COSMOLOGY WITH POWERFUL EXTENDED RADIO SOURCES

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Received 1993 May 6; accepted 1993 November 5

ABSTRACT

Distant powerful radio sources would provide a useful cosmological tool if an intrinsic length or luminosity could be estimated from the observations in a way that is independent of the coordinate distance to the source. A model for the propagation of the radio lobes of powerful extended radio sources is presented here; the model is written in terms of fundamental physical variables such as the luminosity in directed kinetic energy and the ambient gas density, rather than observables such as the radio power. It is shown that the fundamental physical variables may be estimated from observed quantities. The model parameter \( \beta \) that relates the characteristic time the central engine is producing a collimated outflow to the luminosity in directed kinetic energy is constrained by low-redshift observations and extrapolated to high redshift.

The application of the model to powerful extended radio sources allows an estimate of the characteristic source sizes that is nearly independent of the coordinate distance to the source, and thus is basically independent of the deceleration parameter \( q_0 \). The characteristic source sizes estimated in the context of the model may be compared with the observed median source sizes, where each source size is determined directly from the angular extent of the source, the source redshift, and the coordinate distance to the source. The comparison of the intrinsic source sizes estimated using the model, which are nearly independent of \( q_0 \), with the median source sizes estimated using the angular sizes of the sources, which depend on \( q_0 \), constrains the allowed range of \( q_0 \).

The basic model and method is presented and discussed and is applied to one published data set consisting of 10 radio galaxies. Taken at face value, the data favor a low value of \( q_0 \); a low value of \( q_0 \) implies that either space curvature or a cosmological constant is important at the present epoch. Note that Doppler boosting and beaming of radio emission are unlikely to be important in these systems since the relevant flow velocities are small compared with the speed of light.

This analysis has broad implications for the environments of powerful extended radio sources and for models for extracting the luminosity in directed kinetic energy from the central compact object.

Subject headings: cosmology: observations — cosmology: theory — radio continuum: galaxies — radio continuum: general

1. INTRODUCTION

The median angular radio lobe separations of powerful extended radio sources as a function of redshift could be used to estimate \( q_0 \) (Hoyle 1959; Miley 1968, 1971) if the median lobe-lobe size of the sources is a standard yardstick and does not evolve with cosmic epoch, or if the evolution with cosmic epoch can be estimated, as discussed and summarized by Kapahi (1987). The median sizes of 3 CR sources appear to be roughly constant from a redshift of about 0.1 to at least a redshift of about 0.6 independent of the value of \( q_0 \). However, as stressed by Kapahi (1989), the median sizes of 3 CR sources out to redshifts of about 1 should not be assumed to be standard yardsticks since evolution may be important and alter the lobe-lobe separations of radio sources at high redshift.

The question of whether the median angular sizes of powerful extended radio sources as a function of redshift can be used to estimate \( q_0 \) is addressed afresh in this paper. A new way in which the intrinsic radio source sizes can be estimated is introduced here. The principal new ingredients that may allow distant powerful radio sources to be useful cosmological tools are a model for the radio lobe separations written in terms of fundamental physical variables that can account for the properties of radio sources with redshifts less than about 0.6 where the choice of the de/acceleration parameter is not important, and an estimate of the rate of growth of the radio source size, that is, the velocity with which the radio lobe propagates into the ambient medium.

The model, detailed in § 3, is based on fundamental physical variables rather than observables. Observable quantities, such as the radio power, are complicated functions of fundamental physical variables. The fundamental physical variables intrinsic to the active galactic nucleus are quantities such as the luminosity in directed kinetic energy \( L_J \) that powers the radio emission, the characteristic timescale \( t_s \) for which the outflow occurs, and the initial energy available to power the outflow \( E_I \). The fundamental physical variables that are extrinsic to the active nucleus are quantities such as the density of the ambient gas \( n_a \) and the radius of the radio lobe \( a_L \), though the latter could be related to the active nucleus if it is related to the precession of the black hole spin axis, for example.

The evolution of the median sizes of radio sources with radio power and redshift has been discussed by several authors (e.g., Oort, Katgert, & Windhorst 1987; Kapahi 1989). However, neither the radio power nor the redshift constitute fundamental physical variables. The radio power is a complicated function of both intrinsic and extrinsic fundamental physical variables (see §§ 3.2 and 3.3). Evolution with redshift will occur

\[ ^1 \text{National Young Investigator.} \]
if one or more fundamental physical variables, such as the density of the ambient gas in the vicinity of the radio source, evolve with redshift.

The characteristic separation of the radio lobes of an extended powerful radio source is given by the product of the characteristic lobe propagation velocity $v_L$ and the characteristic time for which the outflow occurs $t_\star$. The lobe propagation velocity is observed to increase as the radio power increases (Alexander & Leahy 1987, hereafter AL87; Liu, Pooley, & Riley 1992, hereafter LPR92). The characteristic or median size of radio sources is roughly constant, at least out to redshifts of about 0.5, as detailed in § 2. This suggests that the characteristic time for which the outflow occurs decreases as the radio power increases so that the product $v_L t_\star$ remains constant. The characteristic time $t_\star$ is set by processes near the central engine and thus is related to intrinsic fundamental physical variables, such as the luminosity in directed kinetic energy $L_d$. If the radio power and lobe propagation velocity increase as the luminosity in directed kinetic energy increases, and the time $t_\star$ decreases as the luminosity in directed kinetic energy increases, the product $v_L t_\star$ may remain constant as observed. For this reason, the characteristic time over which the directed kinetic energy powers the radio emission is characterized as a function of the luminosity in directed kinetic energy (see § 3.1). The lobe propagation velocity may be written as a function of the luminosity in directed kinetic energy, the ambient gas density, and the radius of the radio lobe.

The model parameters that cannot be directly observed, such as the luminosity in directed kinetic energy, can be related to observables (see eq. [3c]). One quantity that is difficult to estimate directly from observations is the density of the ambient gas in the vicinity of the radio lobe. When the strong shock jump conditions apply, the radio lobe is ram-pressure-confined by the ambient gas, so the ambient gas density $n_g$ is simply related to the lobe propagation velocity $v_L$ and the magnetic field strength $B$, assuming rough equipartition of energy between the magnetic field and the relativistic electrons that make up the radio-emitting plasma: $n_g \propto (B/v_L)^3$ (see § 3.3). It is interesting to note that the empirically determined relation for radio galaxies between the radio power and the lobe propagation velocity (AL87) is recovered by the model (see §§ 3.2 and 3.3), lending support to the basic picture. In addition, the composite density profile obtained by plotting the ambient gas density estimated as described above versus the radio lobe separations $2r$ implies that the gas in the vicinity of the radio sources has an $r^{-1.5}$ profile (Daly 1994 and § 3.1), indicating that the adopted approximations are good rough estimates; this density profile is similar to that of gas in clusters of galaxies.

The characteristic or median sizes of radio sources with a given radio power or redshift may be written in terms of extrinsic fundamental physical variables such as the radius of the radio lobe, the lobe propagation velocity, and the magnetic field strength, all of which can be estimated directly from observations (see § 3.2 and eq. [9]). This is because intrinsic fundamental physical variables, such as the luminosity in directed kinetic energy, can be written in terms and estimated from observable quantities (see eq. [8]). Fortunately, the characteristic or median separation between the radio lobes given by equation (9) is nearly independent of the coordinate distance to the source, and thus is nearly independent of the de/acceleration parameter $q_0$. The lobe-lobe sizes of sources can be estimated from the angular sizes of sources assuming different values of $q_0$, and the median values of the source sizes determined from the angular sizes can be compared with the intrinsic characteristic source sizes determined using the model to determine which value of $q_0$ best fits the data.

The model source sizes as estimated from the data are presented and discussed in § 4. The dependence of this characteristic source size on $q_0$ and the use of these systems to estimate $q_0$ are detailed in § 4. The possible evolution of relations such as that between the characteristic outflow period $t_\star$ and the luminosity in directed kinetic energy, would be worrisome if quantities such as the luminosity in directed kinetic energy evolve strongly with redshift. Fortunately, the observations suggest that this luminosity does not evolve strongly with redshift, and, in fact, there may be an empirically determined upper bound to $L_d$, as discussed in § 5. The implications that the model application has for the intrinsic properties of the sources and for the environments in which the sources are situated are briefly discussed in § 4 and are discussed in detail by Daly (1994).

The determination of the de/acceleration parameter $q_0$ is fundamental to our understanding of the global structure of the universe, as discussed by Mattig (1958, 1959), and more recently by Sandage (1988) and Peebles (1993). Within the framework of a homogeneous, isotropic, expanding universe, the expansion rate depends on the mean mass density, space curvature, and the cosmological constant, each of which evolves differentially with redshift. If, at the present epoch, only two of these three quantities are important, then the determination of the cosmological parameters $q_0$ and $H_0$ determines the specific cosmological world model that describes our universe. If all three quantities are important at the present epoch, then there are three unknown quantities to be determined: the mean mass density, space curvature, and the cosmological constant $\Omega_0$, in addition to $H_0$ and $q_0$, another observation, such as the present mean mass density, is required in order to be able to use $H_0$ and $q_0$ to determine the other two quantities. The cosmological parameters $\Omega_0$ and $\Lambda_0$, the current mean mass density of the universe relative to the critical value and the present value of the cosmological constant, can be determined from $q_0$, $H_0$, and the present mean mass density.

A determination of the de/acceleration parameter $q_0$ requires the use of a source with a known intrinsic length or luminosity that is located at relatively large redshifts, $z \gtrsim 0.5$–1. This is because the de/acceleration parameter depends on the second derivative of the expansion factor $\alpha(t)$ with respect to time: $q_0 \equiv -\ddot{\alpha}/(\alpha \dot{\alpha})$, where $\dot{\alpha}$ is the cosmological expansion factor, $\ddot{\alpha}$ denotes the derivative with respect to time, and the subscript 0 indicates that the quantity is evaluated at the present epoch.

Unfortunately, it has been difficult to identify an intrinsic length or luminosity to use to determine $q_0$ because systems in the universe evolve with cosmic time, and the universe and systems in the universe at redshifts $\gtrsim 0.5$–1 are significantly younger than their present-day counterparts. A few promising candidates are presently under consideration, such as the use of millisecond radio sources as a standard yardstick (Kellerman 1993; Roland et al. 1993; Gurvits 1994), and Type Ia supernovae as a standard candle (Tammann 1979; Branch & Tammann 1992; Perlmuter et al. 1993).

It is important to determine $q_0$ using several independent methods, since uncertain evolutionary changes may affect any determination of $q_0$. Here, the possibility of using powerful extended radio sources to determine $q_0$ is considered; these sources are characterized by two widely separated edge-
brightened radio lobes. The use of these systems to determine $q_0$ has been discussed by Miley (1968, 1971), Kapahi (1987), Oort et al. (1987), Singal (1988), and Kapahi (1989), to name a few. However, using these systems as a cosmological tool has not been possible because of unknown and unpredictable evolutionary effects. A new model for the propagation of a radio lobe into the ambient medium and new recent observations may now make it possible to estimate an intrinsic length scale of powerful radio sources from the data in a way that is independent of $q_0$, yielding a measure of an intrinsic length scale. Thus, in the model presented and discussed in this paper, the problem of correcting for evolutionary effects is bypassed.

The observational aspects of radio sources relevant to their use as a cosmological tool are discussed in § 2. Radio galaxies emerge as the primary cosmological tool, though radio-loud quasars may be important too. The model for the propagation of the radio source into the ambient medium and a way to estimate the characteristic or median core-lobe or lobe-lobe sizes of the radio source are presented in § 3.1. Interesting relations, such as that between the radio power and the luminosity in directed kinetic energy, are discussed in § 3.3. The model is applied in § 4 to observationally estimate the parameters needed to determine the deacceleration parameter $q_0$. For radio galaxies, the radio source sizes estimated from the model are only weakly dependent on $q_0$. Thus a comparison of the estimated and observed source sizes can be used to estimate $q_0$. The results for radio galaxies are discussed in § 4, and those for radio-loud quasars are discussed in §§ 4 and 4.1. Various tests of whether the model and its application are yielding reliable results are discussed in § 5. The results are summarized and discussed in § 6.

2. THE OBSERVATIONS

2.1. Observations Relevant to Radio Sources as Cosmological Tools

Let us consider a typical high-redshift ($z \sim 1$) double-lobed powerful radio source, such as any of the high-redshift 3 CR sources. Several properties of a source can be determined from observations. In general, the determination of the properties of a source such as the radio power, velocity of propagation of a lobe, and the physical separation between the radio lobes depends on the coordinate distance $r$ to the source through the combination $a_0 r$, where $a_0$ is the value of the cosmic scale factor at the present epoch (see, for example, Mattig 1958, 1959; Sandage 1988; Peebles 1993). The coordinate distance $r$ depends on the present values of the mean mass density, space curvature, and the cosmological constant. If only two of these three quantities are important at the present epoch, the coordinate distance $r$ at a source at redshift $z$ is a function of $q_0$, $H_0$, and $z$, i.e., $a_0 r = f(q_0, H_0, z)$; if all three quantities are important then the coordinate distance depends on another parameter, such as the present value of the mean mass density. At low redshifts, $z \lesssim 0.6$, $(a_0 r)$ is only weakly dependent on specific parameter choices, such as that of $q_0$, so the intrinsic properties of low-redshift sources can be determined independent of the specific cosmological model. The comparison of high-redshift sources with low-redshift sources allows an estimate of $q_0$ that is independent of $H_0$.

Both the maximum and median angular separations of the radio lobes of powerful 3 CR radio galaxies is \( \propto z^{-1} \) for redshifts $z \lesssim 1$ (Miley 1968, 1971; Legg 1970; Wardle & Miley 1974; Kapahi 1987, 1989; Singal 1988). Thus, the maximum and median (or characteristic) sizes of 3 CR radio galaxies are roughly constant at about 150 kpc out to $z \sim 0.6$ where $H_0 = 100$ km s$^{-1}$ Mpc$^{-1}$ is the Hubble constant; the different properties of relatively low power F-R II galaxies found by Singal (1988) are discussed in § 2.2, as are the properties of powerful F-R II quasars.

Observations of 3 CR radio galaxies indicate that if their intrinsic characteristic sizes decrease by about a factor of 2 between redshifts of about 0.5 and 1 a flat matter-dominated universe with $q_0 = 0.5$ would be suggested, and if their intrinsic sizes remain roughly constant to redshifts of about 1, a cosmological model with $q_0 \approx 0$ would be favored (Kapahi 1989; Singal 1988). Note that a negative value of $q_0$ would require a significant cosmological constant at the present epoch.

Powerful 3 CR radio galaxies with a wide range of radio powers have a constant median lobe-lobe size, at least out to redshifts of about 0.6, where the determination of the source size is only weakly dependent on $q_0$. This suggests that the median sizes of the sources are independent of the radio power. This trend holds when sources with lower 178 MHz radio flux densities are studied; 8C radio sources studied by Lacy et al. (1993) have lobe-lobe sizes that are independent of radio power (Lacy et al. 1993). These observational facts motivate the model presented in § 3. The model is designed to explain the properties of radio galaxies at redshifts less than about 0.6, where the source properties are only weakly dependent on $q_0$, and then to extrapolate these properties to high redshift. The model is also designed to explain the lack of dependence of size on power for sources at a given redshift (Lacy et al. 1993).

Observations of radio hotspots and bridges in distant powerful double radio sources indicate that the radio power of a source is likely to have been roughly constant over the lifetime of the source, and thus to first order is independent of the separation of the lobe from the parent galaxy or radio core (Leahy, Muxlow, & Stephens 1989; hereafter LMS89). The observations of AL87 and LPR92 are consistent with a particular source having a roughly constant velocity $v_L$ of propagation of the lobe through the ambient medium. The velocity $v_L$ is determined using the effect of synchrotron aging on the radio spectrum to estimate the time interval $\Delta t$ that has elapsed across a synchrotron-emitting region of size $\Delta x$, $v_L = \Delta x / \Delta t$; for a detailed discussion see AL87 and LMS89. Furthermore, these observations suggest a relation between the radio power $P_r \equiv dE/(dt dv)$ at 178 MHz and the velocity $v_L$ of propagation of the lobe through the ambient medium, of the form $v_L \propto P_r^{1/n}$ with $2 \leq n \leq 5$ (AL87; LPR92). This empirical relation can be explained if the radio lobe is ram-pressure-confined by the ambient medium, as detailed in § 3.1. The typical lifetime of a radio source is $\lesssim 10^7$ to $10^8$ yr, so sources seen at high redshifts are radio-loud for a small fraction of the Hubble time at the redshift of the source. Lobe propagation velocities are small compared with the speed of light, so no relativistic corrections are needed.

LPR92 used the hotspot magnetic field to estimate the lobe propagation velocity and noted that in general the energy density of this field is about 15–25 times higher than that of the microwave background radiation at the redshift of the source. The magnetic field strengths in the lobe and bridge are likely to be smaller than the hotspot field, and the lobe magnetic field appears to be the field strength relevant to the computation of the lobe propagation velocity. For powerful sources in the same redshift range as LPR92, LMS89 found that the magnetic...
energy density of the lobe and bridge typically is comparable to that of the microwave background radiation at the redshift of the source. If this is characteristic of the fields in the lobes and bridges of LPR92, and if inverse Compton cooling with microwave photons is included, the lobe propagation velocities are about a factor of 4 to 5 smaller than the hotspot velocities quoted by LPR92.

2.2. Differences between Radio Galaxies and Quasars

Powerful double edge-brightened sources evolve differently with redshift depending on whether the source is a galaxy or a quasar (Singal 1988). The sources considered by Singal (1988) are all powerful F-R II sources with meter to centimeter radio spectral indices $\alpha > 0.5$, and thus the results of this study are directly comparable to the model discussed in § 3. The lobe-lobe or radio source sizes of powerful radio galaxies, those with 408 MHz radio power $P_{408}(q_0 = 0) \geq 3 \times 10^{26}$ W Hz$^{-1}$, are roughly constant at about 150 h$^{-1}$ kpc at a redshift of about $0.2$ to a redshift of about 0.6–1 (Singal 1988); a radio spectral index of 1 is assumed to convert between 178 and 408 MHz radio powers. Radio galaxies with 178 MHz radio powers $P_{178}(q_0 = 0) \geq 10^{27}$ W Hz$^{-1}$ sr$^{-1}$, such as those presented and discussed by LMS89, have quite regular bridge structures and do not show central distortions characteristic of less powerful radio galaxies.

Lower power radio galaxies, those with 408 MHz radio powers $P_{408}(q_0 = 0)$ between $3 \times 10^{25}$ and $3 \times 10^{26}$ W Hz$^{-1}$, have median source sizes that decrease from about 150 h$^{-1}$ kpc at a redshift of 0.06 to about 50 h$^{-1}$ kpc at a redshift of 0.55. Less powerful sources, such as those studied by Leahy & Williams (1984) that have 178 MHz powers ranging from about $P_{178}(q_0 = 0) \sim (h^{-2} \times 10^{23})$ to $\sim (h^{-2} \times 10^{26})$ W Hz$^{-1}$ sr$^{-1}$, often have distorted radio bridges. Central distortions of the radio-emitting region observed in radio galaxies are thought to indicate a significant backflow velocity of the lobe material (Leahy & Williams 1984). This suggests that different physical processes, for example, whether backflows are present, are important in F-R II radio sources with 178 MHz radio powers above and below $P_{178}(q_0 = 0) \sim (h^{-2} \times 10^{27})$ W Hz$^{-1}$ sr$^{-1}$. Thus, the fact noted by Singal (1988) that the median sizes of radio galaxies with 408 MHz powers in the range from $3 \times 10^{25}$ to about $3 \times 10^{26}$ W Hz$^{-1}$ evolve differently with redshift than those with higher radio powers is reflected by the differences in the bridge structures of sources with different radio powers. It is important that a homogeneous class of sources be used for cosmological purposes. Thus, only very powerful radio galaxies, those with $P_{178}(q_0 = 0) \geq h^{-2} \times 10^{27}$ W Hz$^{-1}$ sr$^{-1}$, should be used as a cosmological tool.

Radio-loud quasars with 408 MHz radio powers in the range from $3 \times 10^{25} \leq P_{408}(q_0 = 0) \leq 3 \times 10^{26}$ W Hz$^{-1}$ seem to have intrinsic sizes that decrease monotonically from about 150 h$^{-1}$ kpc at a redshift of 0.3 to about 40 h$^{-1}$ kpc for $q_0 = 0$ or about 20 h$^{-1}$ kpc for $q_0 = 0.5$ by a redshift of 2 (Singal 1988). Different radio luminosities are present in different redshift bins, so the characteristic size of sources with a given radio power as a function of redshift cannot be determined, though it is interesting that the high-power, high-redshift sources are the smallest.

As pointed out by LMS89, the radio bridges of quasars are quite different from those of radio galaxies; the radio bridges of quasars are shorter and wider than those of radio galaxies at a similar power, and they often show significant distortions near the extremities of the radio source. Thus, the direction of the collimated outflow in extended radio-loud quasars could change direction with time, whereas that in radio galaxies seems to be relatively stable. Alternatively, it could be the collimated outflow in radio-loud quasars emerges as a cone with a nonzero opening angle, whereas in radio galaxies the outflow is highly collimated and emerges as a stream with a very small opening angle.

3. THE MODEL

3.1. Radio Source Sizes

A source with a lobe propagation velocity $v_L$ that has been active to a time $t$ has a core-lobe separation of $l = v_L t$ or a lobe-lobe separation $l = 2v_L t$ when $v_L$ is constant over the time $t$ and when the direction of the collimated outflow is stable; typical velocities are in the range from about $10^3$ to $10^4$ km s$^{-1}$ and typical lobe-lobe separations are about 150 h$^{-1}$ kpc. Observations of $v_L$ are consistent with $v_L$ being roughly constant over the lifetime of the source (see § 2.1). Sources with an average velocity $v_L$ that are active for a characteristic timescale $t_\ast$ will have a characteristic core-lobe size

$$l_\ast = v_L t_\ast$$

(1)

when the direction of the collimated outflow is stable as appears to be the case for powerful radio galaxies but perhaps not for radio-loud quasars (see § 2.2). The characteristic size of the source could be the mean or median size of sources with a particular velocity $v_L$ or radio power $P_r$, since $v_L$ is observed to be a function of $P_r$ (see §§ 2.1 and 3.2).

The lobe is powered by directed kinetic energy released by the central compact object, which is likely to be a massive black hole (see, for example, Begelman, Blandford, & Rees 1984). Thus, the time $t_\ast$ for which a source is active depends only on processes occurring near the active galactic nucleus (AGN) and should be independent of extrinsic quantities such as the redshift of the source and the gas density of the ambient medium with which the jet from the AGN interacts when the lobe is far from the source of the jet. Secondary effects could be important if the properties of the gas near the AGN are related to those of the ambient medium, and the gas density of the ambient medium evolves with redshift. However, the observations suggest that the sources are in environments with similar densities so this is unlikely to be an important effect for the sources considered here, as discussed in § 5 and by Daly (1994).

The characteristic time for which a source will be active is $t_\ast \approx E_\ast / L_\ast$, where $E_\ast$ is the initial energy available to power the jet, most likely the spin energy of the black hole, and $L_\ast$ is the luminosity in directed kinetic energy released from the central compact region; $L_\ast$ is assumed to be roughly constant over the lifetime $t_\ast$ of the source. The characteristic time $t_\ast$ for which the source is actively producing the luminosity in directed kinetic energy $L_\ast$ is likely to be a function of $L_\ast$ and is parameterized by

$$t_\ast \propto L_\ast^{-\beta / 3}.$$  

(2)

The determination of $\beta$ has implications for models to extract the luminosity in directed kinetic energy from the central compact source since $L_\ast \propto E_\ast^{(\beta / 3 - \beta)}$.

The forward region of the radio lobe may be taken to represent the contact discontinuity of a shock front propagating with supersonic velocity into the ambient medium (e.g., De Young & Axford 1967; De Young 1971, 1986, 1991; Scheuer

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The energy input to the hotspot and lobe in the time interval $\delta t$ is $\delta E \approx L_j \delta t$, where $L_j$ is the luminosity in directed kinetic energy carried by the jet to the hotspot and lobe. The radio lobe propagates into the ambient medium with velocity $v_L$ and does $P \cdot dV$ work on the medium.

The pressure $P$ is given by the strong shock jump conditions $P = 0.75 \rho_a v_L^2$, where $\rho_a$ is the mass density of the ambient medium, and the volume element is given by $dV = 4\pi a_L^2 \rho_a v_L^2 \delta t$, where $a_L$ is the radius of the radio lobe. Thus, the energy input to the wave in the time $\delta t = L_j \delta t$ is $0.75 \pi a_L^2 \rho_a v_L^2 \delta t$, or

$$v_L \propto \left( \frac{L_j}{\rho_a a_L^2} \right)^{1/3} \propto kL_j^{1/3},$$

(3a)

$$k \equiv (n_a a_L^2)^{-1/3},$$

(3b)

as discussed by Daly (1990, eq. [2]). Note that the parameter $k$ can be estimated from observables in a way that is independent of the de/acceleration parameter $q_0$ (see §§ 3.1 and 4).

Equations (3a) and (3b) imply

$$L_j \propto n_a a_L^2 v_L^3 \propto (v_L k^{-1})^3.$$

(3c)

Equations (3) are valid when backflows are negligible so most of the energy goes to the forward propagation of the radio lobe, and when the direction of the collimated outflow is relatively stable. Thus, as discussed in § 2.2, equations (3) are applicable to powerful radio galaxies, and powerful radio-loud quasars if the width of the bridges in the quasars is caused by a collimated outflow with a large nonzero opening angle rather than due to a time variation in the direction of the collimated outflow; equations (3) also apply to radio-loud quasars if the velocity of lobe propagation determined empirically is the time average of the propagation velocity of the entire lobe.

The characteristic or median size of radio sources in which the outflow direction is relatively stable and with negligible backflows is $l_a \sim 2 v_L t_*$, so

$$l_a \propto \left( \frac{L_j}{n_a a_L^2} \right)^{1/3} L_j^{-1/3} \propto kL_j^{1/3} a_L^{-1/3}.$$

(4)

The first term $k$ describes the environmental effects on the radio source size and thus is related to extrinsic variables, and the second term $L_j^{-1/3}$ describes the effects of processes near the central compact object on the radio source size and thus is related to intrinsic variables. Substituting $L_j \propto (v_L k)^3$ into equation (4), the characteristic core-lobe or lobe-lobe size of a radio source is given by

$$l_a \propto k^{2/3} v_L^{-1/3}.$$

(5)

Both $k$ and $L_j$ depend on the ambient gas density $n_a$ at the position of the radio lobe (see eqs. [3b] and [3c]). The ambient gas density is difficult to estimate directly. However, for supersonic lobe propagation, the strong shock jump conditions apply and the radio lobe is ram-pressure-confined (De Young & Axford 1967), so that the ram pressure $\propto n_a v_L^2$ is balanced by the pressure of the radio lobe. If the magnetic and relativistic electron pressures in the radio lobe are roughly equal, then the pressure of the radio lobe is $\propto B_L^2$, in which case the ambient gas density can be estimated from

$$n_a \propto (B/v_L)^2.$$

(6)

Usually, the magnetic field strength is estimated by the field strength that minimizes the total energy in the relativistic electrons and the magnetic field, which is nearly equivalent to assuming pressure equilibrium between the relativistic electrons and the magnetic field. The magnetic field strength can be estimated directly when X-ray emission produced by inverse Compton scattering of microwave background photons with relativistic electrons is observed as discussed, for example, by Daly (1992a, b).

There is some observational evidence that the ambient gas density estimated using equation (6) is a good rough approximation. Daly (1994) plots the ambient gas density as a function of radio lobe separation and as a function of redshift; the ambient gas density is obtained from $n_a \propto (B/v_L)^2$, where the magnetic field strength is approximated by the minimum energy value and the velocity is estimated from the synchrotron aging of the relativistic electrons; the data of LMS89 are used. The results are that the ambient gas density is independent of or weakly dependent on redshift, but is proportional to the radio lobe separations $D$, $n_a \propto D^{-1.5}$.

The fact that the density is independent of redshift and does not exhibit large variations implies that the sources are in fairly similar environments. Therefore, a density profile may be estimated by the composite profile $n_a(D)$; generally, the sources from LMS89 have fairly symmetric lobes about the radio source, so the radius is about half of the lobe-lobe separation. Thus, the density estimated using equation (6) and a minimum energy magnetic field has an $r^{-1.5}$ density profile similar to that of gas in clusters of galaxies. This implies that equation (6) is a good way to estimate the ambient gas density in the vicinity of the radio lobe, and that the radio sources sit at the base of a gravitational potential well with a cluster-like density profile. It also implies that the lobe pressure is proportional to the square of the minimum energy magnetic field, and the velocity of lobe propagation estimated from the synchrotron aging of the radio spectrum is good rough approximation. Perhaps the magnetic field and the relativistic electrons in the lobe are in pressure equilibrium, since this would explain why the minimum energy magnetic field is a good rough approximation to the true field strength.

Equation (6) is used in §§ 3.2 and 3.2.1 to derive the relation between the radio power and the lobe propagation velocity. The relation derived is close to that determined empirically, which provides a second line of evidence that equation (6) is a good rough approximation. The relation implied between the luminosity in directed kinetic energy and the radio power is given in § 3.3.

Combining equations (3b) and (6),

$$k \equiv (n_a a_L^2)^{-1/3} \propto \left( \frac{v_L}{B a_L} \right)^{2/3},$$

(7)

is obtained. Equations (3c) and (6) imply

$$L_j \propto (v_L^3 n_a a_L^2) \propto (v_L B^2 a_L^2),$$

(8)

so the intrinsic quantity $L_j$ is related to the extrinsic quantities, and can be estimated from observations. Thus, $l_a \propto kL_j^{1/3} a_L^{-1/3} \propto k^{2/3} v_L^{-1/3}$ becomes

$$l_a \propto \left( \frac{v_L}{B a_L} \right)^{2/3} (v_L B^2 a_L^2)^{1/3} \propto \left( \frac{v_L}{B a_L} \right)^{2/3} (v_L^3)^{1/3}.$$  

(9)

Equation (9) is insensitive to $q_0$ for $\beta \sim 1$ and varies inversely with coordinate distance $(a_\theta n)$ for $\beta \gtrsim 1$, and is applied in § 4 to estimate the de/acceleration parameter $q_0$.  

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The parameter $k$ may be estimated directly from observations of radio sources, as discussed in § 4. When the velocity and redshift dependence of $k$ are known, the parameter $\beta$ can be estimated from the evolution of the median radio source sizes of powerful radio sources with redshifts $z \leq 0.6$, since at these redshifts the source sizes estimated from their angular sizes are only weakly dependent on $q_0$.

A good consistency check will be to compare the values of $q_0$ estimated separately for radio galaxies and for radio-loud quasars, since $k$ and $\beta$ can be estimated for galaxies and quasars separately. It is expected that either the redshift dependence of $k$ or the value of $\beta$ will be different for galaxies and quasars since Singal (1988) finds that the evolution of the median source sizes with redshift for galaxies and quasars exhibit markedly different behaviour. Thus galaxies and quasars provide two independent ways to estimate the differential acceleration parameter $q_0$. As discussed in § 4, $k(z)$ may be estimated from the observations in a way that is nearly independent of $q_0$. Present observations allow an estimate of $\beta$ for radio galaxies, but not for radio-loud quasars; for radio galaxies $\beta \approx 1.5 \pm 0.5$ (see § 4).

The parameter $k$ is related to the mass per unit radius $dM/dr$ heated by the shock front (Daly 1990). The mass per unit radius heated by the shock front is $dM/dr = \rho_s a_r k_s \approx a_r^3 / n_s \propto k^{-3}$. The interpretation of the observed values of $k$ for galaxies and quasars, denoted $k_g$ and $k_q$, respectively, are discussed in § 4, and their implications for the environments of the sources are discussed at length by Daly (1994).

Sections 3.2 and 3.3 are optional, and the reader may skip to § 4 to see the $q_0$ dependence of equation (9) and the value of $q_0$ implied by the data presently available.

### 3.2. The Lobe Propagation Velocity and the Radio Power

As discussed in § 2.1, the lobe propagation velocity $v_L$ is observed to increase as the radio power $P_r$ increases, with $v_L \propto P_r^{1/4}$ and $2 \leq n \leq 5$ (AL87; LPR92). It is shown here, and in § 3.2.1, that this relation follows from the ram-pressure confinement of the radio lobe if the sources are interacting with gas whose density does not vary widely from source to source. That is, the observed relation follows from $v_L \propto B/(n_s)^{1/2}$, with $B$ given by the minimum energy magnetic field. This suggests that equation (6) is a good way to estimate the gas density in the vicinity of the radio lobe.

Let us consider a radio hotspot and lobe with 178 $(1 + z)$ MHz radio power at the frequency $v_L$, $P_r(v_L)$. The radio luminosity $L_r$ at the frequency $v_L$ may be approximated as $L_r(v_L) \approx P_r(v_L)v_L$. The radio luminosity at the frequency $v_L$ may be expressed as the rate of energy injection per relativistic electron $dE_e/\sqrt{dt}$ at the relevant frequency $v_L$ times the total number of relativistic electrons with energy $E_e$ that radiate at frequency $v_L$: $L_r(v_L) \approx N(E_e) dE_e/dt$ (see Daly 1992b). A relativistic electron with a Lorentz factor $\gamma_e$ has an energy $E_1 = \gamma_e m_e c^2$ for $\gamma_L \approx 1$ where $m_e$ is the electron rest mass and $c$ is the speed of light. The relativistic electron spirals about a magnetic field with component $B_L$ perpendicular to the instantaneous direction of propagation of the electron; $B_L$ is parameterized by $B_L = b \times v_L^5$ G.

The rate at which an individual electron radiations synchrotron emission is $dE_e/\sqrt{dt} \approx 1.6 \times 10^{-5} \nu_1^2 b^2$ ergs $s^{-1} \propto B^2 \gamma_1^4 \propto u_b \gamma_1^2$ where $u_b$ is the energy density of the magnetic field. The frequency of the emitted radiation is related to the magnetic field strength and the Lorentz factor of the electron: $v_1 \approx 4.2 \times 10^{-5} b_\gamma^2$ MHz. For a radio spectral index $\alpha$ defined by a radio flux density $f_\nu \propto \nu^{-\alpha}$, the number of relativistic electrons with energy $E_1$ or Lorentz factor $\gamma_1$ is related to the total number of relativistic electrons $N_{rel}$ by $N(E_1) = N_{rel}^\alpha \gamma_1^{-\alpha - 2}$ for $\alpha > 0$, where the low-energy cutoff $\gamma_{\alpha}$ is defined to be the Lorentz factor of the electrons that produce the low-frequency synchrotron emission where the radio spectral index begins to flatten, as discussed in more detail below.

Combining these expressions, the radio power at the frequency $v_1$ is

$$P_r(v_1) \approx N(E_1) \frac{dE_e}{dt} v_1^{-1} \propto N_{rel}^\alpha \gamma_{\alpha}^{-\alpha - 2} B^2 \gamma_1^4 v_1^{-1}.$$  

(10)

The total energy in relativistic electrons is $E_{\nu_1} = N_{rel} \gamma_{\alpha} m_e c^2$ for $\alpha > 0$. Thus, the relevant value of $\gamma_{\alpha}$ is all subsequent expressions is the Lorentz factor of the relativistic electrons that produce the radio emission at the frequency where the radio spectral index changes from values $\geq 0.5$ to values $\leq 0.5$. Noting that the magnetic energy density $u_b \propto B^2$, the radio power depends on the quantities

$$P_r(v_1) \propto E_{\nu_1} \gamma_{\alpha}^2 (1 - 3/2 \alpha - 1) v_1^{-1},$$  

(11)

which is valid for $\alpha > 0.5$ at $v_1$. This may also be written

$$P_r(v_1) \propto E_{\nu_1} \gamma_{\alpha}^2 v_1^{-(3/2 + 1) v_1^{-1}},$$  

(12)

since $v_1 \propto B_{\gamma}^{-1}$. Note that all of the expressions discussed thus far are independent of the volume filling factor of the relativistic electrons and magnetic field.

For a radio spectral index $\alpha \approx 1$, the radio lobe is ram-pressure-confined when $P_r(v_1) v_1 \propto n_L v_1 v_1 V_L$ since $P_r(v_1) v_1 \propto u_b V$ and ram-pressure confinement with the magnetic field near the minimum energy or equipartition value implies $n_L v_1 \propto B^2$, where the total volume $V$ of the radio-emitting plasma occupies some fraction $f$ of the total lobe volume $V_L$. Thus, the relation between the lobe propagation velocity $v_L$, the lobe radio power $P_r$, given that the average magnetic field strength in the radio lobe is $B$, and the lobe volume is $V_L$, for a radio spectral index $\alpha \approx 1$ is

$$v_L \propto \left( \frac{P_r(v_1) v_1}{f V_L} \right)^{1/4} n_s^{-1/4} \gamma_{\alpha}^{-1/4}.$$  

(13)

This relation between the radio power and the lobe propagation velocity is quite close to that found empirically by AL87 and LPR92. This suggests that pressure equilibrium in the lobe may be a good rough approximation, and that the ambient gas density $n_s$ does not vary widely from source to source.

The lobe propagation velocity given by equation (13) depends on the volume-averaged emissivity $\e$ since it depends on $P_r(v_1)v_1/v_1^3 e_i v_1$, where $e_i$ is the efficiency coefficient for synchrotron emission $e_i \equiv dE_i/dE_e/dv_e$ (e.g., Mofett 1975) averaged over the radio lobe. The radio lobe is taken to have a cross-sectional area $n_0 a_0^2$ where $a_0$ is the radius of the radio lobe measured perpendicular to the core-hotspot line; the radio lobe may be taken to have cylindrical or spherical symmetry.

Not all powerful double radio sources have a radio spectral index close to unity. For $\alpha > 0.5$ where $\alpha$ applies to the radio spectral index at the emitted frequency $v_1$, which is related to the observed frequency $v_0$ by $v_1 = v_0(1 + z)$, equation (12) implies

$$P_r(v_1) \propto u_b^{(3 + \gamma_1)/2} \gamma_{\alpha}^{2 - 1} v_1^{(3 + \gamma_1)/2 - 1},$$  

(14)
for \( \alpha \approx 1 \); if the volume of the lobe is \( \propto a_2^3 \) then \( L_r \propto a_2^{3/2} \) and if the lobe volume is \( \propto a_2^3 \) then \( L_r \propto a_2^{3/2} \).

Equations (3a) and (15) can be used to eliminate \( n_s \) and obtain an expression for the luminosity in directed kinetic energy \( L_j \) as a function of the radio power, the lobe propagation velocity, and the radius of the radio lobe \( L_j (P_r, v_L, a_L) \), independent of \( n_s \). For \( \alpha > 0.5 \)

\[
L_j \propto \frac{P_f(v_f)\nu_f^2}{f v_L} \nu_L a_L^2 \gamma_{eo}(2-2\alpha)(3+\alpha) .
\]

Thus, a characteristic value of the luminosity in directed kinetic energy \( L_j \) for a source can be estimated from observations of \( v_{Lj}, a_L \), and \( P_r \). For \( \alpha \approx 1 \) and \( \gamma_{eo} \) roughly constant from source to source, \( L_j \approx v_{Lj} a_L^2 \left( L_r f V_f \right)^{1/2} \), where \( L_r \approx P(v_f) v_f \); this may also be written \( L_j \propto v_{Lj} a_L^2 B^2 \).

A similar relation between the luminosity in directed kinetic energy and the radio power results from another argument. If some substantial fraction, say half or so, of the luminosity in directed kinetic energy goes into relativistic electrons and magnetic fields, the luminosity in directed kinetic energy is related to the total energy in relativistic electrons \( E_{\text{tot}} \) and the time over which the energy is input to the radio-emitting plasma: \( L_j \sim E_{\text{tot}}/t_s \), where \( t_s \) is the characteristic time for which the source with luminosity in directed kinetic energy \( L_j \) is active. Now, the characteristic or median size of a source is \( L_s \approx v_s t_s \) and the total energy in the relativistic electrons is \( E_{\text{tot}} \sim u_B V \), where \( u_B \) is the magnetic energy density and \( V \) is the radio-emitting volume, \( V = f V_f \); thus \( L_j \sim u_B V v_{Lj}/u_B \). The characteristic or median size of powerful 3 CR galaxies is roughly constant, and for \( \alpha \approx 1 \) it is shown in § 3.2. That \( L_j = P(v_f) v_f = u_B^2 V \), so \( L_j \propto \left( L_r f V_f \right)^{0.5} v_f^2 V_f^2 \); this is identical to equation (18) for a lobe volume \( v_L \propto a_L^2 \).

4. DETERMINING THE DE/ACCELERATION PARAMETER

The de/acceleration parameter \( q_0 \) could be estimated if an intrinsic length scale of distant radio sources could be estimated. The characteristic or median sizes of powerful, highly supersonic radio sources can be estimated using the model presented in § 3. The sources used to determine the de/acceleration parameter should have negligible backflows and thus should have radio powers \( P_{178}(q_0 = 0) \sim h^{-2} \times 10^{27} \) W Hz\(^{-1}\) sr\(^{-1}\) as discussed in § 2.

The characteristic or median size of radio sources with a given lobe propagation velocity is given by equation (9): \( L_s \propto k L_r^{1/\alpha} \propto k^{1/2} \gamma_{eo}^{-1} \) where \( k \propto (v_L B^{-1} a_1^{-2})^{1/2} \) (see eq. [7]). The \( q_0 \) dependence of \( k \) is considered, and the value of \( k \) determined empirically; this is followed by a discussion of \( \beta \) for radio galaxies and radio-quiet quasars.

The three quantities \( v_L, B, \) and \( a_1 \) on the right-hand side of equation (7) can be estimated from observations. The lobe propagation velocity can be estimated from the synchrotron aging of relativistic electrons in the radio lobe and bridge \( v_L \sim AD/\Delta t \) where \( AD \) is the length of the region over which the synchrotron age of the relativistic electrons has changed by an amount \( \Delta t \). The time interval \( \Delta t \) can be estimated from the change of the synchrotron spectrum (AL87; Myers & Spangler 1985; LMS89; LPR92). For a given break frequency, \( \Delta t \propto B^{-3/2} \) when the energy density of the magnetic field is greater than or equal to that of the microwave background radiation at the redshift of the source, as is the case for most high-redshift powerful radio sources (LMS89).

The data set presented and discussed by LMS89 can be used to estimate \( k_s \) and \( k_e \); the following parameterization is useful.
for the application of the data presented by LMS89. The size \( \Delta D \) over which the synchrotron aging can be estimated is some fraction \( f_D \) of the total lobe-lobes separation \( D: \Delta D = f_D D \); thus, equation (7) may be written

\[
k_{p,s} \propto \left( \frac{f_D D}{a_s B \Delta t} \right)^{2/3}.
\]

A subscript \( g \) means that the parameter is applied to radio galaxies, and a subscript \( s \) indicates that the parameter is applied to radio-loud quasars. The ratio of the lobe-lobes separation to the full width of the radio lobe \( D/2a_s \) can be approximated by the ratio of the radio source size \( D \) to the width of the bridge, since the bridge width is typically roughly comparable to the width of the radio lobe; this axial ratio is denoted by LMS89 as \( R_T \).

The parameters \( k_p \) and \( k_s \) estimated in this way are very weakly dependent on the de/acceleration parameter \( q_0 \), since \( k_p \) and \( k_s \) are nearly independent of the comoving coordinate distance \( (a_0 r) \) to a source. To obtain the dependence of \( k_{p,s} \) on the coordinate distance \( r \) note that the axial ratio \( D/a_s \) is dimensionless and independent of \( (a_0 r) \); so \( k_{p,s} \propto B^{1/3} \). When the magnetic field strength can be estimated directly by comparing X-ray and radio observations \( k_{p,s} \) is independent of \( (a_0 r) \). When \( B \) is estimated using the minimum energy magnetic field, \( B_{\text{min}} \propto (a_0 r)^{-2/7} \) or \( B_{\text{min}} \propto (a_0 r)^{-1/3} \) depending on how the observations are used to estimate \( B_{\text{min}} \) (see eqs. [16] and [17]). In either case the dependence of \( k_{p,s} \) on \( (a_0 r) \) is extremely weak: \( k_{p,s} \propto (a_0 r)^{-0.1} \).

The information published by LMS89 allows an estimate of \( k_{p,s} \). This parameter has been estimated for radio galaxies and radio-loud quasars using equation (19); the two radio-loud quasars that technically are below the power cut of \( P_{\text{1.5}}(q_0 = 0) \sim h^{-5} \times 10^{37} \) W Hz\(^{-1}\) sr\(^{-1}\) have been included in the analysis. The fraction of the source \( f_D \) over which the synchrotron age listed by LMS89 applies was taken to be the fraction of the length of the bridge over which a radio surface brightness is measured. The average value of the timescale \( t_\text{eq} \) was used, and the axial ratio \( R_T \), which is a measure of the bridge length to the bridge width, was used as an estimate of the ratio \( D/2a_s \).

Figure 1 illustrates the redshift dependence of \( k_p \) and \( k_s \) for the sources listed by LMS89 for radio galaxies with resolved axial ratios (squares), for radio galaxies with lower bounds on axial ratios (diamonds), for radio-loud quasars (open circles), and for radio-loud quasars below the power threshold discussed above (crosses). All of the radio-loud quasars have resolved axial ratios.

The error bars have been estimated from the uncertainties of the observed quantities that go into the determination of \( k \). Ten percent uncertainty has been assumed for the fraction \( f_D \) of the source over which the synchrotron aging has been estimated and twenty percent uncertainty has been assumed for the radius of the radio lobe \( a_s \), the axial ratio \( R_T \), and the break frequency \( v_B \) (see LMS89), which enters because the synchrotron aging time \( \Delta t \) is \( \propto (v_B)^{-5/2} \). Zero uncertainty have been assumed for the magnetic field strength \( B \), and for the linear size, or lobe-lobes separation, \( D \). Combining these error estimates, the fractional uncertainty of the parameter \( k \) is estimated to be about \( \delta k/k = \delta \log k \approx 0.15 \); this estimated error for each point is illustrated on Figure 1. The uncertainty of the lobe propagation velocity is about 15%, and that of the luminosity in directed kinetic energy is about 40%.

The mean value and standard deviation of the mean for the radio galaxies is \( k_p \approx 0.6 \pm 0.04 \) when the lower bounds for the galaxies with unresolved axial ratios are included as detections (10 galaxies are included in the estimate) and is \( k_s \approx 0.5 \pm 0.06 \) when the five galaxies with resolved axial ratios are used to estimate \( k_p \). The mean value for the radio-loud quasars is \( k_p \approx 0.3 \pm 0.03 \); all six of the radio-loud quasars have resolved bridge widths, all have measured axial ratios, and all are included in the estimate. Note that the two quasars from LMS89 that technically are below the power cut of \( h^{-5} \times 10^{37} \) W Hz\(^{-1}\) sr\(^{-1}\) have been included here because the number of quasars is small.

It is interesting to note that the different values of \( k_p \) for radio galaxies and \( k_s \) for radio-loud quasars indicated by Figure 1 are consistent with the results of Singal (1988). Figures 7 and 9 for Singal (1988) show that radio-loud quasars with redshifts from about 0.5 to 0.8 are about a factor of 2 smaller than radio galaxies with similar redshifts and radio powers. The quasar 3C 68.1 at a redshift of about 1.2 tends to lie with the radio galaxies on most figures (see also Daly 1994), and may be a transition case between galaxies and quasars.

The quantities \( k_p \) and \( k_s \) are related to the mass per unit radius \( dM/dR \) swept up by the forward shock front, taken here to be the radio lobe (Daly 1990, eq. [2]): \( dM/dR \propto k_p^{-2} \). The fact that the different sizes of radio galaxies and radio-loud quasars can be accounted for by the difference between \( k_p \) and \( k_s \) may imply that \( \beta_p \sim \beta_p \) but observations of \( k_p \) for many more radio-loud quasars over a broad range of redshifts is required before \( \beta_p \) can be estimated directly.

The number of data points on Figure 1 is small for both the radio galaxies and the radio-loud quasars. If \( k_p \) remains independent of redshift as more points are added to Figure 1, then
we must conclude that the mass per unit radius shock-heated by the forward shock front is not evolving with redshift. When the cross-sectional area of the radio lobe is roughly constant over the lifetime of the source, this is expected when the ambient medium has a constant density. When the cross-sectional area of the radio lobe increases as the distance from the radio core squared, as would be expected if the outflow is over a constant nonzero opening angle, then $k_{\gamma_p}$ will be constant when the density of the ambient medium falls off as the square of the distance from the AGN. Perhaps more importantly, the small scatter of $k_{\gamma_p}$ at a given redshift implies that the media with which the collimated outflows interact are fairly uniform, and the small scatter of $k_\delta$ with redshift suggests that the media (at least for these radio galaxies) do not evolve strongly with redshift; however, many more sources must be investigated before this statement is conclusive. Thus, the redshift dependence of $k_\delta$ and $k_\beta$ and the scatter of these parameters at a given redshift have important implications for the environments of the radio sources and for the evolution of these environments; these implications are addressed in detail by Daly (1994).

There is no evidence that $k_\delta$ is decreasing with redshift, as might be expected if $q_0 = 0.5$: recall from § 2 that the observed median radio source sizes estimated from their angular sizes imply that the intrinsic source size $l_0$ of radio galaxies should decrease by about a factor of 2 over the redshift interval from about $z = 0.5$ to $z = 1$ if $q_0 = 0.5$ (Kapahi 1989; Singal 1988). However, the number of sources plotted on Figure 1 is quite small and many more sources must be plotted before a definitive statement can be made concerning the redshift dependence of $k_\delta$. The intrinsic source sizes depend not only upon $k_\delta$, but also upon $v_\delta(z)$ and $\beta$. The present results for radio galaxies suggest that $v_\delta$ is independent of redshift, so the characteristic source sizes remain roughly constant and independent of redshift for any $\beta$.

The sources investigated by LMS89 were chosen to have large angular sizes, and thus are likely to represent the outer envelope of the distribution of source sizes. At low redshift the envelope or maximum angular size of the sources tracks the median sizes of the sources fairly well, and this trend appears to continue to redshifts of about two for 3 CR sources; a careful study of the behavior of the maximum and median angular sizes of CR galaxies with redshift is currently underway. The ratio $R_\gamma$ will be affected by projection effects since $R_\gamma = D/(2a_\delta)$ and the apparent lobe-lobe separation $D$ will be affected by projection effects; $D = D_{true} \cos \alpha$, where $\alpha$ is the angle between the plane of the sky and the radio lobe axis. It turns out that $l_0 = l_{true}(\cos \alpha)^{4/7}$ independent of $\beta$. The sources with the largest angular sizes are likely to lie in the plane of the sky and thus are probably not strongly affected by projection effects. If the sources are randomly oriented relative to the observer, the projection effects should be the same at all redshifts, and the mean or median value of $k$ should not be seriously affected by projection effects. Note that powerful extended radio sources are unlikely to be significantly affected by Doppler beaming or boosting of radio emission since the relevant flow velocities are small compared with the speed of light.

The characteristic radio source sizes for radio galaxies is given by $l_0 \propto k_\delta L_{343}^{0.33(1-\beta)}$, which may also be written $l_0 \propto k_{\beta}^{4/7(1-\beta)}$. The lobe propagation velocity as a function of redshift is shown in Figure 2; the lobe propagation velocity is independent of the axial ratio $R_\gamma$, so all points represent detections, and the symbols are identical to those in Figure 1. The lobe propagation velocity as a function of redshift is best fit by a line with zero slope (Daly 1994), and value of $v_\gamma/c$ of $\sim 0.06$; the 15% error bars have been estimated as described above. Thus, the present data are consistent with a lobe propagation velocity that is independent of redshift. For the radio galaxies in the present data set, this implies $l_\delta \propto 1$ from a redshift of zero to a redshift greater than one, and this statement is independent of the value of $\beta_\delta$. For the construction of Figure 2, a value of $q_0 = 0$ has been assumed, and $v_\gamma \propto (a_\delta r)^{4/7}$. The sources have been normalized to Cygnus A, and the lobe propagation velocity of Cygnus A is estimated to be about 0.6e assuming a Hubble constant of 75 km s$^{-1}$ Mpc$^{-1}$ (Carilli et al. 1991).

The value of $\beta_\delta$ is important for estimating the dependence of $l_\delta$ on the coordinate distance $(a_\delta r)$ to the source. For the radio galaxies in the LMS89 data set, $k_\delta \propto \beta_\delta^{\gamma}$ (Daly 1994), so $l_\delta \propto k_{\delta}^{4/7(1-\beta_\delta)} \propto v_\delta^{-2(1-\beta_\delta)}$. The characteristic or median sizes of radio galaxies at different redshifts appears to be independent of redshift, and that at a given redshift appears to be independent of radio power (Lacy et al. 1993). In order to be able to estimate $\beta_\delta$, some assumptions must be adopted. Let us assume that the relation between $k_\delta$ and $v_\delta$ given above holds for radio galaxies in general; the fact that the characteristic or median source sizes are independent of radio power suggests that they are also independent of lobe propagation velocity (note that the radio galaxies in the LMS89 data set show no dependence of $v_\delta$ on the radio lobe separations; Daly 1994). If $l_\delta$ is independent of $v_\delta$, then $\beta_\delta \propto 1.5 \pm \sigma_{\beta_\delta}$, where $\sigma_{\beta_\delta}$ indicates the allowed range of values of $\beta_\delta$. The allowed range of $\beta_\delta$ can be estimated by considering radio galaxies with redshifts less than $\sim 0.5$ and comparing the scatter about the median source sizes with the range of lobe propagation velocities. In order that the scatter of $l_\delta$ not exceed the observed value we require $\beta_\delta = 1.5 \pm \sigma_{\beta_\delta}$, where $\sigma_{\beta_\delta} \approx 1.5 \delta \log l_\delta/\delta \log v_\delta$. For sources with redshifts less than $\sim 0.5$, $\delta \log l_\delta \approx 0.07$ (Kapahi 1989), which we will take to be $\pm 0.1$ to be conservative, and $\delta \log v_\delta \approx 0.5$ (AL87). For comparison, the radio galaxies included in LMS89 data set and used here have a range of about 0.5 in $\log v_\delta$. Using $\delta \log l_\delta \approx 0.1$ and $\delta \log v_\delta \approx 0.3$ to be conservative, $\sigma_{\beta_\delta} \approx 0.5$, so $\beta_\delta \approx 1.5 \pm 0.5$.

For a value of $\beta_\delta = 1.5$, the characteristic source size estimated using the model described here is $l_\delta \propto (a_\delta r)^{3/7}$ since $l_\delta \propto (a_\delta r)^{4/7-2(1-\beta_\delta)}$. This follows since the synchrotron aging timescale $t$ depends on the magnetic field strength $B$, $t \propto B^{3/2}$ when the magnetic energy density is comparable to or larger than that of the microwave background radiation at the redshift of the source, as is the case for all of the sources investigated by LMS89, and the minimum energy magnetic field varies as $B \propto (a_\delta r)^{-7/2}$. Thus, for $\beta \geq 6/7$, $l_\delta$ estimated using the model described in § 3 is independent of or inversely proportional to the coordinate distance $(a_\delta r)$ to the source. The median source size estimated from the angular sizes of the sources is proportional to $a_\delta r$ since $D = \theta(a_\delta r)(1+z)$, where a source at redshift $z$ has an angular extent $\theta$. Thus, the ratio of the characteristic source sizes estimated from their angular sizes to the model source sizes is at least proportional to $a_\delta r$, and may be as strong as $(a_\delta r)^{1.5}$. It is this fact that allows the method presented here to provide a means of estimating the de/acceleration parameter $q_0$, since $a_\delta r$ depends on $q_0$.

The present data set strongly favors a value of $q_0 \leq 0$ since both $k_\delta$ and $v_\delta$ for radio galaxies are constant out to redshifts of
COSMOLOGY WITH RADIO SOURCES

\[ \log(\nu/c) \] vs. \( \log(1+z) \)

Fig. 2—log-log plot of the lobe propagation velocity as a function of \((1+z)\). Symbols are identical to those in Fig. 1. The lobe propagation velocity is independent of the axial ratio.

\(~1.5\). Note that a negative value of \(q_0\) implies a cosmological constant \(\Lambda_0\) (see Peebles 1993, eq. [13.71]). The lobe propagation velocities at high redshift would decrease if the data shown in Figure 2 were plotted assuming \(q_0 = 0.5\) rather than \(q_0 = 0\), but this would indicate that the characteristic source sizes estimated using the model increase with redshift for \(\beta_\nu \gtrsim 0.9\), while the median source sizes estimate from the angular sizes assuming \(q_0 = 0.5\) decrease with redshift for \(z \gtrsim 0.5\).

A value of \(q_0 = 0.5\) is not ruled out since the radio galaxies in the LMS89 data set may not be representative. These sources were selected to be of large angular extent, and the redshift behavior of this outer envelope relative to the median source sizes is at present under investigation. In addition, numerous other radio sources are being gathered and will be analyzed to increase the number of intrinsic source sizes estimated using the model described here to estimate \(q_0\).

It is interesting to note that for radio-loud quasars either \(k_\nu\) decreases with increasing redshift or \(v_\nu\) increases with redshift and \(\beta_\nu > 1\). The radio luminosity \(L_\nu\) and redshift are constant. If \(\beta_\nu > 1\), the radio-loud quasars will provide an important estimate of \(q_0\), since the predicted lobe-source size depends on \((a_\nu r)^{-1/2}\), where the dependence on \(B\) is ignored since it is relatively small, while the lobe-source size determined from the observed angular size depends on \((a_\nu r)^2\).

A change of the characteristic or median radio source sizes is expected if there are changes in the ambient medium or the radio lobe radius, which will be reflected in the redshift dependence of the parameters \(k_\nu\) and \(k_s\). The source sizes could also change if the average lobe propagation velocity changes as a function of redshift and the characteristic time for which a source is active \(t_* \propto L_\nu^{-1/2}\) is parameterized by a \(\beta_\nu\) value that is not exactly equal to 1. The quantity \(\beta_\nu\) may be estimated at redshifts \(z \lesssim 0.5\), where the effects of an assumed value of \(q_0\) are small, and extrapolated to high redshifts. The present data suggest \(\beta_\nu \approx 1.5 \pm 0.5\) and \(k_s \approx k_\nu\) constant for radio galaxies, though these numbers are preliminary. If the trend shown by newly analyzed sources indicates that \(v_\nu\) increases with redshift while \(k_\nu\) is constant, then the data could be consistent with \(q_0 = \frac{1}{3}\).

The direction of the collimated outflow from radio-loud quasars may change with time, as indicated by the data and as discussed in more detail in § 2.2. The use of radio-loud quasars to substantiate the result obtained using radio galaxies is addressed in § 4.1 and 5.

The reason that the characteristic or median lobe-lobe sizes of radio galaxies is independent of the radio power and apparently independent of redshift is because \(k_\nu\) and \(v_\nu\) are independent of redshift. The fact that there is no sign of a decrease of \(k_\nu\) at redshifts greater than \(~0.6 \pm 1\) suggests that \(q_0 \approx 0\) since a decrease of the intrinsic radio source sizes is expected over the redshift interval from about 0.6 to 1 if \(q_0 = 0.5\), which is not seen in Figure 1 for radio galaxies. However, the number of galaxies analyzed is small and \(q_0 = 0.5\) may be consistent with larger data sets.

The typical velocities of lobe propagation reported to date for radio sources are \(\lesssim 0.1c\). When the velocity of lobe propagation exceeds about 0.1c corrections to the observed lobe separations should be applied due to fact that the observed radio emission from each lobe is not emitted simultaneously in the galaxy frame (Leahy 1991); Doppler boosting and beaming of the radio emission is unlikely to be important for these sources since the typical lobe propagation velocities are usually \(\lesssim 0.1c\), thus \(v_\nu \ll c\). In the future, it would extend the data sets and avoid problems of simultaneity if core-lobe separations rather than lobe-lobe separations are considered, and the \(v_\nu(P)\) or \(v_\nu(P/V)\) relation determined for each lobe separately, as done by AL87.

It is interesting to note that if the lobe propagation velocity reaches a maximum value and then remains constant a comparison of the velocity of lobe propagation with the angular separation of the lobes could indicate a value of \(q_0\). This, however, would require that \(v_\nu \approx \text{constant}\) over a significant redshift interval, since both \(L_\nu\) estimated from the median angular size of radio sources and \(v_\nu\) are \(\propto C x; v_\nu\) is determined via \(v_\nu \propto \Delta x/\Delta t \propto C x B^{-1.5}\) for a given break frequency. Thus \(v_\nu\) scales as \(C x\) when the magnetic field strength can be estimated by a comparison of radio and X-ray observations.

Another potentially useful constraint arises from the luminosity in directed kinetic energy \(L_k\). If there is a maximum value for the rate of energy extraction from the black hole, the envelope of the distribution of \(L_k\) with redshift might be a useful distance indicator. Dimensionally, \(L_k \propto \rho \nu_\nu^{2.5} \propto (a_\nu r)^2\) when the magnetic field strength is estimated using the average minimum energy value for a source. However, the upper bound on \(L_k\) could increase with redshift, and it is better to use the model described in §§ 3 and 4 to estimate \(q_0\) and use this estimate to study the evolution of \(L_k\) with redshift.

### 4.1. Radio-Loud Quasars

The characteristic core-lobe size of radio-loud quasars is given by \(r_s = r_s + t_s\), where \(t_s\) is the characteristic timescale that the collimated outflow from the AGN points in a given direction. The facts that the radio bridges of quasars are much shorter and wider than those of galaxies (LMS89), and their characteristic lobe radii are much larger than those of galaxies, as illustrated by Daly (1994), suggests that either the direction of the collimated outflow changes with time, as discussed in § 2.2, or the luminosity in directed kinetic energy is released into a fairly large solid angle. For the present, it will be assumed that the radio-loud quasars can be described by an expression that is identical to that derived for powerful extended radio galaxies: \(L_\nu \propto k_\nu L_{1.3}^{3.45} \propto k_\nu L_c^{1.45}\). The intrinsic lobe-lobe sizes of radio-loud quasars decreases by about a factor of 4 for \(q_0 = 0\) or a factor of about 10 for...
$q_0 = 0.5$ over the redshift interval from about 0.2 to 2 (Singal 1988). Thus, either $\psi(z)$ increases with redshift and $\beta > 1$ or $k_s$ decreases as the redshift increases. Note that if $\beta_s$ is greater than one, radio-loud quasars will provide a very good constraint on $q_0$; for example, if $\beta_s \approx 2$, the characteristic source size estimated using the model varies approximately as $(a_0 r)^{-1}$ while the median source sizes estimated from the observed angular sizes vary as $(a_0 r)$. More data are needed to be able to determined the relative contributions of these two possible effects that could cause the characteristic lobe-lobe sizes of radio-loud quasars to decrease with redshift so dramatically, while the characteristic lobe-lobe sizes of the radio galaxies remain roughly constant.

The parameter $\beta_s$ could be determined for the radio-loud quasars in a way that is analogous to the way $\beta_p$ is estimated. At the present time there are too few sources to determine the redshift dependence of $k_s$. The fact that the difference between the values of $k_p$ and $k_s$ on Figure 1 can explain the ratio of the median radio galaxy size to the median radio-loud quasar size for sources with similar powers and redshifts suggests that the different redshift evolution of the radio source sizes of radio galaxies and radio-loud quasars at relatively low redshifts, $z \lesssim 0.5$, is accounted for by environmental differences, that is, by the parameter $k$, and suggests that $\beta_s$ may be $\sim \beta_p$, though this remains to be determined.

5. CHECKS AND BALANCES

Two primary types of checks and balances should be considered. The first is whether the relations that form the basis of the model continue to hold at high redshift and at a given redshift as more sources are considered, and the second is whether the value of $q_0$ estimated using the model is reliable.

A key assumption that underlies the model, motivated by both high- and low-redshift observations, is that the characteristic time $t_e$ for which the central source is active and producing a highly collimated outflow that powers the radio emission has a power-law dependence on the luminosity in directed kinetic energy $L_j$: $t_e \propto L_j^{-\beta/2}$. Note that this means that the initial energy available to power the outflow $E_j$ is related to the energy extraction rate $L_j$ by $E_j \propto L_j^{-\beta/2}$; the initial energy available to power the outflow is not assumed to be constant from source to source. One concern is that the constant of proportionality relating $t_e$ and $L_j$ may evolve with redshift. This would imply that either the characteristic time $t_e$ has a non-power-law dependence on $L_j$, or that it also depends upon other variables. Clearly, it is possible that the power-law dependence of $t_e$ on $L_j$, suggested by both high- and low-redshift data, is applicable only over a particular range of $L_j$.

The reliability of the power-law relation between $t_e$ and $L_j$ can be tested by assembling a sample of sources at a given redshift with a broad range of $L_j$, or at relatively low redshifts, $z \lesssim 0.6$, with a large range of $L_j$ to test the range of $L_j$ for which the relation is valid. The source sizes can be estimated using the model and compared with the medium intrinsic sizes estimated from angular sizes for different ranges of $L_j$. Recall that at low redshifts, $z \lesssim 0.6$, the source sizes estimated from angular sizes are fairly insensitive to the assumed value of $q_0$, so the median intrinsic source sizes are known. This is essentially the exercise carried out in § 4 to estimate the range of $\psi$ allowed for powerful extended radio galaxies. In addition, the validity of the model and the range of allowed values of $\beta$ can be checked by seeking correlations between the radio power and the source size at a given redshift, as done by Lacy et al. (1993).

Another important test is whether the range and maximum value of $L_j$ vary systematically with redshift. If the characteristic or maximum value of the luminosity in directed kinetic energy varies systematically with redshift, there is a worry that other properties of the systems may vary systematically with redshift, indicating that unpredictable evolutionary effects may be important. The luminosity in directed kinetic energy $L_j$ is plotted as a function of redshift $z$ in Figure 3, where $L_j$ has been estimated using equation (8) for the data of LMS89; the symbols are identical to those in Figure 1. For the radio galaxies, the range of $L_j$ does not appear to vary systematically with redshift, and there seems to be a fairly tight upper bound on the maximum value of $L_j$; a value of $q_0 = 0$ has been assumed for the points in Figure 2. These points have been normalized using Cygnus A, assuming that for this source the ambient gas density is about $7 \times 10^{-3}$ cm$^{-3}$ at the location of the radio lobe (Arnaud et al. 1984), the lobe radius is $3.8$ kpc (LMS89), and the lobe propagation velocity is 0.06c (Carilli et al. 1991); to convert the numbers to a common scale a value of Hubble's constant of 75 km s$^{-1}$ Mpc$^{-1}$ has been adopted. Note that if the lobe propagation velocity decreases by a factor of 6, as suggested by Carilli et al. (1991), the normalization of $L_j$ decreases by a large factor, $6^3$, so the relative values of $L_j$ are much more well determined than the absolute values.

Another concern is whether the relation assumed between the ambient gas density, the minimum energy magnetic field, and the velocity of lobe propagation, given by equation (6), continues to hold for a larger sample of sources, and for sources at higher redshifts. The best test of this is to have an independent estimate of the ambient gas density. Such an estimate is very unlikely since the best way to obtain $n_e$ is through X-ray measurements of thermal bremsstrahlung emission from a hot intergalactic medium; however, both the active nucleus and the radio lobes may also be X-ray emitters, and X-ray measurements of distant radio sources usually are not well resolved.

A test of whether equation (6) is a good rough approximation for $n_e$ is the composite density profile, and whether this

![Fig. 3.—log-log plot showing the distribution of luminosities in directed kinetic energy $L_j$ estimated using eq. (8) as a function of $(1 + z)$ for the data set of LMS89; the symbols are identical to those in Fig. 1. The error bars for each point have been estimated from the uncertainties of the observables that go into the construction of the value of $L_j$; see § 5 of the text for details. Note that $L_j$ is proportional to the square of the axial ratio.](image-url)
profile continues to exhibit an $\sim r^{-1.5}$ dependence on the core-lobe separation indicating that the sources are in similar environments and that the gas in the vicinity of the radio source has an $\sim r^{-1.5}$ density profile, characteristic of gas in clusters of galaxies. The similarity of the environments, even over a broad range of redshifts, is indicated by the independence of $n_e$ on redshift, where $n_e$ is estimated using equation (6), and by the lack of large variations in $n_e$, as discussed by Daly (1994).

The fact that the ambient gas density is quite similar from source to source suggests that secondary relations will not pose difficulties. For example, if the AGN fueling rate is related to the ambient gas density in the vicinity of the radio lobe, strong evolution of the density of the ambient medium with redshift would be cause for concern. The lack of any such evolution implies that the relation assumed between the time for which the source is active $t_*$ and the luminosity in directed kinetic energy $L_*$ will not be subject to secondary effects associated with the evolution of the ambient gas with redshift.

How can the validity of the value of $q_0$ estimated using the model and method described in §§ 3 and 4 be tested? Each of the values of $q_0$ estimated separately for galaxies and quasars should remain constant out to large redshifts. Radio sources are detected at significant redshifts, and the dependence of the median source sizes estimated from angular source sizes on $q_0$ becomes stronger with redshift. If the model can be used to estimate $q_0$, reliably, a single value of $q_0$ should be indicated by the application of the model, independent of the redshift range of the sources. In addition, radio galaxies and radio-loud quasars each provide an independent estimate of $q_0$, and each is based upon the same model. The values estimated using each type of source can be compared to check for consistency.

6. DISCUSSION

Distant powerful radio sources would be a useful cosmological tool if the evolution of the sources could be accounted for, or if an intrinsic length-scale could be identified. A simple model for the propagation of the radio lobe into the ambient medium is investigated here. This model assumes ram pressure confinement of the radio lobe; that is, the strong shock jump conditions apply. Unless the magnetic field strength of the radio lobe can be estimated from a combination of X-ray and radio observations, the minimum energy magnetic field must be used as an estimate of the lobe magnetic field, which is equivalent to assuming rough pressure equilibrium between the magnetic field and relativistic electrons. These assumptions lead to a relation between the radio power and the lobe propagation velocity that is close to that determined empirically, indicating that ram pressure confinement of the lobe is a good rough approximation and that the minimum energy magnetic field is a good rough approximation to the magnetic field strength in the radio lobes of very powerful radio sources. These assumptions are also supported by the composite density profile: the density estimated using equation (6) plotted against radio lobe separation indicates that the gas in the vicinity of the radio lobes has an $\sim r^{-1.5}$ density profile (see Daly 1994).

The characteristic or median radio source size, that is, the core-lobe or lobe-lobe separations, of sources with a particular radio power may be written $l_* = v_t_1 t_*$, where $v_t_1$ is the average lobe propagation velocity and $t_*$ is the characteristic time for which the AGN produces a significant luminosity in directed kinetic energy. Thus, the characteristic radio source size may be written as the product of two functions: $l_* \propto k L_1^{1/3(1-p)} \propto k^{1/2} t_*^{-1/2}$. The first term, $k$, depends on extrinsic variables and represents the effect of the environment of the radio source size, and the second term, $L_1^{1/3(1-p)}$, depends on the intrinsic variables and represents the effect of the luminosity in directed kinetic energy released by the AGN on the radio source size.

The first term may be estimated assuming that the radio lobe is ram-pressure–confined by the ambient medium so $n_e \propto (B v_t_1^2)/(\eta_1$). This determination of $k \propto (n_e_0 a_0^{-1})^{-1/2} \propto (\eta_1 B_1^{-1} a_1^{-1})^{1/2}$ is largely independent of the deceleration parameter $q_0$. Once the redshift and velocity dependence of $k$ is known, the parameter $\beta$ can be estimated. $\beta$ can be estimated by comparing the predicted and observed median radio source sizes at redshifts less than about 0.5 where the effects of the chosen value of $q_0$ are minimal.

There is a published data that lists the parameters necessary to be able to estimate $k_*$ for radio galaxies and $k_*$ for radio-loud quasars; these are the data of LMS89. Other published data sets will be analyzed and used to estimate these parameters. The mean values and the standard deviation from the mean of $k_*$ and $k_*$ estimated from the LMS89 data set are $k_* \approx 0.6 \pm 0.04$ and $k_* \approx 0.3 \pm 0.03$. The scatter about $k_*$ and $k_*$ at a given redshift appears to be fairly small, though many more sources must be investigated to confirm this trend. The radio-loud quasars in the LMS89 sample are drawn from a small range of redshifts; thus no statement about the redshift dependence of $k_*$ can be made. The radio galaxies have a large range of redshifts, and the present data suggest that $k_*$ is independent of redshift. The sources observed by LMS89 were chosen to be those with large angular sizes and thus are likely to be drawn from the outer envelope of the radio source size distribution.

Once the redshift and velocity dependence of $k$ is known, the parameter $\beta$ can be estimated from radio sources with redshifts less than about 0.5; that is, the dependence of $k(z)$ and $k(v_1)$ must be determined before the parameter $\beta$ can be estimated. It appears that $k_*$ is roughly independent of redshift, and that $\beta_1 \approx 1.5 \pm 0.5$; however, many more radio galaxies must be plotted on Figure 1 so that the redshift dependence of $k_*$ and scatter at a given redshift can be evaluated. A consistency check follows from the comparison of the predicted and observed ratio of the radio galaxy sizes to the radio-loud quasar sizes as a function of redshift.

The results presented in § 4 favor a low value of $q_0$. However, the LMS89 sources may not be representative and larger data sets must be analyzed before $q_0$ can be determined. Thus, the results may be consistent with a flat matter-dominated universe, but at present favor a universe with significant space curvature or cosmological constant at the present epoch. The constraints on $q_0$ will become stronger as more data are analyzed.

In order to be able to estimate $\beta_1$ for radio-loud quasars, the redshift and velocity dependence of $k_*$ must be determined. It is interesting to note that, for radio sources corresponding to the radio powers of the sources on Figure 1, the difference of the median radio source sizes between radio galaxies and radio-loud quasars reported by Singal (1988) can be explained by the different values of $k_*$ and $k_*$, indicated on Figure 1. This suggests that $\beta_1$ may be $\sim \beta_1 \sim 1.5$, and that the different redshift behavior of the median sizes of radio galaxies and radio-loud quasars (discussed in § 2.2) may be due to environmental effects, that is, due to their different values of $k$. However, sources with a broad range of redshifts must be evaluated and
the velocity dependence of $k_\alpha$ estimated before a value of $\beta_\gamma$ can be surmised.

The determinations of $k_\alpha$ and $k_\beta$ have important implications for the environments of the radio sources, as discussed at length by Daly (1994). The parameter $k_\beta$ depends both on the average density and density profile of the gas in the vicinity of the radio source and on whether the collimated outflow has a fairly small opening angle (close to zero), or a larger, constant opening angle. The dispersion of these parameters at a given redshift is related to both the dispersion of the gas density of the environments at that redshift and the dispersion of the opening angles. Similarly, the evolution of $k_\alpha$ and $k_\beta$ with redshift has implications for the evolution of the sources and of their environments. For example, the fact that the lobe radii are larger for radio-loud quasars than for radio galaxies may suggest that the collimated outflow in radio-loud quasars changes direction with time (LMS89), or is ejected into a large solid angle.

This could be understood if the quasars have a massive accretion disk that is the source of the radiation from the quasar, and that causes the spin axis of the black hole to precess, while radio galaxies may not have a massive accretion disk and thus have outflows that are fairly stable with time, and have low radiant luminosities associated with their AGN. The determination of the parameter $\beta_\gamma$ will place important constraints on models to extract the spin energy of the black hole, or otherwise power the luminosity in directed kinetic energy, since $L_\gamma \propto \epsilon_1^{13-\beta}$ (see § 3.1), suggesting that $L_\gamma \propto \epsilon_1^2$ for radio galaxies, where $\epsilon_1$ is the initial energy available to power the collimated outflow.

Thus, the determination of the parameters $k$ and $\beta$ will place important constraints on the unified model suggested by Daly (1992b) and currently under investigation. In this model the luminosity in directed kinetic energy is related to the extraction of the spin energy of the central compact object, and the radiative energy (observed as a quasar) is powered by the gravitational binding energy of matter falling onto the black hole.

It is a pleasure to thank Dave De Young, Paddy Leahy, Simon Lilly, Ed Groth, Alan Marscher, Jerry Ostriker, John Peacock, Jim Peebles, Rick Perley, Saul Perlmutter, Suzanne Staggs, Lin Wan, Dave Wilkinson, and Ned Wright for helpful discussions. This work was supported in part by the US National Science Foundation.

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