

A limit on the amplitude of the initial perturbations of the energy density of the radiation field on a particular scale translates into a limit on the initial amplitude of the perturbations of the mass distribution on the same scale, given that the perturbations are of the isocurvature or adiabatic type. If the initial amplitude of the perturbations of the mass distribution on another scale can be estimated, these two points may be combined to estimate an upper bound to the index of the initial power spectrum of the mass distribution, since, given the index and given the amplitude of the initial perturbations on one scale, the initial amplitude of the perturbations on any other scale can be determined assuming that the initial power spectrum is well approximated by a power law over the scales of interest.

The damping of pressure waves over the redshift interval from about 5×10^6 to about 5×10^5 results in a chemical potential, and the damping of pressure waves over the redshift interval from about 5×10^5 to 10^3 results in a Compton distortion, as discussed in § 2. The magnitude of each distortion is proportional to the square of the amplitude of the waves that are damped.

The initial amplitudes of perturbations of the energy density of the radiation field for waves with relatively large comoving wavelengths (i.e., wavelengths on the order of the horizon size at recombination) are constrained by existing limits on the anisotropy of the microwave background radiation to be small, less than about 10^{-3}. If the amplitudes of the waves when the waves enter the horizon is constant (i.e., independent of wavenumber), or decreasing with increasing wavenumber, the magnitude of the distortions that will result from the damping of pressure waves will be so small as to be unlikely ever to be observed: dimensionless chemical potential or Compton distortion less than about 10^{-7}. It is therefore assumed that the amplitude of the perturbations of the energy density of the radiation field when these enter the horizon are larger on small scales than they are on larger scales, that is, the amplitude of the waves is increasing with increasing comoving wavenumber. Note that, while the universe is radiation-dominated, the amplitudes of the waves remain constant until the damping processes become important.

Photon diffusion damps waves with progressively longer comoving wavelengths, as does nonlinear dissipation, in the case in which the initial amplitudes of the waves increase as the comoving wavelength decreases (since in this case the largest amplitude waves have undergone the most oscillations), as described in § 3. At a redshift of about 5×10^6 the damping of pressure waves begins to leave its signature in a chemical potential, whereas damping at higher redshifts cannot be observed (see § 2). The waves damped at this redshift have amplitudes larger than those of any other waves that will be subsequently damped, in the case in which the initial amplitude of the waves increases as the comoving wavelength decreases. Therefore, the primary contribution to a chemical potential results from the waves that are damped at the redshift $z_{\rm th}$, the largest possible redshift at which damping will result in a chemical potential. Similarly, the damping of waves at the redshift z_y (about 5×10^5), the largest redshift at which the damping of the waves leads to a Compton distortion, will contribute most to the Compton distortion.

The limits described in this paper are obtained assuming that the primary contribution to the chemical potential results from the damping of pressure waves at the redshift $z_{\rm th}$, and that the primary contribution to a Compton distortion results from damping at the redshift z_y. These assumptions are valid if the initial amplitude of the waves (defined to be the amplitude of the perturbation of the energy density of the radiation field when it crosses the horizon) is increasing as the comoving wavenumber increases. As noted above, if this is not the case, the damping of waves will produce such small distortions that they cannot be observed.

Given the redshift of the damping and the damping mechanism, the damping scale can be determined (§ 3.4). If the

damping mechanism is photon diffusion, the damping scale is determined by the redshift of the damping and is independent of the magnitude of the distortion. So, the damping scale relevant for the production of a chemical potential at the redshift $z_{\rm th}$ is set, as is that relevant for the production of a Compton distortion at the redshift z_y. If the damping mechanism is nonlinear dissipation, the damping scale depends both on the redshift of the damping and on the magnitude of the distortion, as discussed in § 3.4.

2. DISTORTIONS

The type of distortion resulting from the dissipation of pressure (i.e., acoustic) waves in the early universe depends primarily upon the ratio of the electron number density to the photon number density; three primary epochs may be identified (e.g., Danese & De Zotti 1980; Sunyaev & Zel'dovich 1980; Danese et al. 1990). During the first epoch, scattering between photons and electrons is efficient and causes the radiation field to relax to statistical equilibrium, and bremsstrahlung radiation and double Compton scattering produce photons efficiently so that the radiation field is in thermal equilibrium (Peebles 1971, § VII; Lightman 1981; Danese & De Zotti 1982). During this epoch the spectrum will be that of a blackbody irrespective of the damping of pressure waves, or any energy input to the radiation field. This epoch corresponds to redshifts greater than $z_{\rm th}$, where

$$z_{\rm th} \simeq 10^7 \eta_{10}^{-0.35} \tag{1}$$

(Danese et al. 1990), where $\eta_{10} = 10^{10} n_b/\bar{n}_\gamma$; n_b is the number density of nucleons, which may be related to the current mass density of baryons ρ_b and the proton rest mass m_p: $n_b = \rho_b/m_p$; and $\bar{n}_\gamma$ is the mean number density of photons of the primeval radiation field, $\bar{n}_\gamma = a_{\rm SB} T^3/(2.7k)$, $a_{\rm SB}$ being the Stefan-Boltzmann constant and k being the Boltzmann constant.

The current mean mass density in baryons relative to the critical density is related to the baryon-to-photon ratio:

$$\Omega_b = 3.7 \times 10^{-3} \eta_{10} h^{-2} \left(\frac{T}{2.75}\right)^3 , \tag{2}$$

where Hubble's constant is $H_0 = 100h$ km s^{-1} Mpc^{-1}. In the application of this equation the current temperature T of the primeval radiation field will be taken to be 2.75 K.

Evidently, the redshift $z_{\rm th}$ above which the radiation field will be in thermal equilibrium, even if there is an energy input, depends primarily on η_{10}, and that dependence is fairly weak. Observationally η_{10} is constrained to lie between about 2.5 and 4.5 if there are three effectively massless neutrino species with equilibrium abundances during nucleosynthesis (i.e., at an energy of about 1 MeV) (Olive et al. 1990), but may be as large as 10 if only two neutrino species are effectively massless during nucleosynthesis, or if one of the neutrino species has an adundance significantly below the equilibrium value (Yang et al. 1984). If three effectively massless neutrino species with equilibrium abundances are present during primordial nucleosynthesis, then Ω_b must lie in the range from $9.3 \times 10^{-3} h^{-2}$ to $1.7 \times 10^{-2} h^{-2}$. This means that Ω_b may be too small to account for the mass density indicated by dynamics, unless there are only two massless neutrino species present with equilibrium abundance during nucleosynthesis, or $\Omega_{t,0} \lesssim 0.07(h/0.5)^{-2}$, $\Omega_{t,0}$ being the total mass-energy density relative to the critical value at the current epoch.

Note that equations (1) and (3) are only strictly valid for $\Omega_b \lesssim 0.3(h/0.5)^{-2}$, and are slightly altered for larger values of Ω_b (Danese et al. 1990). Note also that if some process causes the redshift $z_{\rm th}$ of z_y to differ from the values given by equations (1) and (3), the results are altered, as discussed in § 4.5.

The second epoch corresponds to the redshift interval from $z_{\rm th}$ to z_y. During the second epoch photon production is not efficient but scattering is, so the radiation field relaxes to statistical equilibrium. If nonlinear dissipation, photon diffusion, or some energy input to the radiation field occurs during this epoch, scattering will cause the radiation field to relax to a Bose-Einstein distribution function; there will be too few photons for the spectrum to be that of a blackbody. The redshift z_y above which scattering causes the radiation field to relax to statistical equilibrium is

$$z_y \simeq 6.4 \times 10^5 \eta_{10}^{-0.5} \tag{3}$$

(Danese et al. 1990). This epoch, too, is weakly dependent on parameter choices, as it depends primarily on η_{10} and this dependence is not strong.

Finally, during the third epoch, if the radiation field deviates from that of a blackbody as a result of the damping of pressure waves, or some form of energy input, the radiation field will not relax to statistical equilibrium, but Compton scattering will distort the spectrum. This epoch corresponds to the redshift interval from z_y to recombination, $z_{\rm rec} \simeq 10^3$. The distortion of the spectrum which results from the damping of pressure waves during this epoch will be very similar to a Compton y-distortion.

The properties of the radiation field at the end of the first epoch are considered in § 2.1. The chemical potential resulting from the damping of pressure waves during the second epoch, over the redshift interval from $z_{\rm th}$ to z_y, is discussed in § 2.2. In § 2.3 the distortion resulting from the damping of pressure waves during the third epoch, over the redshift interval from z_y to $z_{\rm rec}$, is presented.

2.1. *Properties of the Radiation Field at Redshifts Greater than $z_{\rm th}$*

It is assumed that the initial inhomogeneities of the energy density of the radiation field are imprinted at a redshift greater than $z_{\rm th}$, so that at the redshift $z_{\rm th}$ the radiation field is at every point a blackbody with the temperature dependent on position: $T = T(\boldsymbol{x})$. At any point the photon distribution function is $N = (e^{h\nu/kT(\boldsymbol{x})} - 1)^{-1}$. When variations of the energy density of the radiation field on a particular scale enter the horizon, pressure gradients cause pressure waves to be established. The oscillation of the waves is adiabatic, so that during the oscillation of a wave the distribution function remains Planckian but the temperature is adiabatically varied. The amplitudes of the waves will quickly become smoothly distributed about the rms value $\langle|\delta_r|^2\rangle^{1/2}$ (which, for convenience, is denoted by $\delta_{r,\rm rms}$), where $\delta_r(\boldsymbol{x}) = [\rho_\gamma(\boldsymbol{x}) - \bar{\rho}_\gamma]/\bar{\rho}_\gamma$ is the fractional perturbation of the energy density of the radiation field at the point $\boldsymbol{x}$, $\rho_\gamma(\boldsymbol{x})$ is the energy density of the radiation field at that point, and $\bar{\rho}_\gamma$ is the average density of the radiation field. Perturbations of the energy density of the radiation field may be related to those of the temperature, since $\rho_\gamma \propto T^4$. The quantity $\langle|\delta T/\bar{T}|^2\rangle^{1/2}$, denoted for simplicity $\sigma/\bar{T}$, satisfies $\sigma/\bar{T} \simeq 0.25\, \delta_{r,\rm rms}$, for $\sigma/\bar{T} \ll 1$, where $\bar{T}$ is the mean temperature of the radiation field. It will be assumed that $\sigma/\bar{T} \ll 1$, which is consistent with equation (5), since y is observationally constrained

to be $y \lesssim 10^{-3}$ (Mather et al. 1990; Gush, Halpern, & Wishnow 1990).

The small-scale random motions of the waves will have a Maxwellian distribution of velocities, and hence the temperature of the radiation field will be Gaussian-distributed, since the Doppler-shifted temperature is proportional to the velocity of the waves (see Zel'dovich, Illarionov, & Sunyaev 1972). The total distribution function is given by

$$N = \frac{1}{\sqrt{2\pi}\sigma} \int_{-\infty}^{\infty} \frac{\exp\left[-x^2/(2\sigma^2)\right]dx}{\exp\left[h\nu/k(\bar{T}+x)\right]-1} . \tag{4}$$

This is the *initial* distribution function characterizing the spectrum; it characterizes the spectrum that would be observed on scales much larger than the scale of the perturbations. The distribution function has the form given by equation (4) *prior* to the damping of waves that will lead to spectral distortions.

Note for future reference that the distribution function given by equation (4) is identical to that characterizing a Compton y-distorted spectrum when $y \ll 1$ (Zel'dovich, Illarionov, & Sunyaev 1972), where $\bar{T} = \bar{T}_{\rm RJ}$ corresponds to the temperature in the Rayleigh-Jeans region of the spectrum, and

$$\left(\frac{\sigma}{\bar{T}}\right)^2 = 2y . \tag{5}$$

The average number density of photons, $\bar{n}_\gamma$, computed prior to the damping of waves relevant for a spectral distortion, is

$$\bar{n}_\gamma = k'_N \int_0^{\infty} \int_{-\infty}^{\infty} \frac{\exp\left[-x^2/(2\sigma^2)\right]dx}{\exp\left[h\nu/k(\bar{T}+x)\right]-1} \nu^2 \, d\nu , \tag{6}$$

where k'_N is a constant (different from k_N). As $\sigma \to 0$, $\bar{n}_\gamma \to k_N \bar{T}^3$, which defines the constant k_N. Interchanging the order of integration and integrating, we find

$$\bar{n}_\gamma = k_N \bar{T}^3[1 + 3(\sigma/\bar{T})^2] . \tag{7}$$

The average energy density $\bar{\rho}_\gamma$ of the radiation field, computed prior to the damping of waves relevant for a spectral distortion, is

$$\bar{\rho}_\gamma = k'_E \int_0^{\infty} \int_{-\infty}^{\infty} \frac{\exp\left[-x^2/(2\sigma^2)\right]dx}{\exp\left[h\nu/k(\bar{T}+x)\right]-1} \nu^3 \, d\nu , \tag{8}$$

where k'_E is a constant (different from k_E). As $\sigma \to 0$, $\bar{\rho}_\gamma \to k_E \bar{T}^4$, which defines the constant k_E. Interchanging the order of integration and integrating, we find

$$\bar{\rho}_\gamma = k_E \bar{T}^4[1 + 6(\sigma/\bar{T})^2 + 3(\sigma/\bar{T})^4] \simeq k_E \bar{T}^4[1 + 6(\sigma/\bar{T})^2] . \tag{9}$$

There will be no signature from the damping of pressure waves during the first epoch, at redshifts greater than $z_{\rm th}$. At this redshift the radiation field is characterized by the mean temperature $\bar{T}$ of the radiation field and the rms fluctuations of the temperature about this mean, $\sigma/\bar{T}$, which specify the total distribution function (eq. [4]) and determine the mean photon number and energy densities given by equations (7) and (9).

2.2. *The Second Epoch: The Production of a Chemical Potential*

During the second epoch, over the redshift interval from $z_{\rm th}$ to z_y, scattering will cause the radiation field to relax to statistical equilibrium but photon production is insignificant, hence the distribution of the occupation numbers at the end of this epoch will be of the form

$$N = [\exp(h\nu/kT - \mu/kT) - 1]^{-1} , \quad \mu < 0$$

(e.g., Peebles 1971, § VII); μ is the chemical potential. The average photon number density and energy density are

$$\bar{n}_\gamma \simeq k_N \bar{T}^3[1 + 1.368(\mu/k\bar{T})][1 + 3(\sigma/\bar{T})^2] \tag{10}$$

and

$$\bar{\rho}_\gamma \simeq k_E \bar{T}^4[1 + 1.11(\mu/k\bar{T})][1 + 6(\sigma/\bar{T})^2] , \tag{11}$$

which are obtained using a straightforward generalization of the methods applied by Sunyaev & Zel'dovich (1980).

At the redshift $z_{\rm th}$, the average number density and energy density of the radiation field are given by equations (7) and (9) with $\sigma = \sigma_i$ and $\bar{T} = T_i$. At the redshift z_y the average number density and energy density of the radiation field are given by equations (10) and (11) with $\sigma = \sigma_f$ and $\bar{T} = T_f$. Since bremsstrahlung and double Compton scattering are inefficient at producing photons during this epoch, both the total number of photons within some large finite volume V, $\bar{n}_\gamma V$, and the quantity $\bar{\rho}_\gamma V^{4/3}$ are conserved. Equations (7), (9), (10), and (11) determine μ/kT_f in terms of σ_i/T_i and σ_f/T_f. When $|\mu/kT| \ll 1$, and $\sigma/\bar{T} \ll 1$,

$$\frac{\mu}{kT} \simeq -2.8\left[\left(\frac{\sigma_i}{T_i}\right)^2 - \left(\frac{\sigma_f}{T_f}\right)^2\right] \simeq -\frac{\delta^2_{r,\rm rms}}{5.7} , \tag{12}$$

where $\delta_{r,\rm rms}$ is understood to be the summed or integrated value of the mean amplitudes of the perturbations which have been damped during this epoch (averaged over space). If the damping of waves on one particular scale (with amplitude δ_r) is the primary contributor to the chemical potential (the case considered in this paper), then the amplitude of the waves δ_r on this scale averaged over space should be substituted for $\delta_{r,\rm rms}$; the amplitudes of the waves on this scale will have some distribution, and the mean value of these amplitudes is referred to as the amplitude of the waves on this scale.

Since the observationally determined upper limit on a chemical potential is small, $|\mu/kT| \lesssim 10^{-2}$ (Danese & De Zotti 1978; Smoot et al. 1988; Mather et al. 1990; Gush et al. 1990), $\delta_{r,\rm rms} \lesssim 0.25$ on scales which are damped during this epoch.

Note that equation (12) is different from that which would be obtained if it were assumed that the damping of the waves mimics a dimensionless energy input to the radiation field with $\delta\epsilon/\epsilon \simeq (\frac{1}{3})\delta_r^2$, where δ_r is the amplitude of the waves with a wavelength on the order of the damping scale. As discussed in § 1, the damping of pressure waves does not constitute an energy input to the radiation field; it is a redistribution of the energy initially present in that field, which may be seen in the following way.

Consider the radiation field during an early epoch, just before the inhomogeneities which will be damped to produce a distortion enter the horizon. The total energy of the radiation field within some large finite volume, and hence the mean energy density of the radiation field, is set. When the inhomogeneities enter the horizon, they begin to oscillate as pressure waves with an amplitude equal to that which they have at the time they enter the horizon. This means that the mean energy density of the radiation field may be obtained by integrating over the distribution of the amplitudes of the perturbations, as done above. The damping of the waves distributes the energy density of the radiation field smoothly (i.e., washes out the inhomogeneities). Equation (12) indicates that the damping of the pressure waves may be said to mimic an energy input to the

radiation field with the dimensionless amplitude of the waves δ_r related to the dimensionless energy input $\delta\epsilon/\epsilon \simeq (\frac{1}{8})\delta_r^2$. Then the familiar relation between the magnitude of the chemical potential and the dimensionless energy input results: $\mu/kT \simeq 1.4\,\delta\epsilon/\epsilon$.

2.3. *The Third Epoch: A Compton-distorted Spectrum*

Nonlinear damping (i.e., shock formation) causes the kinematic energy of the waves to be transferred back to the thermal energy of the radiation field. Scattering causes the radiation field to be Compton-distorted, and a Compton-distorted spectrum is characterized by equation (4) (since $y \ll 1$). Waves with an initial amplitude of $\delta_{r,\mathrm{rms}} \simeq 4(\sigma/\bar{T})$ will lead to a Compton y-parameter $y \simeq 0.5(\sigma/\bar{T})^2 \simeq \delta_{r,\mathrm{rms}}^2/32$ (see eq. [5]) when they are damped.

The damping of pressure waves by photon diffusion leads to a similar result, which may be seen as follows. Since the chemical potential must be small, we shall assume that the photon occupation number at the beginning of this epoch is well approximated by equation (4). If scattering during diffusion is neglected (i.e., the instanteous diffusion approximation), the final distribution function is again given by equation (4). This is because, prior to diffusion, each region at temperature $(\bar{T} + x)$ is weighted by the volume occupied by regions at this temperature, and after diffusion the photon number density is diluted by precisely this same factor. The diffusion causes the inhomogeneity of the photon temperature (photon occupation number described by a Planck function; temperature dependent on position) to be replaced by an isotropic distribution of photons with a distorted spectrum. Scattering during this epoch causes the radiation field to be Compton-distorted. However, the initial and final occupation numbers describing the radiation field (obtained neglecting scattering) are precisely the same as expected to result from Compton scattering (when $y \ll 1$); that is, equation (4) describes a Compton-distorted spectrum (when $y \ll 1$), and if the spectrum deviates from a Compton-distorted spectrum, scattering will cause it to become Compton-distorted. Therefore, the final occupation number is given by equation (4).

In summary, during the third epoch, the damping of acoustic waves with an amplitude $\delta_{r,\mathrm{rms}}$ will lead to a Compton-distorted spectrum. When $(\sigma/\bar{T}) \ll 1$, $\delta_{r,\mathrm{rms}} \simeq 4(\sigma/\bar{T})$, and, substituting this in equation (5), the Compton y-parameter is

$$y \simeq \frac{1}{32}\,\delta_{r,\mathrm{rms}}^2 \,. \tag{13}$$

Here $\delta_{r,\mathrm{rms}}$ is understood to be the summed or integrated value of the mean amplitude of the perturbations which have been damped during this epoch (i.e., averaged over space). If the primary contribution of the Compton distortion results from the damping of waves with one particular length scale (the case considered in this paper), the mean amplitude of the waves on this scale δ_r (averaged over space) should be substituted in equation (13) for $\delta_{r,\mathrm{rms}}$.

Note that equation (13), like equation (12), differs from the expression obtained for a Compton distortion resulting from an energy input to the radiation field from some external source. As noted in § 1 and discussed in § 2.2, the damping of pressure waves does not constitute an energy input to the radiation field but a redistribution of the energy of the radiation field. The familiar relationship between the Compton y-parameter and the dimensionless energy input to the radiation field if the distortion results from an energy input of $y \simeq (\frac{1}{4})\delta\epsilon/\epsilon$ is obtained from equation (13) when $\delta_r^2 \simeq (\frac{1}{8})\delta\epsilon/\epsilon$; δ_r is the amplitude of the waves with a wavelength on the order of the damping scale averaged over space. In this sense, it may be said that the damping of the pressure wave is like an energy input to the radiation field, with the amplitude of the waves that are damped related to the dimensionless energy input via the expression given above.

3. DAMPING MECHANISMS

Two mechanisms cause the damping of acoustic oscillations: nonlinear dissipation (i.e., shock formation) and photon diffusion. If the initial amplitudes of the waves are large enough, the primary damping mechanism will be nonlinear dissipation.

As discussed in § 1, if the amplitudes of the waves are not increasing with increasing wavenumber k, the distortions produced by the damping of the waves will not be observable, since the amplitudes of the perturbations are constrained to be small on large scales by microwave anisotropy experiments. Therefore, it is assumed that the initial (i.e., at horizon crossing) amplitudes of the waves are increasing with comoving wavenumber k.

The damping causes the amplitudes of the waves to turn over and decrease at a wavenumber corresponding to the damping scale. And the scale being damped moves toward progressively smaller wavenumbers, that is, larger scales. Hence, the maximum value of the wave amplitudes is always decreasing in magnitude as time progresses (and the redshift decreases), and the wavenumber with the largest amplitude is decreasing as the redshift decreases. Therefore, the maximum contribution to a distortion, be it a chemical potential or a Compton distortion, results from the damping which occurs at the largest possible redshift, since it is at that redshift that waves with the largest amplitudes are damped. So the maximum contribution to a chemical potential results from damping of waves at a redshift z_{th} (see § 2.2), and the maximum contribution to a Compton distortion results from damping at a redshift z_y (see § 2.3).

The damping scale depends on the damping mechanism. The domains of the damping processes, that is, the conditions which determine the primary damping mechanism, are discussed in § 3.3.

3.1. *Nonlinear Dissipation*

The propagation of waves in a relativistic fluid, and the steepening of the waves into shocks, has been considered in detail (e.g., Peebles 1970; Liang & Baker 1977; Peebles 1980, § 89; Zel'dovich & Novikov 1983). The dissipation of the waves results because the fluid speed depends on the density of the fluid; the fluid speed is the time rate of change of the position of the fluid element in proper Cartesian coordinates. The fluid speed increases as the fluid density increases, so sinusoidal waves become sawtooth waves as the crests overtake the troughs. This leads to the damping of the wave, and part of the wave energy is dissipated.

This process of nonlinear dissipation limits the dimensionless amplitude δ_r of the oscillations: $\delta_r \lesssim N_{\mathrm{os}}^{-0.5}$ where N_{os} is the number of oscillations that the wave has undergone (Peebles 1980; § 89 Zel'dovich & Novikov 1983, § 16.3). The number of oscillations a wave with period P undergoes is $N_{\mathrm{os}} = \int P^{-1}\,dt$. The dimensionless amplitude of the waves considered here is well below unity, so the nonlinear dissipation decreases the wave amplitude gradually as the wave oscillates; the shock formed during each oscillation is weak. In this case

the velocity of the wave is always close to the sound speed, and the relationship between the frequency $\nu(\nu = P^{-1})$, the wavelength λ, and the velocity of the wave may be approximated by $\nu \simeq c_s/\lambda$. When the scale factor a is less than $a_{\rm eq}$, the dominant component of the fluid is relativistic and the sound speed is well approximated by $c_s \simeq c/(3)^{1/2}$; $a_{\rm eq}$ is the cosmic scale factor when the energy density of the radiation field is equal to that of the matter. The physical wavelength of the wave increases as the universe expands, $\lambda = \lambda_e(a/a_e)$, where the subscript e refers to the value of the parameter when the wave enters the horizon, $\lambda_e = ct_e$, and a is the cosmic scale factor. Combining these expressions, a wave which enters the horizon at the redshift z_e undergoes $N_{\rm os}$ oscillations from the redshift z_e to the redshift z:

$$N_{\rm os} = \int_{t_e}^{t} \frac{dt}{P} \simeq \left(\frac{t}{t_e}\right)^{0.5} - 1 \simeq \sqrt{\frac{t}{t_e}} = \frac{a}{a_e} = \frac{1+z_e}{1+z} . \quad (14)$$

This results because $a = a_e(t/t_e)^{0.5}$ for $a < a_{\rm eq}$.

A wave which enters the horizon at a redshift z_e has a comoving wavelength

$$\lambda = ct_e(a_0/a_e) = c(a_0/a_e)(a_e/a_{\rm eq})^2 t_{\rm eq} = l_{\rm eq}(a_e/a_{\rm eq}) ,$$

where $l_{\rm eq}$ is the comoving horizon size at $z_{\rm eq}$. The comoving wavelength of a wave which enters the horizon at a redshift z_e for $z_e > z_{\rm eq}$ is

$$\lambda(z_e > z_{\rm eq}) = 4 \times 10^5 (1+z_e)^{-1} \text{ Mpc} . \quad (15)$$

This results because the comoving horizon size at $a_{\rm eq}$ is $l_{\rm eq} = ct_{\rm eq}(a_0/a_{\rm eq}) = 10h^{-2}\Omega_{t,0}^{-1}$ Mpc, where $t_{\rm eq} = (\frac{2}{3})H_0^{-1}\Omega_{t,0}^{-0.5} z_{\rm eq}^{-1.5}$ and $z_{\rm eq} = 4 \times 10^4 h^2 \Omega_{t,0}$, $\Omega_{t,0}$ being the total mass-energy density relative to the critical value at the current epoch.

A wave with comoving wavelength λ at a redshift z has undergone $N_{\rm os}$ oscillations, and has a maximum amplitude $\delta_{r,\max} = N_{\rm os}^{-0.5}$. Equations (14) and (15) indicate that the relationship between the comoving wavelength of the wave and its maximum amplitude is

$$\lambda = 4 \times 10^5 \frac{\delta_{r,\max}^2}{1+z} \text{ Mpc} . \quad (16)$$

This expression will be used in § 3.3 to determine when the primary damping mechanism is photon diffusion and when it is nonlinear dissipation. A wave with a comoving wavelength λ at a redshift z with an amplitude which exceeds $\delta_{r,\max}$ given by equation (14) will have been damped by the process of nonlinear dissipation.

Photon diffusion also damps the acoustic oscillations, and the damping scale for this process is discussed below. It is important to note that, when the wavelength of the perturbations is greater than the damping scale for photon diffusion, if damping occurs at all, the primary mechanism will be nonlinear dissipation.

3.2. *Photon Diffusion*

The comoving scale on which photon diffusion is important, $\lambda_{\gamma d}$, is determined by the photon mean free path l or by the collision time, t_c: $l = ct_c = (n\sigma_{\rm T})^{-1}$, where n is the electron number density and $\sigma_{\rm T}$ is the Thomson cross section. Photon diffusion decreases the amplitude to e^{-1} of its initial value for waves with a comoving wavenumber k_d at a redshift z (Peebles 1980, § 92; see also Peebles & Yu 1970; Silk 1974):

$$\lambda_{\gamma d} = \frac{2\pi}{k_d} \simeq 2.3 \times 10^4 h^{-1.5} \Omega_b^{-0.5} \Omega_{t,0}^{-0.25} (1+z_{\rm eq})^{0.25} (1+z)^{-1.5} \text{ Mpc} , \quad (17a)$$

where $z_{\rm eq}$ is the redshift at which the radiation and matter have equal energy densities. With Ω_b given by equation (2) and $z_{\rm eq} \simeq 4 \times 10^4 h^2 \Omega_{t,0}$, this simplifies to

$$\lambda_{\gamma d} \simeq 5.3 \times 10^6 \eta_{10}^{-0.5} (1+z)^{-1.5} \text{ Mpc} . \quad (17b)$$

Note that, like the wavelength λ given by equation (15), this scale is independent of most cosmological parameters, and depends primarily upon η_{10} and the redshift z.

For reference, the mass scale corresponding to any comoving length scale λ is

$$M = 2.5 \times 10^{11} M_\odot \left(\frac{\lambda}{\text{Mpc}}\right)^3 \left(\frac{h}{0.5}\right)^2 \left(\frac{\Omega_{t,0}}{1.0}\right) , \quad (18)$$

where $\Omega_{t,0}$ is the ratio of the total mass-energy density to the critical value at a redshift of zero. The damping scale set by photon diffusion at recombination, $z_{\rm rec} \sim 10^3$, is ~ 50 Mpc (taking $\eta_{10} = 10$), which corresponds to a mass scale of $\sim 7 \times 10^{16} (\Omega_{t,0}/1.0)\, M_\odot$.

3.3. *Domains of the Damping Processes*

Consider waves with wavelengths from λ_* to $2\lambda_*$. These waves have some mean amplitude δ_*, and, if they are damped, they will produce a spectral distortion with a magnitude proportional to δ_*^2. Given λ_*, the process that will lead to the damping of the waves at any particular redshift z_* may be determined in terms of δ_*; if $\lambda_* > \lambda_{\gamma d}(z_*)$ (eq. [17b]), then the only process that could have damped the waves is nonlinear dissipation. When $\lambda_* \sim \lambda_{\gamma d}(z_*)$, the damping mechanism will be photon diffusion. If photon diffusion becomes important on the scale λ_* at z_*, the mean amplitude of these waves must be less than $\delta_{r,\max}$ given by equation (16) with $\lambda = \lambda_*$ and $z = z_*$, since waves with larger amplitudes would have been diminished to this value at earlier epochs via nonlinear dissipation. Therefore, given λ_* and z_*, the damping mechanism is determined by δ_*: if δ_* is greater than some set value, the damping mechanism will be nonlinear dissipation, and if it is less than this value the damping mechanism will be photon diffusion. Since the magnitude of the distortion is proportional to δ_*^2 (eqs. [12] and [13]), values of μ_* and y greater than the said set value can only result from nonlinear dissipation, and lower values can only result from photon diffusion.

Consider an energy input resulting in a chemical potential. A value of $|\mu/kT|$, denoted for simplicity by μ_*, implies a value for $\delta_{r,\rm rms}$ (see eq. [12]), which implies a value for $N_{\rm os}$, assuming that the initial amplitude of the wave was larger than $N_{\rm os}^{-0.5}$, which implies a value for λ (see eq. [16]), assuming that the chemical potential results from dissipation at a redshift $z_{\rm th}$ (see §§ 1 and 3). The ratio of this value of λ to the damping length due to photon diffusion $\lambda_{\gamma d}$ may then be used to determine the values of μ_* above which the primary damping mechanism will be nonlinear dissipation and below which the primary damping mechanism will be photon diffusion.

Equation (12) indicates that $\mu_* \simeq 0.18\delta_{r,\rm rms}^2$, and the scale which has undergone significant damping contributes most

to $\delta_{r,\mathrm{rms}}$ so that $\delta^2_{r,\mathrm{rms}} \sim N_{\mathrm{os}}^{-1}$, with $N_{\mathrm{os}} \simeq a_{\mathrm{th}}/a_e$, hence $\mu_* \simeq 0.18(1 + z_{\mathrm{th}})/(1 + z_e)$. Note that the chemical potential is assumed to result from dissipation at the largest redshift at which a chemical potential could be produced, given an energy input to the mean radiation field; this approximation is valid when the amplitudes of the perturbations in the radiation field are rising toward smaller scales (see §§ 1 and 3), and this is assumed, since otherwise the distortions will be well below any observational bounds; the power spectra for which this assumption is valid are discussed in § 4.2. Using equation (15) to obtain $(1 + z_e)^{-1}$ in terms of λ and substituting into equation (12), $\mu_* \simeq 4.5 \times 10^{-7} z_{\mathrm{th}}(\lambda/\mathrm{Mpc})$ is obtained, and, substituting z_{th} into equation (1), $\mu_* \simeq 4.5\eta_{10}^{-0.35}(\lambda/\mathrm{Mpc})$ results. Solving this equation for λ and dividing by equation (17), the ratio $\lambda/\lambda_{\gamma d} \simeq 1.4 \times 10^3 \mu_* \eta_{10}^{0.325}$ is obtained. Therefore, $\lambda \gtrsim \lambda_{\gamma d}$ when $\mu_* \gtrsim 7.4 \times 10^{-4}\eta_{10}^{0.325}$; when $\lambda > \lambda_{\gamma d}$, if damping occurs at all, it must be due to nonlinear dissipation, and when $\lambda \simeq \lambda_{\gamma d}$ the relevant damping mechanism is photon diffusion. For η_{10} in the interval from 2.5 to 10, $\eta_{10}^{-0.325}$ lies between 0.7 and 0.5, so the transition from nonlinear dissipation to photon diffusion occurs at about $\mu_* \simeq 4.5 \times 10^{-4}$. Hence, $\lambda \gtrsim \lambda_{\gamma d}$ for $\mu_* \gtrsim 4.5 \times 10^{-4}$; in this case the process that could account for the chemical potential is nonlinear dissipation. If μ_* is observationally determined to be less than about 4.5×10^{-4}, then the process leading to the chemical potential could not be nonlinear dissipation but could be photon diffusion.

A similar analysis may be used to determine the primary damping mechanism that could lead to a Compton y-parameter. The Compton y-parameter is $y \simeq 0.03\delta_{r,\mathrm{rms}}$ (eq. [13]). Assuming that nonlinear dissipation is the damping mechanism, $\delta^2_{r,\mathrm{rms}} \sim N_{\mathrm{os}}^{-1}$, and $N_{\mathrm{os}} \simeq (1 + z_e)/(1 + z)$, so that at the redshift when the "Compton epoch" commences, $z = z_y$, the ratio $\lambda/\lambda_{\gamma d}$ is approximately $2 \times 10^3 \eta_{10}^{0.25} y$, where equations (3), (15), and (17) have been used. Hence, the process of photon diffusion takes over from that of nonlinear dissipation in producing a Compton y-parameter as y decreases through $y = 5.1 \times 10^{-4}\eta_{10}^{-0.25}$. As η_{10} is varied from 2.5 to 10, $\eta_{10}^{-0.25}$ varies from 0.8 to 0.6, so the primary damping mechanism transfers from nonlinear dissipation to photon diffusion at about $y \simeq 3.5 \times 10^{-4}$, that is, $\lambda \gtrsim \lambda_{\gamma d}$ for $y \gtrsim 3.5 \times 10^{-4}$. Therefore, for values of the Compton y-parameter greater than about 3.5×10^{-4}, the damping mechanism that could lead to the Compton y-parameter is nonlinear dissipation, and for values of the Compton y-parameter below about 3.5×10^{-4}, the relevant damping mechanism is photon diffusion.

3.4. *Damping Lengths and Mass Scales*

The damping of pressure waves during the redshift interval from z_{th} to z_y will result in a chemical potential (see § 2.2). As detailed in §§ 1, 3, and 4, larger amplitude waves are damped at larger redshift, so the chemical potential resulting from the damping of waves during the second epoch is, to a good approximation, equal to that resulting from the damping of waves at a redshift z_{th}.

As shown in § 3.3, when the chemical potential is larger than about $7.4 \times 10^{-4}\eta_{10}^{-0.325} \simeq 4.5 \times 10^{-4}$, the damping mechanism that could produce the chemical potential is nonlinear dissipation. The damping length is given by equation (16), with $z = z_{\mathrm{th}}$, and $\delta^2_{r,\max} \simeq 5.7\mu_*$ (see eq. [12]). The damping scale is $\lambda \simeq 0.23\mu_* \eta_{10}^{0.35}$ Mpc, and the mass scale (eq. [18]) is

$$M \simeq 3 \times 10^9 \mu_*^3 \eta_{10}^{1.05}\left(\frac{h}{0.5}\right)^2\left(\frac{\Omega_{t,0}}{1.0}\right) M_\odot . \qquad (19a)$$

The current bound on μ_* of about $\mu_* \lesssim 10^{-2}$ indicates a mass scale less than about $10^3\ M_\odot$. If a chemical potential greater than about 4.5×10^{-4} is observed, it could be due to nonlinear dissipation of waves with a mass scale given by equation (19a).

A chemical potential smaller than about 4.5×10^{-4} could result from photon diffusion. Then, the damping scale is given by equation (17b) evaluated at $z \simeq z_{\mathrm{th}}$. This length scale is $\lambda_{\gamma d}(z_{\mathrm{th}}) \simeq 1.7 \times 10^{-4}\eta_{10}^{-0.025}$ Mpc, implying a mass scale (see eq. [18]) of

$$M \simeq 2.5\left(\frac{\eta_{10}}{4.5}\right)^{-0.075}\left(\frac{h}{0.75}\right)^2\left(\frac{\Omega_{t,0}}{1.}\right) M_\odot . \qquad (19b)$$

Hence, a chemical potential will result from the damping of perturbations with a mass scale greater than about 1 $M_\odot$ (eq. [19]).

Similarly, the primary contribution to a Compton y-parameter resulting from the damping of pressure waves, if it is to be observable at all, arises from the damping of waves at the largest redshift included in the third epoch (see §§ 1, 3, and 4.2). Hence, the redshift z_y, given by equation (3), should be used to determine the damping scale relevant for a Compton y-parameter. As shown in § 3.3, a value of $y \gtrsim 5.1 \times 10^{-4}\eta^{-0.25} \simeq 3.5 \times 10^{-4}$ could result from nonlinear dissipation; photon diffusion will produce y's below this value.

The damping scale for nonlinear dissipation is given by equation (16) with $z = z_y$ and $\delta^2_{r,\max} \simeq 32y$ (see eq. [13]). The damping scale is $\simeq 20y\eta_{10}^{0.5}$ Mpc. The corresponding mass scale (eq. [18]) is

$$M \simeq 2 \times 10^{15} y^3 \eta_{10}^{1.5}\left(\frac{h}{0.5}\right)^2\left(\frac{\Omega_{t,0}}{1.0}\right) M_\odot . \qquad (20a)$$

The mass scale corresponding to a Compton y-parameter of 10^{-3} is $\sim 10^6\ M_\odot$.

Values of y less than about 3.5×10^{-4} will be produced by photon diffusion. The damping scale is given by equation (17b) with $z = z_y$. The damping scale is $\lambda_{\gamma d}(z_y) \simeq 10^{-2}\eta_{10}^{0.25}$ Mpc $\sim 1.5 \times 10^{-2}$ Mpc; the corresponding mass scale (eq. [18]) is

$$M \simeq 2 \times 10^6\left(\frac{\eta_{10}}{4.5}\right)^{0.75}\left(\frac{h}{0.75}\right)^2\left(\frac{\Omega_{t,0}}{1.}\right) M_\odot . \qquad (20b)$$

Hence, an observable Compton distortion will result from the damping of perturbations with mass scales greater than about $10^5\ M_\odot$.

At recombination, the scale on which photon diffusion is operating is $\lambda_{\gamma d}(z_{\mathrm{rec}}) \simeq 170\eta_{10}^{-1/2}$ Mpc (see eq. [17b] with $z = z_{\mathrm{rec}} \simeq 10^3$). The corresponding mass scale may be obtained from equation (18): $M \simeq 3 \times 10^{16}(\eta_{10}/4.5)^{-1.5}(h/0.75)^2(\Omega_{t,0}/0.1)\ M_\odot$, (see eq. [17b] with $z = z_{\mathrm{rec}} \simeq 10^3$). For $\Omega_{t,0} \simeq 1$, this mass scale is $\sim 10^{17}\ M_\odot$, which is very large. The primordial perturbations of the energy density of the radiation field on mass scales less than that given above (10^{16}–$10^{17}\ M_\odot$) will be erased and will be replaced by spectral distortions. (Secondary anisotropies may be imprinted on scales less than those given above, as a result of the Sachs-Wolfe effect and/or scattering between photons and moving electrons.)

4. CONSTRAINTS ON THE INITIAL POWER SPECTRUM

The amplitude of the perturbations of the radiation field on any given scale may be related to those in the matter, assuming that the perturbations are of the adiabatic or isocurvature type, as discussed in § 4.1. When the perturbations enter the horizon they oscillate as pressure waves, and when $a < a_{\mathrm{eq}}$, the ampli-

tude of the waves remains constant before the damping of the waves becomes important. The relationships between the initial power spectrum of the perturbations of the matter and the fluctuations of the radiation are given in § 4.2. Constraints on the power spectrum in the isocurvature model are discussed in § 4.3. A similar discussion applied to adiabatic perturbations is given in § 4.4.

4.1. *Types of Initial Inhomogeneities*

Two types of inhomogeneities which may be present in the early universe are adiabatic and isocurvature perturbations (e.g., Peebles 1980 § 94). Adiabatic perturbations are those in which the matter and radiation are perturbed such that $\delta_r = (4/3)\delta_m$, where δ is the fractional perturbation of the mass-energy density on a particular length scale: $\delta_\alpha \equiv \rho_\alpha(x)/\bar{\rho}_\alpha - 1$. In this model, perturbations with an amplitude greater than about unity will form black holes when they enter the horizon, so if the amplitude of the perturbations is increasing toward small scales, a small-scale cutoff must be imposed.

Isocurvature perturbations are those in which the perturbation of the radiation is compensated by an equal and opposite perturbation of the matter, so that everywhere the space curvature is unperturbed—hence the name "isocurvature." Initially, $\delta_r = -\delta_m(a/a_{\rm eq})$, where a is the scale factor and $a_{\rm eq}$ refers to the scale factor when the average energy density of the radiation is equal to that of the matter. Because the total perturbation is negligible, large-amplitude isocurvature fluctuations do not lead to the formation of black holes.

4.2. *Relation to the Initial Power Spectrum*

Given constraints on μ_* and y, the amplitude of the perturbations of the energy density of the radiation field with wavelengths comparable to the damping scale are constrained (see eqs. [12] and [13]). Because the perturbations of the energy density of the radiation field are related to those in the matter, these constraints will lead to constraints on the initial power spectrum of the mass distribution if the amplitude of the perturbations of the mass density on some other scale can be estimated (see § 1).

Consider waves with comoving wavenumbers between k_1 and k_2, where $\lambda_2 = 2\pi/k_2 \simeq 2\lambda_1$. The mean amplitude of the perturbations of the radiation field on the scale $\sim\lambda_r$ is

$$\delta_r^2(\lambda_r) \simeq \frac{V_u}{(2\pi)^3}\int_{k_r}^{2k_r} |\delta_r'|^2 k^2\, dk\ ,$$

where δ_r' are the Fourier components of the energy density of the radiation field and V_u is a suitably large volume within which the universe is periodic (see Peebles 1980, § 26). A similar equation applies to the mean amplitude of the perturbations of the mass distribution on the scale $\sim\lambda_m$,

$$\delta_m^2(\lambda_m) \simeq \frac{V_u}{(2\pi)^3}\int_{k_m}^{2k_m} |\delta_m'|^2 k^2\, dk\ ,$$

where δ_m' are the Fourier components of the mass distribution. Similarly, we could have multiplied the integrated by a window function and integrated over all wavenumbers.

Let us consider the mean amplitude of the fluctuations of the mass density $\delta_m(\lambda_m)$ on the comoving scale λ_m, and the mean amplitude of the fluctuations of the energy density of the radiation field $\delta_r(\lambda_r)$ one some other scale λ_r. The initial power spectrum of the mass distribution is assumed to be a power law: $P(k) = Ak^n = |\delta_m'|^2$.

In the isocurvature model, fluctuations which enter the horizon when $a \lesssim a_{\rm eq}$ have not grown significantly (Peebles 1983). These perturbations satisfy $\delta_r \simeq -\delta_m \lambda_r/\lambda_{\rm eq}$. Therefore,

$$\frac{\delta_r^2(\lambda_r)}{\delta_m^2(\lambda_m)} \sim \left(\frac{\lambda_r}{\lambda_{\rm eq}}\right)^2\left(\frac{\lambda_m}{\lambda_r}\right)^{n+3} = \left(\frac{\lambda_m}{\lambda_{\rm eq}}\right)^2\left(\frac{\lambda_m}{\lambda_r}\right)^{n+1}\ ; \qquad (21a)$$

this equation is a good approximation when λ_m and λ_r are less than $\lambda_{\rm eq} \simeq 10h^{-2}\Omega_{t,0}^{-1}$ Mpc; all length scales λ discussed here are in comoving coordinates.

In the adiabatic model $\delta_r \simeq (4/3)\delta_m$, and the amplitude of the fluctuations outside the horizon grow as $t \propto a^2$ for scales which enter the horizon at $a < a_{\rm eq}$ (e.g., Peebles 1980, § 86). Hence, $\delta_r^2(\lambda_r)$ and $\delta_m^2(\lambda_m) \propto a_e^4 k^{n+3} \propto k^{n-1}$, since $a_e \propto k^{-1}$, where a_e is the cosmic scale factor when the perturbation enters the horizon. Therefore,

$$\frac{\delta_r^2(\lambda_r)}{\delta_m^2(\lambda_m)} \sim 2\left(\frac{\lambda_m}{\lambda_r}\right)^{n-1}\ . \qquad (22a)$$

Equations (21a) and (22a) may be used to estimate the chemical potential μ/kT and the Compton y-parameter which will result from the damping of acoustic oscillations, since these equations may be used to estimate $\delta_r(\lambda_r)$ on a given comoving scale λ_r, assuming that $\delta_m(\lambda_m)$ is known on some other comoving scale λ_m.

Davis & Peebles (1983) find that at the current epoch $\delta_m(\lambda_m)(z = 0) \simeq 1$ on the scale of $\lambda_m \simeq 8h^{-1}$ Mpc. If the galaxy distribution is biased, then $\delta_m(8h^{-1}$ Mpc) is less than unity; the case of a biased galaxy distribution is discussed in § 4.5. Assuming for the present that the galaxy distribution is unbiased, the initial value of $\delta_m(\lambda_m)$ on this scale λ_m, relevant for equations (19) and (20), is $\delta_m(\lambda_m) \simeq (1 + z_f)/(1 + z_i)$, z_i being the redshift at which the perturbations on this scale begin to grow and z_f being that at which growth ceases: $(1 + z_f) \simeq \Omega_{t,0}^{-1}$. The value of z_i depends on whether nonbaryonic (i.e., uncharged, weakly interacting) dark matter exists in the universe; for $\Omega_{t,0} = \Omega_b$, $z_i = \min(z_{\rm eq}, z_{\rm rec})$, and for $\Omega_{t,0} \gtrsim 2\Omega b$, $z_i \simeq z_{\rm eq}$. The cosmological constant is assumed to be zero; the case of a nonzero cosmological constant is discussed by Daly (1991).

The initial value of $\delta_m(\lambda_m)$, as determined from the current value of ~ 1 on the scale of $\lambda_m \simeq 8h^{-1}$ Mpc, z_i, and z_f, is independent of the ionization history of the universe. Hence, the bounds obtained here are unaffected by whether the universe was reionized for a substantial period subsequent to recombination.

The comoving length scale λ_r relevant for equations (21a) and (22a) depends upon the epoch of damping and the damping mechanism. If the damping mechanism is photon diffusion, then equation (17b) is used to determine the damping length $\lambda_r = \lambda_{\gamma d}$. The epoch z at which $\lambda_{\gamma d}$ is evaluated depends on whether the chemical potential or the Compton y-parameter is to be determined. It is assumed that $\delta_r(\lambda_r)$ is increasing with increasing wavenumber k, hence the waves damped at the largest redshift have the largest amplitude and make the largest contribution to the distortion. This is the case for $n \gtrsim -1$ in the isocurvature model, and $n \gtrsim 1$ in the adiabatic model; it is for this reason that the redshift $z_{\rm th}$ is used to determine the damping scale relevant for a chemical potential, and z_y is used to determine that relevant for a Compton distortion. If the index of the initial power spectrum is smaller than the limits indicated above (-1 for isocurvature and 1 for adiabatic perturbations), then the constraints obtained by considering the magnitude of the chemical potential and Compton

y-parameter resulting from the damping of waves at the respective redshifts $z_{\rm th}$ and z_y are lower limits. If, in the adiabatic model, $n < 1$, then the distortions will be larger than those obtained from the damping of waves at the redshifts $z_{\rm th}$ and z_y. This is relevant for open models. However, these models are severely constrained by microwave background anisotropy experiments on relatively large scales ($\sim 10^{17}\ M_\odot$). Since the object here is to constrain these models by a second, independent method, it is undesirable to use scales close to those relevant to the anisotropy constraints. And, since the lower bounds on the chemical potential and Compton y-parameter seriously constrain these models, these lower bounds suffice.

To determine the magnitude of the chemical potential, the redshift $z_{\rm th}$, given by equation (1), is used in equation (17b); $\delta_r(\lambda_r)$ is then determined from equation (21a) for isocurvature perturbations and from equation (22a) for adiabatic perturbations; and μ/kT is then determined using equation (12). If the value of $\mu_* = |\mu/kT|$ is greater than about 4.5×10^{-4}, then the damping mechanism that will lead to the chemical potential is not photon diffusion but is nonlinear dissipation (see § 3.3). The wavelength λ_r must be obtained from equation (16) instead of from equation (17b). In this case, the relevant length scale λ_r to be substituted in equation (21a) or (22a) (depending on whether isocurvature or adiabatic perturbations are under consideration) is obtained from equation (16) with $z = z_{\rm th}$ and $\delta_{r,\max} = \delta_r(\lambda_r)$; equation (21a) or (22a) is then solved for $\delta_r^2(\lambda_r)$, and μ/kT is determined from equation (12).

Similarly, to estimate the value of the Compton y-parameter, it is assumed that the damping mechanism is photon diffusion. The comoving damping scale λ_r is obtained from equation (17b) evaluated at the redshift z_y (eq. [3]), the value of $\delta_r(\lambda_r)$ is determined from equations (21a) and (21b), and that of the Compton y-parameter from equation (13). If the Compton y-parameter is greater than about 3.5×10^{-4}, then the mechanism that leads to the distortion is not photon diffusion but is nonlinear dissipation. The length scale λ_r is determined by equation (16) evaluated at the redshift z_y, and $\delta_{r,\max} = \delta_r(\lambda_r)$. This is subtituted in equations (21a) and (21b), each of which is solved for $\delta_r(\lambda_r)$, and the value of the y-parameter is obtained from equation (13).

To summarize, equation (21a) may be written

$$\delta_r^2(\lambda_r) \simeq \frac{0.64h^2}{z_i^2}\left(\frac{8h^{-1}\ \mathrm{Mpc}}{\lambda_r}\right)^{n+1} \tag{21b}$$

for the isocurvature model; λ_r is the comoving scale of the waves which are damped. When $\Omega_{t,0} = \Omega_b$, $z_i = \min(z_{\rm eq}, z_{\rm rec})$, and when $\Omega_{t,0} \gtrsim 2\Omega_b$, $z_i = z_{\rm eq}$, where $z_{\rm eq} = 4 \times 10^4 h^2\ \Omega_{t,0}$. Consider the production of a chemical potential at $z = z_{\rm th}$: the procedure is to assume that the damping mechanism is photon diffusion, so that $\lambda_r = \lambda_{\gamma d}(z_{\rm th})$ (see eq. [17b]). The cosmological parameters $\Omega_{t,0}$, h, and η_{10}, and a value for n, are chosen; then $\delta_r^2(\lambda_r)$ may be determined from equation (21b), and μ_* from equation (12). If μ_* is greater than $7.4 \times 10^{-4}\eta_{10}^{-0.325}$, the computation must be redone with λ_r determined by equation (16) with $z = z_{\rm th}$ and $\delta_{r,\max} = \delta_r(\lambda_r)$. Equation (21b) is then used to solve for $\delta_r^2(\lambda_r)$, and equation (16) is used to obtain μ_*.

The expected value of y is obtained in a similar way. The damping mechanism is assumed to be photon diffusion, and $z = z_y$ is used in equation (17b) to obtain λ_r: $\lambda_r = \lambda_{\gamma d}(z_y)$. Equation (21b) is used to determine $\delta_r^2(\lambda_r)$ and equation (13) relates $\delta_r^2(\lambda_r)$ to y. If the value of y is greater than $5.1 \times 10^{-4}\eta_{10}^{-0.25}$, then the damping mechanism is not photon diffusion but is nonlinear dissipation. Then the length scale λ_r is obtained from equation (16) with $z = z_y$ and $\delta_r(\lambda_r) = \delta_{r,\max}$, and equation (21b) is used to solve for $\delta_r^2(\lambda_r)$; the Compton y-parameter is then obtained from equation (13).

In the adiabatic model, equation (22a) reads

$$\delta_r^2(\lambda_r) \simeq \frac{2}{z_i^2\,\Omega_{t,0}^2}\left(\frac{8h^{-1}\ \mathrm{Mpc}}{\lambda_r}\right)^{n-1}. \tag{22b}$$

In the case of the production of a chemical potential, λ_r is obtained from equation (17b) with $z = z_{\rm th}$, assuming that the distortion results from photon diffusion, and $\delta_r^2(\lambda_r)$ is determined using equation (22b); μ_* is obtained from equation (12). If $\mu_* \gtrsim 7.4 \times 10^{-4}\eta_{10}^{-0.325}$ (see § 3.3), λ_r must be obtained from equation (16) with $z = z_{\rm th}$ and $\delta_{r,\max} = \delta_r(\lambda_r)$. Then equation (22b) is used to solve for $\delta_r^2(\lambda_r)$, and μ_* is obtained from equation (12).

The value of the Compton y-parameter is similarly determined. The damping mechanism is assumed to be photon diffusion. The length scale λ_r is obtained from equation (17b) with $z = z_y$, $\delta_r^2(\lambda_r)$ is obtained from equation (22b), and y is determined using equation (13). If the value of y is greater than $5.1 \times 10^{-4}\eta_{10}^{-0.25}$ (see § 3.3), then the damping mechanism is not photon diffusion but is nonlinear dissipation. The length scale λ_r is then obtained from equation (16) with $z = z_y$ and $\delta_{r,\max} = \delta_r(\lambda_r)$. Equation (22b) is used to solve for $\delta_r^2(\lambda_r)$, and y is obtained from equation (13).

Note that it is assumed that the galaxy distribution traces the underlying mass distribution in a one-to-one fashion, that is, the galaxy distribution is not "biased." In the event that the galaxy distribution is biased, the normalization $\delta_m(\lambda_m)(8h^{-1}$ Mpc) would decrease by the bias factor, and the magnitude of the distortion would decrease for a given value of the index of the initial power spectrum and a given set of cosmological parameters $\Omega_{t,0}$, η_{10}, and h. The effect of a bias parameter is addressed in detail in § 4.5.

4.3. *Constraints in the Isocurvature Model*

4.3.1. *Constraints Neglecting Photon Production*

Figure 1*a* shows the predicted magnitude of the Compton y-parameter resulting from the damping of pressure waves as a function of the index n of the initial power spectrum of isocurvature perturbations of the matter for various choices of the cosmological parameters $\Omega_{t,0}$, h, and η_{10}. The observational bound on the Compton y-parameter is presently $y \lesssim 10^{-3}$ (Mather et al. 1990), and is expected to reach at least 10^{-4} and possibly 10^{-5} (J. C. Mather 1990, private communication). Scattering of photons from the primeval radiation field with hot gas as they traverse the universe from the last scattering surface may lead to a Compton y-parameter, in which case the applicability of the bounds discussed here will be limited by the magnitude of the observed value. An upper bound on (or detection of) y implies a bound on $\delta_r(\lambda_r)$ (see eq. [12]), which is taken to be the mean amplitude of the perturbations on the damping scale at z_y (see § 1). The bound on $\delta_r(\lambda_r)$ (on the damping scale) implies a bound on the index of the initial power spectrum (see eqs. [21a], [21b], [22a], and [22b]), assuming values for the cosmological parameters η_{10}, h, and $\Omega_{t,0}$.

The isocurvature baryon model (Peebles 1987) is obtained when $\Omega_b = \Omega_{t,0}$. When the primary damping mechanism is photon diffusion, that is, when the Compton y-parameter is $\lesssim 3.5 \times 10^{-4}$, equations (21) indicate that $y \propto \eta_{10}^{-(9+n)/4}h^{1-n}$ for $\Omega_{t,0} = \Omega_b$ assuming that $\min(z_{\rm rec}, z_{\rm eq}) = z_{\rm eq}$; y depends

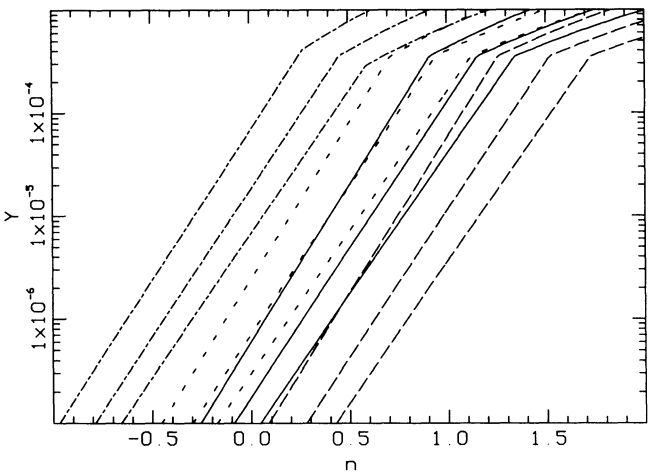

FIG. 1*a*

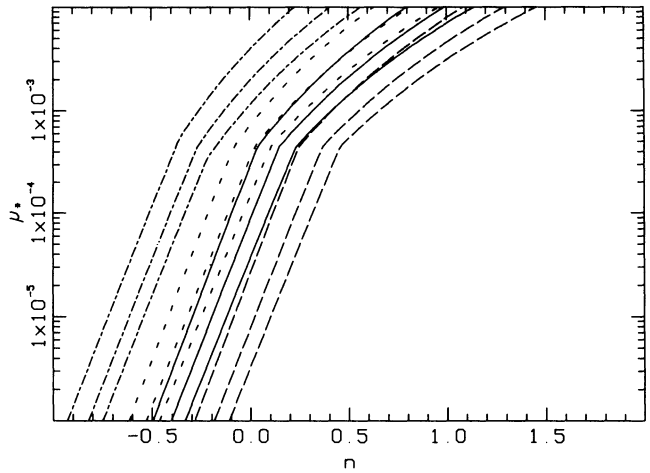

FIG. 1*b*

FIG. 1.—Values of (*a*) the Compton *y*-parameter and (*b*) the chemical potential μ_* computed in the isocurvature model as a function of *n*, the index of the initial power spectrum of the perturbations of the mass distribution. The three curves at the left in each panel are obtained with $\Omega_{t,0} = \Omega_b$, with values of η_{10} (moving from left to right) of 2.5, 4.5, and 10, and a value of $h = 0.75$; *y* and μ_* are very weakly dependent on *h* when $\Omega_{t,0} = \Omega_b$. For $\Omega_{t,0} \neq \Omega_b$, *y* and μ_* are very weakly dependent on η_{10}, and are more strongly dependent on *h* and $\Omega_{t,0}$; for the three sets with $\Omega_{t,0} \neq \Omega_b$, $\eta_{10} = 4.5$ has been assumed. The short-dashed curves are obtained with $\Omega_{t,0} = 0.15$, the solid lines with $\Omega_{t,0} = 0.3$, and the long-dashed curves with $\Omega_{t,0} = 1.0$; in each of these sets *h* takes on the values 0.5, 0.75, and 1.0, going from left to right.

primarily on η_{10} and is only weakly dependent on *h*. Figure 1*a* shows that the current limit of $y \lesssim 10^{-3}$ (Mather et al. 1990; Gush et al. 1990) yields an upper bound on *n* of $n \lesssim 0.9$ for $\eta_{10} \simeq 4.5$. If the observational bound is tightened to $y \lesssim 10^{-4}$, then the index of the initial power spectrum must satisfy $n \lesssim 0.4$ for $\eta_{10} = 10$ and $n \lesssim 0.1$ for $\eta_{10} = 2.5$; if the observational bound on *y* becomes $y \lesssim 10^{-5}$, then $n \lesssim 0.1$ for $\eta_{10} = 10$ and $n \lesssim -0.3$ for $\eta_{10} = 2.5$. Note that these constraints on *n* are relaxed if the cosmological constant is nonzero, as discussed by Daly (1991).

Note that the value of η_{10} is constrained to lie between about 2.5 and 4.5 if there are three effectively massless neutrino species present with equilibrium abundances during primordial nucleosynthesis (Olive et al. 1990), when the energy scale is $\sim$MeV. However, if there are only two effectively massless neutrino species present with equilibrium abundances during primordial nucleosynthesis, then η_{10} may be as large as 10 (Yang et al. 1984). A value of *h* of 0.75 indicates values of $\Omega_{t,0} = \Omega_b$ of 0.02, 0.03, and 0.07 for values of η_{10} of 2.5, 4.5, and 10 (see eq. [2]). Hence, it may be that there are only two massless neutrino species present with equilibrium abundances during primordial nucleosynthesis, or that $h = 0.5$, or that we are in a region of high overdensity and $\Omega_{t,0}$ is fairly small, $\sim$0.03.

The predicted magnitude of the Compton *y*-parameter decreases as $\Omega_{t,0}$ increases. When $\Omega_{t,0} \neq \Omega_b$, equations (21) indicate that

$$y \propto \Omega_{t,0}^{-2} h^{-(n+3)} \eta_{10}^{-(n+1)/4} .$$

Hence, *y* is only weakly dependent on η_{10}, and is much more strongly dependent on *h* and $\Omega_{t,0}$. As shown in Figure 1, for $y \lesssim 10^{-4}$ and $\Omega_{t,0} = 0.15$, $n \lesssim 0.9$ for $h = 1.0$ and $n \lesssim 0.6$ for $h = 0.5$. And, with $\Omega_{t,0} = 1.0$, Figure 1*a* shows that $n \lesssim 1.5$ $h = 1.0$ (this choice of parameters, though, is unrealistic, as it suffers from the "age problem": the current age of the universe is too small), and $n \lesssim 1.1$ for $h = 0.5$, which is a much more realistic choice for *h* when $\Omega_{t,0} = 1.0$. The intermediate case, $\Omega = 0.3$, is shown in Figure 1.

4.3.2. *Photon Production*

Photon production may be significant in the isocurvature model because the baryons may be very strongly clumped. However, the photon production rate cannot be uniquely determined because it depends not only on $\delta_m^2(\lambda_m)$ but also on the detailed distribution of the baryons, which is unknown. In addition, $\delta_m(\lambda_m)$ is increasing toward smaller scales, and it is reasonable to expect that there is a small-scale cutoff. The scale of this cutoff, and the value of $\delta_m^2(\lambda_m)$ on this scale, would determine the properties of the "elementary building blocks" of the larger scale perturbations. Some simple assumptions may be made to determine whether photon production is significant, such as the scale of the cutoff, and the nature of the clumping of the matter, but, as discussed below, if these are altered, the photon production rate will change. The very simple picture considered below maximizes the extent of the photon production.

The dimensionless frequency, $x_0 = h\nu/kT$, below which free-free emission produces a blackbody spectrum at temperature *T* is

$$x_0 \simeq 100 z^{-1.5} [\Omega_b \delta_m(\lambda_m) h^2]^{0.5} \tag{23}$$

(Peebles 1971 § VII; Illarionov & Sunyaev 1975b), where $\delta_m(\lambda_m)$ is the amplitude of the perturbations of the matter on the scale λ_m (i.e., from $\sim\lambda_m$ to $\sim 2\lambda_m$),

$$\delta_m(\lambda_m) \simeq \delta_m(8h^{-1}\ \mathrm{Mpc}) \left(\frac{8h^{-1}\ \mathrm{Mpc}}{\lambda_m}\right)^{(n+3)/2} . \tag{24}$$

The regions where this photon production is occurring occupy some small fraction *f* of the volume of the universe, $f \sim [\delta_m(\lambda_m)]^{-1}$ for $\delta_m(\lambda_m) \gg 1$. If it is assumed that the baryons in the regions of high density have a constant number density, and that their number density between these regions is negligibly small, we can determine whether the photon production from these regions will be sufficient to fill the universe with photons characterized by a blackbody spectrum at frequencies

below x_0. This condition is

$$[\delta_m(\lambda_m)]^{-1}\left(\frac{\lambda_{\rm rec}}{\lambda_m}\right) \gtrsim 1 , \qquad (25)$$

since the emission will occur until recombination, $z_{\rm rec} \simeq 10^3$. The wavelengths in equation (25) are evaluated at recombination and are in either comoving or physical coordinates; the wavelength $\lambda_{\rm rec}$ refers to the horizon size at recombination, and comoving coordinates are used.

Equation (24) may be used to eliminate $\delta_m(\lambda_m)$ from equation (25), which results in a lower bound on λ_m;

$$\lambda_m \gtrsim [0.1h^{0.5}\eta_{10}^{-2}(8h^{-1})^{n/2}]^{2/(n+1)} \text{ Mpc} , \qquad (26)$$

which has been obtained assuming that

$$\delta_m(8h^{-1} \text{ Mpc}) \simeq (\Omega_{t,0}\, z_{\rm eq})^{-1},$$

that is, that min $(z_{\rm eq}, z_{\rm rec}) = z_{\rm eq}$, as discussed in detail in §§ 4.3. and 4.4, and that $\Omega_{t,0} = \Omega_b$, so that $z_{\rm rec} \gtrsim z_{\rm eq}$; $\lambda_{\rm rec}$ is obtained from equation (15) with $z_e = z_{\rm rec} \simeq 10^3$. Note that this means that the subsequent discussion is relevant for the isocurvature baryon ($\Omega_{t,0} = \Omega_b$) model. Photon emission from perturbations on scales that satisfy equation (26) will "fill" the universe with photons, and hence will produce a blackbody spectrum at frequencies $h\nu/kT \lesssim x_0$. The lower bound on λ_m (eq. [26]), results in an upper bound on x_0. For the case $n = 0$,

$$\lambda_m \gtrsim (0.1h^{0.5}\eta_{10}^{-2})^2 \text{ Mpc} , \quad \delta_m(\lambda_m) \lesssim 4 \times 10^4\eta_{10}^4 h^{-1} ,$$

which implies $x_0 \lesssim 4 \times 10^{-2}\eta_{10}^{2.5}h^{-0.5}$, so that $x_0 \lesssim 15$, $x_0 \lesssim 2$, and $x_0 \lesssim 0.5$ for $\eta_{10} = 10$, 4.5, and 2.5, respectively, where a value of $h = 0.75$ has been assumed. When $x_0 \gtrsim 1$, the photon production has been copious enough to completely erase the Compton y-distortion. However, if the small-scale cutoff of the fluctuations occurs at a wavelength greater than that given by equation (26), so that the scale of the fundamental building blocks is larger than λ_m, then this scale (the small-scale cutoff) should be used to determine x_0, and the value of x_0 will be smaller than that obtained above. If the value of x_0 is large ($x_0 \gtrsim 1$ in the case of a Compton distortion, and $x_0 \gtrsim \mu_*$ in the case of a chemical potential), the photon production will render the distortion unobservable, and the bounds on the index of the initial power spectrum in the isocurvature model are invalidated. However, if a distortion was observed, and/or the jump due to free-free emission in the Rayleigh-Jeans region of the spectrum was observed, much could be learned if it were interpreted in the context of the isocurvature model.

Note that the value of λ_m obtained above for the case $n = 0$, $\eta_{10} = 4.5$, and $h = 0.75$ is $\lambda_m \sim 2 \times 10^{-5}$ Mpc, which indicates a value of $\delta_m \sim 10^7$ (assuming that $\Omega_{t,0} = \Omega_b$), which is quite large. (The current overdensity in the Galactic disk is $\sim 10^6$.) Given this large value, it is reasonable to expect that the small-scale cutoff occurs at a wavelength very much greater than 10^{-5} Mpc, in which case $x_0 \ll 1$. So, in the context of the isocurvature model, it is not unreasonable to expect that a Compton y-parameter produced prior to recombination would be observable.

It is instructive to consider photon production in the isocurvature baryon model ($\Omega_{t,0} = \Omega_b$) from the baryons clumped on the same scale as the scale whose damping leads to the distortion. First, photon production from the clumped baryons on the scale whose damping via photon diffusion would lead to a Compton y-parameter is considered. Then, the value of the small-scale cutoff such that $x_0 = 1$ is determined: if the small-scale cutoff is greater than this value but less than the length scale of the perturbations whose damping leads to the distortion, then the Compton y-parameter produced by the damping will be observable. Finally, photon production from the perturbations whose damping would lead to a chemical potential is considered. It appears that photon production from the perturbations of the mass distribution associated with the perturbations of the energy density of the radiation field responsible for producing the chemical potential will render that distortion unobservable.

Consider the Compton distortion resulting from photon diffusion at the redshift z_y. The damping scale is $\lambda \simeq 10^{-2}\eta_{10}^{0.25}$ Mpc (see eqs. [17b] and [3]); the corresponding mass scale, equation (18), is

$$M \sim 3 \times 10^5(\eta_{10}/10)^{0.75}(h/0.75)^2(\Omega_{t,0}/0.1)\, M_\odot .$$

The mean amplitude of the perturbations on this scale is

$$\delta_m(\lambda_m) \simeq 2h^2\eta_{10}^{-2}(800h^{-1}\eta_{10}^{-0.25})^{(n+3)/2}$$

(eq. [24]); it has been assumed that min $(z_{\rm eq}, z_{\rm rec}) = z_{\rm eq}$, since we are considering the isocurvature baryon model and $\Omega_{t,0} \lesssim 0.1$. For $n = 0$, $\delta_m(\lambda_m) \sim 10^2$–$10^3$ for $\eta_{10} \simeq 10$–4.5. Equation (23) evaluated at $z = z_{\rm rec} \simeq 10^3$ yields

$$x_0 \simeq 2.5 \times 10^{-4}\eta_{10}^{-0.5}h^{-1}(800h^{-1}\eta_{10}^{-0.25})^{(n+3)/4} .$$

For $n = 0$, $x_0 \simeq 4 \times 10^{-2}\eta_{10}^{-11/16}h^{-7/4}$, so

$$x_0 \simeq 2 \times 10^{-2}(h/0.75)^{-7/4}(\eta_{10}/4.5)^{-11/16} .$$

Photon production will only lead to a blackbody spectrum at frequencies $h\nu/kT \lesssim x_0 \sim 10^{-2}$. This means that photon production from the clumped baryons associated with the perturbations of the energy density of the radiation field that will produce the Compton distortion will not render that distortion unobservable.

It is of interest to calculate the scale at which $x_0 = 1$; if the small-scale cutoff of the matter perturbations occurs at a scale greater than this scale, but less than the scale of the perturbations whose damping leads to the distortion, then the distortion will remain imprinted in the background. Equation (23) may be written

$$x_0 \simeq 2 \times 10^{-5}\sqrt{\Omega_b/\Omega_{t,0}^2}\,(8h^{-1} \text{ Mpc}/\lambda_m)^{(n+3)/4} .$$

With $\Omega_{t,0} = \Omega_b$ and $n = 0$, requiring that $x_0 \lesssim 1$ implies $\lambda_m \gtrsim 4 \times 10^{-5}\,(\eta_{10}/4.5)^{-2/3}h^{1/3}$ Mpc. Therefore, if the scale of the small-scale cutoff is greater than $\lambda_{\rm co} \sim 5 \times 10^{-5}(\eta_{10}/4.5)^{-2/3}h^{1/3}$ Mpc, photon production from baryons clumped on scales greater than that of the small-scale cutoff will not produce a blackbody spectrum all the way up to the peak of the spectrum, and the Compton distortion will be observable. This value of $\lambda_{\rm co}$ implies a rms density contrast on this scale of $\delta_m(\lambda_{\rm co}) \sim 3 \times 10^6(\eta_{10}/10)^{-1}$ (see eq. [24]) for $\Omega_{t,0} = \Omega_b$, min $(z_{\rm eq}, z_{\rm rec}) = z_{\rm eq}$, and $n = 0$. Since this density contrast is large, it is likely that the small-scale cutoff occurs at a length scale greater than $\lambda_{\rm co}$ indicated above, so that the rms density contrast on the scale of the cutoff is smaller than $\sim 10^6$. It also seems reasonable that the scale of the small-scale cutoff lies below about $10^5\, M_\odot$, so that globular clusters could form from the growth of initial perturbations, in which case the associated perturbations of the energy density of the radiation field will be present to produce the Compton distortion in the first place (at a redshift of z_y). Hence, it is reasonable to expect that a Compton distortion will be produced, and that it will be

observable. The limits on the index of the initial power spectrum are illustrated in Figure 1*a*.

Figure 1*b* illustrates the magnitude of the chemical potential that will result in the isocurvature model neglecting photon production and a small-scale cutoff. However, it is unlikely that a chemical potential will be observable in this model even if the cutoff is at a small length scale: photon production from the clumped baryons associated with (i.e., on the scale as) the perturbations in the energy density of the radiation field that produce the chemical potential will produce a blackbody spectrum at least up to frequencies $h\nu/kT \simeq \mu_*$, so the chemical potential will not be observable.

This may be seen in the following way. The damping scale for $\mu_* \gtrsim 4.5 \times 10^{-4}$ is $\lambda \simeq 0.23\mu_* \eta_{10}^{0.35}$ Mpc (see eq. [16] evaluated at $z = z_{\rm th}$, eq. [12], and eq. [1]). The corresponding mass scale is

$$M \simeq 3 \times 10^9 \mu_*^3 \eta_{10}^{1.05} (h/0.5)^2 (\Omega_{t,0}/1.0)\ M_\odot\ .$$

The mean amplitude of the perturbations of the mass density on this scale is (eq. [24])

$$\delta_m(\lambda_m) \simeq 2h^2 \eta_{10}^{-2} (35h^{-1}\mu_*^{-1}\eta_{10}^{-0.35})^{(n+3)/2}\ ;$$

assuming that $\Omega_{t,0} = \Omega_b$, and that min $(z_{\rm eq}, z_{\rm rec}) = z_{\rm eq}$, i.e., the isocurvature baryon model. The value of x_0 (eq. [23]) is

$$x_0 \simeq 2.5 \times 10^{-4} \eta_{10}^{-0.5} h (35h^{-1}\mu_*^{-1}\eta_{10}^{-0.35})^{(n+3)/4}\ .$$

The photon production will only produce a blackbody up to the dimensionless frequency x_0, so when $x_0 \lesssim \mu_*$ the chemical potential μ_* will remain observable. Hence, to be able to observe the chemical potential requires that $x_0 \lesssim \mu_*$. For $n = 0$ this condition reduces to $\mu_* \gtrsim 0.015(\eta_{10}/10)^{-0.436} h^{1/7}$. In the isocurvature baryon models μ_* is predicted $\mu_* \sim 10^{-3}$ (see Fig. 1*b*). However, as shown above, this chemical potential will not remain imprinted in the microwave background radiation because photon production from baryonic clumps on the same scale as the perturbations in the energy density of the radiation field which produced the distortion will "fill in" the distortion. And, indeed, these photons will "fill" the universe, as can be seen from equation (25), which is satisfied for $\mu_* \gtrsim 10^{-3}\eta_{10}^{-2.35}h$; this expression is obtained by substituting the values for $\delta_m(\lambda_m)$ and λ given above into equation (25) for the case $n = 0$.

4.4. *Constraints in the Adiabatic Model*

Figure 2*a* shows the Compton y-parameter resulting from the damping of pressure waves as a function of n, the index of the initial power spectrum of adiabatic perturbations of the mass distribution, for various choices of the cosmological parameters $\Omega_{t,0}$, h, and η_{10}. The Compton y-parameter determined by equations (22) and (13) when photon diffusion is the primary damping mechanism, so that $\lambda_r = \lambda_{yd}(z = z_y)$ (see eqs. [3] and [17b]) is $y \propto \Omega_{t,0}^{-4} h^{-(3+n)} \eta_{10}^{(1-n)/4}$, and when $\Omega_{t,0} = \Omega_b$ this becomes $y \propto \eta_{10}^{-(15+n)/4} h^{5-n}$, for min $(z_{\rm rec}, z_{\rm eq}) = z_{\rm eq}$. As illustrated in Figure 2*a*, the current observational limit on the Compton y-parameter of 10^{-3} indicates that models with $n \geq 0.9$, $\Omega_{t,0} = \Omega_b$, and $\eta_{10} = 2.5$ or 4.5 are ruled out for all values of the Hubble constant considered.

In the adiabatic model the damping of the acoustic oscillations by photon diffusion washes out the perturbations in the baryon distribution; perturbations in any component that is coupled to the radiation field, such as a charged dark matter particle (DeRujula, Glashow, & Sarid 1989; Dimopoulos et al. 1990), are also likely to be washed out. In the adiabatic model, nonbaryonic (that is, uncharged, weakly interacting matter) is required if galaxies and clusters of galaxies are to form from the gravitational growth of initial inhomogeneities in the mass distribution. Hence, models with $\Omega_{t,0} \neq \Omega_b$ are more realistic, albeit for completeness the Compton y-parameter has also been determined for the case $\Omega_b = \Omega_{t,0}$.

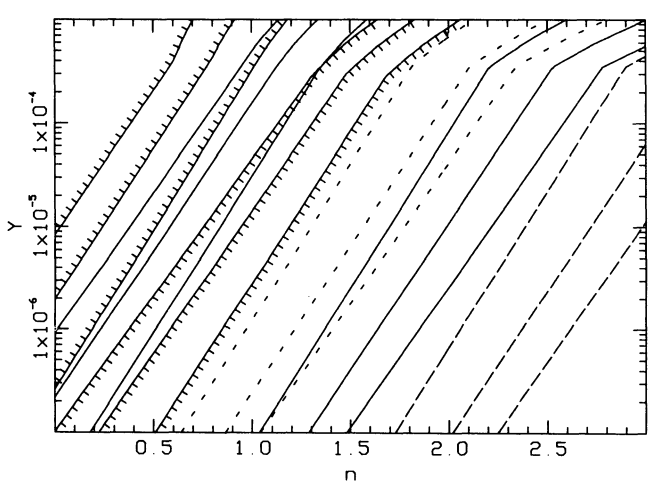

FIG. 2*a*

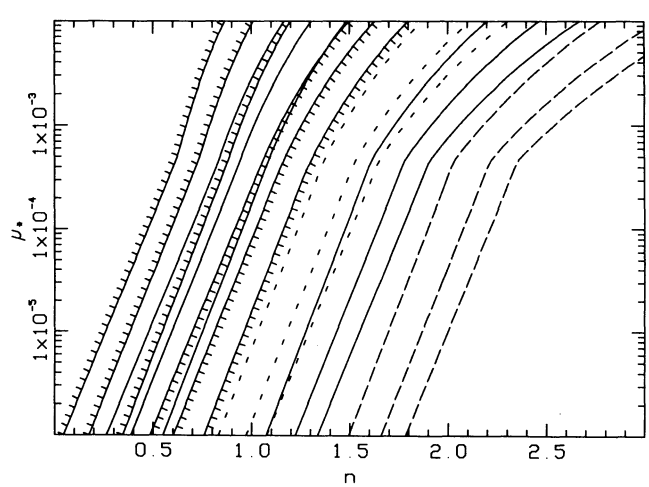

FIG. 2*b*

FIG. 2.—Values of (*a*) the Compton y-parameter and (*b*) the chemical potential μ_* computed in the adiabatic model as a function of n, the index of the initial power spectrum of the mass distribution. The lines with tick marks pointing to the upper left are obtained with $\Omega_{t,0} = \Omega_b$, $\eta_{10} = 2.5$ and values of h of 1.0, 0.75, and 0.5 decreasing from left to right. Similarly, moving from left to right, the three solid lines are for $\eta_{10} = 4.5$, $\Omega_{t,0} = \Omega_b$, with values of h of 1.0, 0.75, and 5.0 decreasing from left to right, and the lines with the tick marks pointing toward the lower right result when $\Omega_{t,0} = \Omega_b$ and $\eta_{10} = 10$, with the three values of h indicated above, decreasing from left to right. For $\Omega_{t,0} \neq \Omega_b$, y and μ_* are very weakly dependent on η_{10}, and are more strongly dependent on h and $\Omega_{t,0}$; for the three sets with $\Omega_{t,0} \neq \Omega_b$, $\eta_{10} = 4.5$ has been assumed. Continuing to move from left to right, the short-dashed curves are obtained with $\Omega_{t,0} = 0.15$, the solid lines with $\Omega_{t,0} = 0.3$, and the long-dashed curves with $\Omega_{t,0} = 1.0$; in each of these sets h takes on the values 0.5, 0.75, and 1.0 and is increasing from left to right. Note that for $n \leq 1$ the values of y and μ_* indicated are lower bounds.

Open models with $\Omega_{t,0} \neq \Omega_b$ lead to the following limits. The current limit on y of 10^{-3} implies that an open model with $\Omega_{t,0} = 0.15$ and $h = 0.5$ must have a value of $n \lesssim 2.1$; and an open model with $\Omega_{t,0} = 0.3$ and $h = 0.5$ must have $n \lesssim 2.6$. The index n of the initial power spectrum will be constrained to be $n \lesssim 2.2$ and $n \lesssim 1.7$ for $\Omega_{t,0} = 0.15$ and $h = 1.0$ and 0.5, respectively, for $y \lesssim 10^{-4}$; these constraints will be tightened to $n \lesssim 1.8$ and $n \lesssim 1.3$ for $h = 1.0$ and 0.5, respectively, if the observational limit on y becomes $y \lesssim 10^{-5}$.

The chemical potential determined by equations (22) and (12) when photon diffusion is the primary damping mechanism so that $\lambda_r = \lambda_{\gamma d}(z = z_{\rm th})$ (see eqs. [1] and [17b]) is

$$|\mu/kT| = \mu_* \propto \Omega_{t,0}^{-4} h^{-(3+n)} \eta_{10}^{(1-n)/40} ,$$

so that the chemical potential is nearly independent of η_{10} except when $\Omega_{t,0} = \Omega_b$.

The current observational limit on the chemical potential, μ_*, is $\mu_* \lesssim 10^{-2}$ (Danese & De Zotti 1978; Smoot et al. 1988, Mather et al. 1990). The limit on μ_* from *COBE* is expected to be 10^{-3} to 10^{-4} (J. C. Mather 1990, personal communication). The limit on μ_* may significantly constrain models because the only epoch during which a chemical potential can be produced is from $z_{\rm th}$ to z_y (see § 2). Aside from the presence of confusing backgrounds, such as Galactic radio emission, the only mechanism which can alter a chemical potential once it has been imprinted in the primeval radiation field is free-free emission which may produce photons in the Rayleigh-Jeans region of the spectrum. However, this has a distinct spectral signature and could not fill in the dip in the spectrum of the background if the perturbations are adiabatic. Hence, any deviation of the spectrum from that of a blackbody can unambiguously be determined to be due to either a chemical potential, free-free emission, or both, if other backgrounds can be subtracted off, which generally can be done at wavelengths less than about 10 cm.

Figure 2*b* shows the value of μ_* expected from the damping of pressure waves at the redshift $z_{\rm th}$ for adiabatic perturbations as a function of n, the index of the initial power spectrum of the mass distribution for various choices of the cosmological parameters $\Omega_{t,0}$, h, and η_{10}. Motivated by observations indicating that $\Omega_{t,0} \sim 0.1$, consider the constraints which arise when $\Omega_{t,0} = 0.15$. As illustrated in Figure 2*b*, the current limit on μ_* constrains n to be $n \lesssim 2.3$ and $n \lesssim 1.9$ for $h = 1.0$ and 0.5, respectively, where η_{10} has been assumed to be 4.5. Note that μ_* is only weakly dependent on η_{10} when $\Omega_{t,0} \neq \Omega_b$ (see the legend to Fig. 2). If the observational limit on μ_* becomes $\mu_* \lesssim 10^{-3}$, then $n \lesssim 1.8$ and $n \lesssim 1.5$ for $h = 1.0$ and 0.5, respectively, and for $\mu_* \lesssim 10^{-4}$, $n \lesssim 1.5$ and $n \lesssim 1.3$ for $h = 1.0$ and 0.5, respectively.

Baryonic models will be significantly constrained, as is evident from Figure 2*b*. However, because the damping of the waves washes out adiabatic perturbations of the baryon distribution, the first structures to form would have to have length scales greater than the damping scale at recombination. This corresponds to a mass scale of about 10^{16}–10^{17} $M_\odot$. In addition, smaller scale structures such as galaxies would have to form from the growth of secondary fluctuations, since the initial perturbations on galactic scales will be washed out.

Adiabatic models with $\Omega_{t,0} = 1.0$ and $h = 0.5$ motivated by the aesthetic prejudices of theorists will be constrained as the limits on a chemical potential decrease. For $\mu_* \lesssim 10^{-2}$, $n \lesssim 2.8$; for $\mu_* \lesssim 10^{-3}$, $n \lesssim 2.2$; and for $\mu_* \lesssim 10^{-4}$, $n \lesssim 1.9$.

4.5. *Effects That Weaken the Constraints*

There are two effects that could weaken the constraints presented here. First, the limits on the index of the initial power spectrum will be weakened if the galaxy distribution is biased (see § 4.2), since this will decrease the initial amplitude of the perturbations of the mass distribution needed to account for the galaxy distribution at the current epoch. Second, the limits will be weakened if the redshift $z_{\rm th}$ or z_y is decreased from the values adopted here. This will be the case, for example, if small distortions imprinted at these epochs are subsequently altered by photon production and electron scattering, as suggested by the recent work of Burigana, Danese, & De Zotte (1990).

4.5.1. *Effect of a Bias Parameter*

If the current galaxy distribution is biased so that the variance of the galaxy number counts is larger than that of the underlying mass distribution, the value of $\delta_m(\lambda_m)$ used to normalize the initial amplitude of the perturbations on the scale λ_m is decreased by the inverse of the bias parameter b^{-1}, that is, $\delta_m(\lambda_m) \to b^{-1}\delta_m(\lambda_m)$. The equations (eqs. [21a] and [22a] or equivalently [21b] and [22b]) used to estimate limits on the index of the initial power spectrum will be altered, hence a different limit will result. The new limit on the index of the initial power spectrum is denoted by n', while that obtained assuming that $b = 1$, and exhibited in Figures 1 and 2, is denoted by n. Equations (21a) and (21b) become

$$\delta_r^2(\lambda_r) \simeq \frac{\delta_m^2(\lambda_m)}{b^2}\left(\frac{\lambda_m}{\lambda_{\rm eq}}\right)^2\left(\frac{\lambda_m}{\lambda_r}\right)^{n'+1} \simeq \frac{0.64h^2}{b^2 z_i^2}\left(\frac{8h^{-1}\ {\rm Mpc}}{\lambda_r}\right)^{n'+1} . \tag{27}$$

Equations (22a) and (22b) become

$$\delta_r^2(\lambda_r) \simeq \frac{2\delta_m^2(\lambda_m)}{b^2}\left(\frac{\lambda_m}{\lambda_r}\right)^{n'-1} \simeq \frac{2}{b^2 z_i^2\,\Omega_{t,0}^2}\left(\frac{8h^{-1}\ {\rm Mpc}}{\lambda_r}\right)^{n'-1} . \tag{28}$$

Solving each of these equations for n' in terms of n, an identical result obtains for isocurvature and adiabatic fluctuations: $n' = n + \epsilon$, where ϵ is the correction term

$$\epsilon = \frac{2 \log b}{\log (8h^{-1}\lambda_r^{-1}\ {\rm Mpc})} . \tag{29}$$

Therefore, given a value of the bias parameter and the damping scale λ_r, the correction term ϵ can be computed from equation (29). Then the limit on the index of the initial power spectrum may be obtained by adding ϵ to the limit given in Figures 1 and 2.

Consider damping by photon diffusion at the redshifts $z_{\rm th}$ and z_y. The damping scale λ_r is obtained from equation (17b) evaluated at $z_{\rm th}$ or z_y when a chemical potential or Compton distortion (respectively) is under consideration. For a chemical potential produced by photon diffusion, and for a bias parameter $b = 2$, the correction to the index n is about 0.13, so $n' = n + 0.13$; values of $h = 0.75$ and $\eta_{10} = 4.5$ have been assumed, but the results are only weakly dependent on these choices. For a Compton y-distortion produced by photon diffusion, the correction to the index n is about 0.2, so $n' = n + 0.2$ for a bias parameter of 2, again assuming $h = 0.75$ and $\eta_{10} = 4.5$. Corrections relevant for nonlinear dissipation may be obtained from equation (29) with λ_r relevant for nonlinear dissipation, discussed in §§ 3.1 and 3.4.

4.5.2. *Effect of Decreasing the Damping Redshift*

The second effect which will weaken the constraints presented in Figures 1 and 2 is the decrease of the redshift $z_{\rm th}$ or z_y. Consider damping by photon diffusion where the redshift at which the damping occurs is decreased by a factor α, so the damping scale determined from equation (17b) becomes $\lambda_r \to \lambda_r'$, where $\lambda_r' = \alpha^{-1.5}\lambda_r$. Equation (27), relevant for isocurvature perturbations, becomes

$$\delta_r^2(\lambda_r') \simeq \delta_m^2(\lambda_m)\left(\frac{\lambda_m}{\lambda_{\rm eq}}\right)^2\left(\frac{\lambda_m}{\lambda_r'}\right)^{n'+1} \simeq \frac{0.64h^2}{z_i^2}\left(\frac{8h^{-1}\ {\rm Mpc}}{\alpha^{-1.5}\lambda_r}\right)^{n'+1} . \tag{30}$$

Using equations (22a) and (30) to determine n' in terms of n, we obtain

$$n' = \frac{n - f(\alpha, \lambda_r)}{1 + f(\alpha, \lambda_r)} , \tag{31}$$

where

$$f(\alpha, \lambda_r) = \frac{1.5 \log \alpha}{\log (8h^{-1}\lambda_r^{-1}\ {\rm Mpc})} . \tag{32}$$

Similarly, for adiabatic perturbations damped by photon diffusion,

$$\delta_r^2(\lambda_r') \simeq 2\delta_m^2(\lambda_m)\left(\frac{\lambda_m}{\lambda_r'}\right)^{n'-1} \simeq \frac{2}{z_i^2\,\Omega_{t,0}^2}\left(\frac{8h^{-1}\ {\rm Mpc}}{\alpha^{-1.5}\lambda_r}\right)^{n'-1} . \tag{33}$$

With $\lambda_r' = \alpha^{-1.5}\lambda_r$, indicated by equation (17b), equations (22b) and (33) imply

$$n' = \frac{n + f(\alpha, \lambda_r)}{1 + f(\alpha, \lambda_r)} . \tag{34}$$

This is the same function $f(\alpha, \lambda_r)$ given by equation (32).

To evaluate the effect on decreasing $z_{\rm th}$, let $\alpha = \alpha_{\rm th}$ when λ_r is obtained from equation (17b) evaluated at $z_{\rm th}$, which is given by equation (1). That is, λ_r' is obtained by evaluating equation (17b) at the redshift $\alpha_{\rm th} z_{\rm th}$. Adopting $h = 0.75$ and $\eta_{10} = 4.5$ (this choice affects the results only weakly), $f(\alpha_{\rm th}, \lambda_r) \simeq -0.15$ for $\alpha_{\rm th} = \frac{1}{3}$.

Hence, for isocurvature perturbations the following results are obtained: for $n = 0$, $n' \simeq 0.2$; for $n = 1$, $n' \simeq 1.4$; for $n = 2$, $n' \simeq 2.5$. For adiabatic perturbations the following results are obtained: for $n = 1$, $n' \simeq 1$; for $n = 2$, $n' \simeq 2.2$; for $n = 3$, $n' \simeq 3.4$.

To evaluate the effect of decreasing the redshift z_y, let $\alpha = \alpha_y$ when λ_r is obtained from equation (17b) evaluated at z_y; that is λ_r' is obtained by evaluating equation (17b) at the redshift $\alpha_y z_y$. Assuming for definiteness $h = 0.75$ and $\eta_{10} = 4.5$ (the results are weakly dependent on these parameter choices), $f(\alpha_y, \lambda_r) \simeq 0.5 \log \alpha_y \simeq -0.25$ for $\alpha_y = \frac{1}{3}$. Hence, for isocurvature perturbations the following results are obtained: for $n = 0$, $n' \simeq 0.3$; for $n = 1$, $n' \simeq 1.7$; for $n = 2$, $n' \simeq 3$. And for adiabatic perturbations the following results are obtained: for $n = 1$, $n' \simeq 1$; for $n = 2$, $n' \simeq 2.3$; for $n = 3$, $n' \simeq 3.7$.

Similar results are obtained when damping by nonlinear dissipation is considered. In this case the damping scale λ_r' is obtained from equation (16) evaluated at the redshift $z \to \alpha z$.

For isocurvature perturbations the first part of equation (30) indicates that

$$n' = \frac{n - g(\alpha, \lambda_r)}{1 + g(\alpha, \lambda_r)} , \tag{35}$$

where the function $g(\alpha, \lambda_r)$ is given by

$$g(\alpha, \lambda_r) = \frac{\log \alpha}{\log (8h^{-1}\lambda_r^{-1}\ {\rm Mpc})} . \tag{36}$$

A similar result obtains for adiabatic perturbations. The first part of equation (32) with $\lambda_r' = \alpha^{-1}\lambda_r$, as indicated by equation (16), may be used to determine the corrected value of the limit on the index n':

$$n' = \frac{n + g(\alpha, \lambda_r)}{1 + g(a, \lambda_r)} , \tag{37}$$

where the function $g(\alpha, \lambda_r)$ is given by equation (36).

The value of λ_r is determined from equation (16) evaluated at the redshift $z_{\rm th}$ if the production of a chemical potential is under consideration, or at the redshift z_y if the production of a Compton y-parameter is under consideration. The decrease of the redshifts is accounted for by the parameter α, as in the previous discussion. The value of λ_r depends on the magnitude of the distortion under consideration in addition to a dependence on h and η_{10}.

The value of λ_r at redshift $z_{\rm th}$ relevant for the production of a chemical potential follows from equations (1) and (12): $\lambda_r \simeq 0.2\mu_* \eta_{10}^{0.35}$. For a value of $\mu_* = 10^{-2}$, $\eta_{10} = 4.5$, and $h = 0.75$, $g(\alpha_{\rm th}, \lambda_r) \simeq 0.3 \log \alpha_{\rm th} \simeq -0.14$ for $\alpha_{\rm th} = \frac{1}{3}$.

Equation (35) indicates that for isocurvature perturbations the following results obtain: for $n = 0$, $n' \simeq 0.2$; for $n = 1$, $n' \simeq 1.3$; for $n = 2$, $n' \simeq 2.5$. And for adiabatic perturbations, the following results are obtained: for $n = 1$, $n' \simeq 1$; for $n = 2$, $n' \simeq 2.2$; for $n = 3$, $n' \simeq 3.3$.

The value of λ_r evaluated at the redshift z_y relevant for the production of a Compton distortion follows from equations (16), (3), and (13); $\lambda_r \simeq 20y\eta_{10}^{0.5}$ Mpc. For the parameter choices $y = 10^{-3}$, $h = 0.75$, and $\eta_{10} = 4.5$, $g(\alpha_y, \lambda_r) \simeq 0.42 \log \alpha_y \simeq -0.2$ for $\alpha_y = \frac{1}{3}$.

For isocurvature perturbations this alters the constraints on the index of the initial power spectrum determined from the constraints on the magnitude of a Compton y-parameter: for $n = 0$, $n' \simeq 0.3$; for $n = 1$, $n' \simeq 1.5$; for $n = 2$, $n' \simeq 2.8$. And for adiabatic perturbations the results are, for $n = 1$, $n' \simeq 1$; for $n = 2$, $n' \simeq 2.3$; for $n = 3$, $n' \simeq 3.5$.

5. SUMMARY

Initial perturbations of the energy density of the radiation field associated with initial perturbations of the mass density on scales relevant for galaxies and clusters of galaxies are not observable as anisotropies of the temperature of the microwave background radiation. By the epoch of recombination, $z_{\rm rec} \sim 10^3$, the initial inhomogeneities in the photon distribution will be smoothed out by the processes of nonlinear dissipation and photon diffusion on mass scales $\lesssim 10^{17}(\Omega_{t,0}/1.0)$ $M_\odot$, as discussed in § 3.4. Any initial inhomogeneities of the energy density of the radiation field on scales less than the damping scale at recombination, $\sim$50–100 Mpc (eq. [17b]), will not be seen as an anisotropy in the microwave background but will be seen as a spectral distortion of the background. Observable spectral distortions may result from the damping of pressure waves with mass scales between about 1 and 10^6 $M_\odot$ (see § 3.4).

The damping of pressure waves over the redshift interval from $z_{\rm th}$ to z_y (see § 2.2), which roughly corresponds to the redshift interval from 6×10^6 to 3×10^5, will leave its signature in a chemical potential. In the adiabatic model, photon

production is not efficient, and this chemical potential will be observable (see § 4.4). Photon production in the isocurvature model may be significant (see § 4.3), hence a chemical potential may not be observable if the initial perturbations are of the isocurvature type.

The damping of pressure waves over the redshift interval from z_y to recombination, roughly corresponding to the redshift interval from 3×10^5 to 10^3, will leave its signature in a Compton distortion of the spectrum of the microwave background radiation. Photon production may be efficient in the isocurvature baryon model, which may or may not erase the Compton distortion (see § 4.3.2). Photon production in the adiabatic model is inefficient, and is unlikely to affect the Compton distortion produced by the damping of pressure waves.

The processes of nonlinear dissipation and photon diffusion produce a different range of values of μ_* and y; this result was obtained under the assumption that the initial value (i.e., at horizon crossing) of the amplitudes of the waves increases with increasing wavenumber k (see §§ 1 and 3). A chemical potential $\mu_* \gtrsim 4.5 \times 10^{-4}$ will result from nonlinear dissipation (see § 3.3). Photon diffusion will produce a chemical potential $\mu_* \lesssim 4.5 \times 10^{-4}$. The current bound on a chemical potential is $\mu_* \lesssim 10^{-2}$, and this bound is unlikely to be reduced to $\mu_* \lesssim 5 \times 10^{-4}$; hence, if a chemical potential is observed to be $\gtrsim 5 \times 10^{-4}$, it could result from nonlinear dissipation but is unlikely to be due to photon diffusion.

Nonlinear dissipation will produce a Compton y-parameter that is $y \gtrsim 3.5 \times 10^{-4}$ (see § 3.3); if y is observed to be greater than about 3×10^{-4}, it could result from nonlinear dissipation. If the upper bound on y drops below this value, then it is unlikely to result from nonlinear dissipation, but it could result from photon diffusion.

The observationally determined upper bounds on spectral distortions of the microwave background imply upper bounds on the amplitudes of the waves which have been damped (see eqs. [12] and [13]). The damping scale is determined by the redshift of the damping, and the damping mechanism, as discussed in § 3.4. The amplitude of the perturbations of the radiation field on a given scale may be related to those of the matter on the same scale, and those in the matter can be determined as a function of n, the index of the initial power spectrum (see § 4.2), given that the amplitude of the perturbations of the mass density can be estimated on some other scale (galaxy counts at a redshift of zero are used to obtain this point; see §§ 1 and 4.2). Therefore, the upper bounds on a chemical potential and Compton y-parameter lead to bounds on the amplitude of the waves which are damped, which lead to bounds on the corresponding fluctuations in the mass distribution, which, when combined with the amplitude of the fluctuations of the mass density on some other scale, indicate constraints on the index n of the power spectrum characterizing the initial perturbations of the mass distribution. These bounds have been obtained in the isocurvature model (see § 4.3) and in the adiabatic model (see § 4.4) for several sets of cosmological parameters. Two effects which may weaken the constraints, a biased galaxy distribution and an alteration of the redshift $z_{\rm th}$ or z_y, are discussed in § 4.5.

In the event that a distortion is observed, it may be due to the damping of pressure waves prior to recombination, in which case it would provide information on the initial perturbations in the mass distribution on small scales (see § 3.4). If the perturbations are of the adiabatic type, the values of μ_* and y are not independent, and a value for one implies a value for the other. This is not the case for isocurvature perturbations because photon production is likely to be important, and this may alter the chemical potential and not the Compton distortion.

Upper bounds on the value of the chemical potential and on a Compton distortion may be used to constrain the index of the initial power spectrum of the perturbations in the mass distribution. These constraints have been determined and are illustrated in Figures 1 and 2.

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