Direct Constraints on the Properties and Evolution of Dark Energy

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Abstract.

We describe a method to derive the expansion and acceleration rates directly from the data, without the need for the specification of a theory of gravity, and without adopting an a priori parameterization of the form or redshift evolution of the dark energy. If one also specifies a theory of gravity we can also determine the pressure, energy density, and equation of state of the dark energy as functions of redshift. We then apply this methodology on a modern data set of distances to Supernovae and to radio galaxies. We find that the universe transitions from deceleration to acceleration at a redshift of $z_T \approx 0.4$, and the present value of deceleration parameter is $q_0 = -0.35 \pm 0.15$. The standard "concordance model" with $\Omega_0 = 0.3$ and $\Lambda = 0.7$ provides a reasonably good fit to the dimensionless expansion rate as a function of redshift, though it fits the dimensionless acceleration rate as a function of redshift less well. Adopting General Relativity as the theory of gravity, we obtain the redshift trends for the pressure, energy density, and equation of state of the dark energy out to $z \sim 1$. They are generally consistent with the concordance model, at least out to $z \sim 0.5$, but the existing data preclude any stronger conclusions at this point. For the present values of these quantities we obtain $p_0 = -0.6 \pm 0.15$, $f_0 = 0.62 \pm 0.05$, and $w_0 = -0.9 \pm 0.1$. Application of this methodology to richer data sets in the future may provide valuable new insights into the physical nature and evolotion of the dark energy.

1. Introduction

One way to determine the expansion and acceleration rates of the universe as functions of redshift is through studies of the coordinate distance to sources at different redshift. This has been accomplished with a variety of techniques including the use of type Ia supernovae (e.g. Riess et al. 1998; Perlmutter et al. 1999; Tonry et al. 2003; Knop et al. 2003; Barris et al. 2004; Riess et al. 2004) and powerful classical double radio galaxies (e.g. Daly 1994; Guerra & Daly 1998; Guerra, Daly, & Wan 2000; Podariu et al. 2003).

Luminosity or coordinate distances can then be used to study the expansion history of the universe and the properties of the dark energy in a variety of ways. The analysis techniques fall into two broad categories: the integral ap-

proach and the differential approach. The former, traditional approach involves the integration of a theoretically predicted expansion rate over redshift to obtain predicted coordinate distances to different redshifts; the difference between these predicted coordinate distances and the observed coordinate distances is then minimized to obtain the best fit model parameters. This approach usually requires the specification of a theory of gravity (generally taken to be General Relativity; GR) and a parameterization of the redshift evolution of the dark energy. Maor, Brustein, & Steinhardt (2001) and Barger & Marfatia (2001) discuss the difficulties involved with the use of this method to extract the properties and redshift behavior of the dark energy. Different approaches have been developed to extract the redshift behavior of the dark energy using the integral method (e.g. Starobinsky 1998; Huterer & Turner 1999, 2001; Saini et al. 2000; Chiba & Nakamura 2000; Maor, Brustein, & Steinhardt 2001; Golaith et al. 2001; Wang & Garnavich 2001; Astier 2001; Gerke & Efstathiou 2002; Weller & Albrecht 2002; Padmanabhan & Choudhury 2002; Tegmark 2002; Daly & Guerra 2002; Huterer & Starkman 2003; Sahni et al. 2003; Alam et al. 2003; Podariu et al. 2003; Wang & Freese 2004; Wang et al. 2004; Wang & Tegmark; Nessier & Perivolaropoulos 2004; Gong 2004a,b; Zhu, Fujimoto, & He 2004; Elgaroy & Multamaki 2004; Huterer & Cooray 2004; Alam, Sahni, & Starobinsky 2004).

2. The Methodology

The differential approach has been investigated by Daly & Djorgovski (2003, 2004a), and is briefly summarized here. It is well known (e.g. Weinberg 1972; Peebles 1993; Peebles & Ratra 2003) that the dimensionless expansion rate $E(z)$ can be written as the derivative of the dimensionless coordinate distances $y(z)$; the dimensionless coordinate distance $y(z)$ is simply related to the coordinate distance $(a_0 r)$ through the equation $y = (H_0/c)(a_0 r)$, where c is the speed of light and H_0 is Hubble's constant. The expression for $E(z)$ is particularly simple when the space curvature term is equal to zero. In this case,

$$\left(\frac{\dot{a}}{a}\right) \; H_0^{-1} \equiv E(z) = (dy/dz)^{-1} \; , \tag{1}$$

where a is the cosmic scale factor. This representation follows directly from the Friedman-Robertson-Walker line element, and does not require the specification of a theory of gravity. Similarly, in a spatially flat universe (which is supported by CMBR measurements, Spergel et al. 2003), it is shown in Daly & Djorgovski (2003) that the dimensionless deceleration parameter

$$-\left(\frac{\ddot{a}a}{\dot{a}^2}\right) \equiv q(z) = -[1 + (1+z)(dy/dz)^{-1} \; d^2y/dz^2] \tag{2}$$

also follows directly from the FRW line element, and is independent of any assumptions regarding the dark energy or a theory of gravity. Thus, measurements of the dimensionless coordinate distance to sources at different redshifts can be used to determine dy/dz and d^2y/dz^2, which can then be used to determine $E(z)$ and $q(z)$, and these direct measures are completely model-independent, as discussed by Daly & Djorgovski (2003).

In addition, if a theory of gravity is specified, the measurements of dy/dz and d^2y/dz^2 can be used to determine the pressure, energy density, and equation of state of the dark energy as functions of redshift (Daly & Djorgovski 2004a,b); we assume the standard GR for this study. *These determinations are completely independent of any assumptions regarding the form or properties of the dark energy or its redshift evolution.* Thus, we can use the data to determine these functions directly, which provides an approach that is complementary to the standard one of assuming a physical model, and then fitting the parameters of the chosen function.

In a spatially flat, homogeneous, isotropic universe with non-relativistic matter and dark energy Einstein's equations are $(\ddot{a}/a) = -(4\pi G/3)\ (\rho_m + \rho_{DE} + 3P_{DE})$ and $(\dot{a}/a)^2 = (8\pi G/3)\ (\rho_m + \rho_{DE})$, where ρ_m is the mean mass-energy density of non-relativistic matter, ρ_{DE} is the mean mass-energy density of the dark energy, and P_{DE} is the pressure of the dark energy. Combining these equations, we find $(\ddot{a}/a) = -0.5[(\dot{a}/a)^2\ + (8\pi G)\ P_{DE}]$.

Defining the critical density at the present epoch in the usual way, $\rho_{0c} = 3H_0^2/(8\pi G)$, it is easy to show that $p(z) \equiv (P_{DE}(z)/\rho_{0c}) = (E^2(z)/3)\ [2q(z)-1]$. Combining this expression with eqs. (1) and (2) we obtain the pressure of the dark energy as a function of redshift in terms of first and second derivatives of the dimensionless coordinate distance y (Daly & Djorgovski 2004a)

$$p(z) = -(dy/dz)^{-2}[1 + (2/3)\ (1+z)\ (dy/dz)^{-1}\ (d^2y/dz^2)]\ . \tag{3}$$

Thus, the pressure of the dark energy can be determined directly from measurements of the coordinate distance. In addition, this provides a direct measure of the cosmological constant for Friedmann-Lemaitre models since in these models $p = -\Omega_\Lambda$. If more than one new component is present, this pressure is the sum of the pressures of the new components.

Similarly, the energy density of the dark energy can be obtained directly from the data

$$f(z) \equiv \left(\frac{\rho_{DE}(z)}{\rho_{0c}}\right) = (dy/dz)^{-2} - \Omega_0(1+z)^3\ , \tag{4}$$

where $\Omega_0 = \rho_{0m}/\rho_{0c}$ is the fractional contribution of non-relativistic matter to the total critical density at zero redshift, and it is assumed that this non-relativistic matter evolves as $(1+z)^3$. If more than one new component is present, then f includes the sum of the mean mass-energy densities of the new components.

The equation of state $w(z)$ is defined to be the ratio of the pressure of the dark energy to it's energy-density $w(z) \equiv P_{DE}(z)/\rho_{DE}(z)$. As shown by Daly & Djorgovski (2004a), the equation of state is

$$w(z) = -\frac{[1 + (2/3)\ (1+z)\ (dy/dz)^{-1}\ (d^2y/dz^2)]}{[1 - (dy/dz)^2\ \Omega_0\ (1+z)^3]}\ . \tag{5}$$

Here, w is the equation of state of the dark energy; if more than one new component contributes to the dark energy, w is the ratio of the sum of the total pressures of the new components to their total mean mass-energy densities.

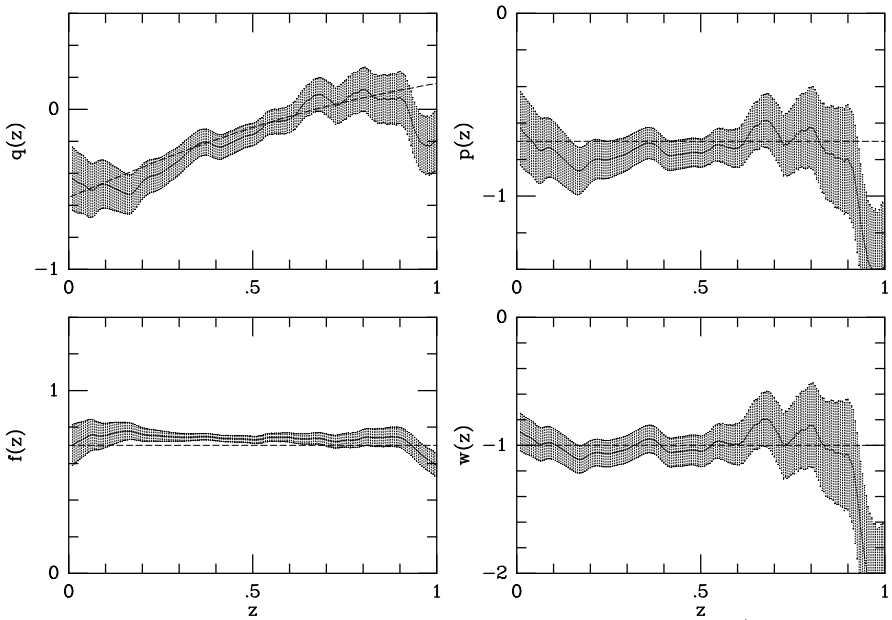

Figure 1. Application of our methods on the simulated (pseudo-SNAP) data set, obtained with equations (2), (3), (4), and (5) respectively as described in the text, using a window function with $\Delta z = 0.4$. The dotted/hatched regions show the recovered trends for the quantities of interest. The assumed cosmology is a standard Friedmann-Lemaitre model with $\Omega_0 = 0.3$ and $\Lambda_0 = 0.7$, and the theoretical (noiseless) values of the measured quantities are shown as dashed lines. There is a good correspondence (typically well within $\pm 1\sigma$) up to $z \sim 0.9$, except in the case of $f(z)$ where a small systematic bias is present, and the formally evaluated errors may be too small as an artifact of the numerical procedure.

3. Results and Conclusions

The results presented here follow those presented by Daly & Djorgovski (2003, 2004a), where more details can be found. We perform a test of the procedure using a simulated data set which mimics the anticipated SN measurements from the SNAP/JDEM satellite (see http://snap.lbl.gov), with a known assumed cosmology, namely the standard Friedmann-Lemaitre model with $\Omega_0 = 0.3$ and $\Lambda_0 = 0.7$ (see Daly & Djorgovski 2003, 2004a for more details on this simulated data set). The results for the dark energy parameters as functions of redshift are shown in Figure 1. We see that our method can recover robustly the assumed parameters, at least out to $z \approx 0.9$. Reassured by this test, we turn to the analysis of actual data.

The data used here includes 20 radio galaxies (RG) compiled by Guerra, Daly, & Wan (2000) and the "gold" supernova (SN) sample compiled by Riess et al. (2004). We note that in the redshift interval where the two sets of coordinate distances (RG and SN) overlap, the agreement is excellent (see Figure 2), suggesting that neither one is affected by some significant bias, and allowing us to combine them for this study.

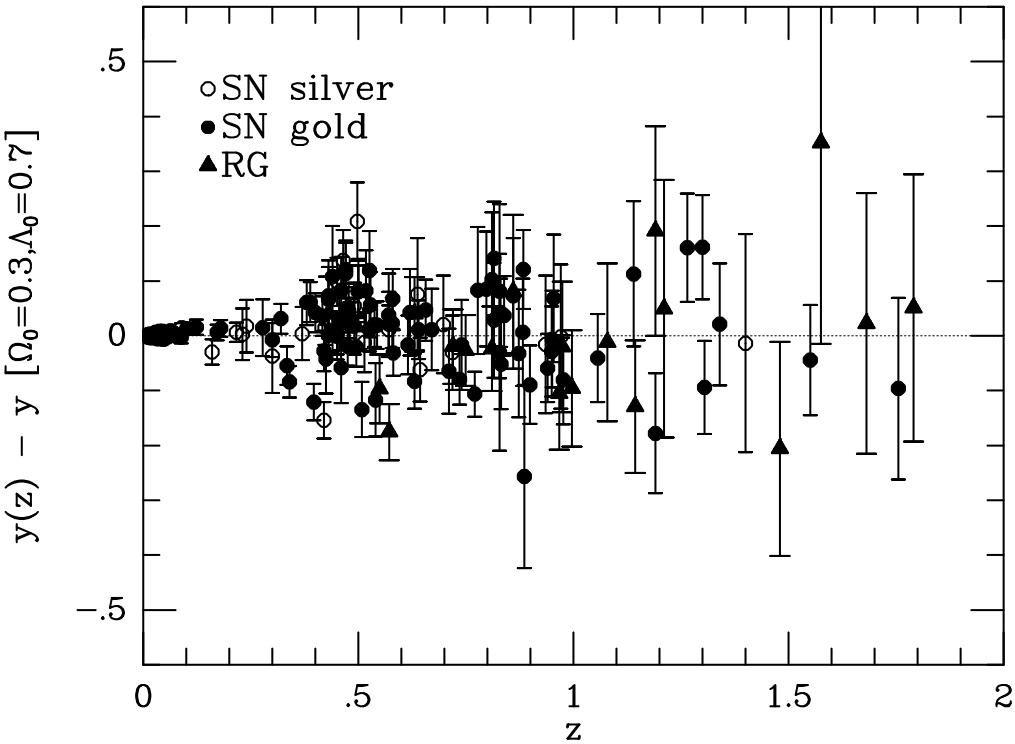

Figure 2. The difference between the dimensionless coordinate distances and those expected in a spatially flat universe with a cosmological constant and $\Omega_0 = 0.3$. SNe and RGs are plotted with different symbols as indicated. There is no significant systematic offset between them in the redshift range where there is an overlap.

Measurements of luminosity distances and angular size distances are easily converted to coordinate distances, $y(z)$, and these are shown in Figure 3. The dimensionless expansion rate obtained from these data using equation (1) is shown as a function of redshift in Figure 4. The result is consistent with the standard "concordance model," of $\Omega_0 = 0.3$ and $\Lambda = 0.7$. The dimensionless acceleration rate obtained from these data and equation (2) is shown in Figure 5.

We see that the universe transitions from acceleration to deceleration at a redshift of about 0.4 (consistent with determinations by Daly & Djorgovski 2003, 2004a,b, Riess et al. 2004, and Alam, Sahni, & Starobinsky 2004); and our determination only depends upon the assumption that the universe is homogeneous, isotropic, and spatially flat.

Assuming GR, we solve for the pressure (see equation 3 and Figure 6), energy density (see equation 4 and Figure 7), and equation of state (see equation 5 and Figure 8), of the dark energy. Each is generally consistent with remaining constant to a redshift of about 0.5 and possibly beyond, but determining their behavior at higher redshifts is severely limited by the available data.

As more and better data become available, particularly more coordinate distance determinations for sources with redshifts between 0.4 and 1.5, this

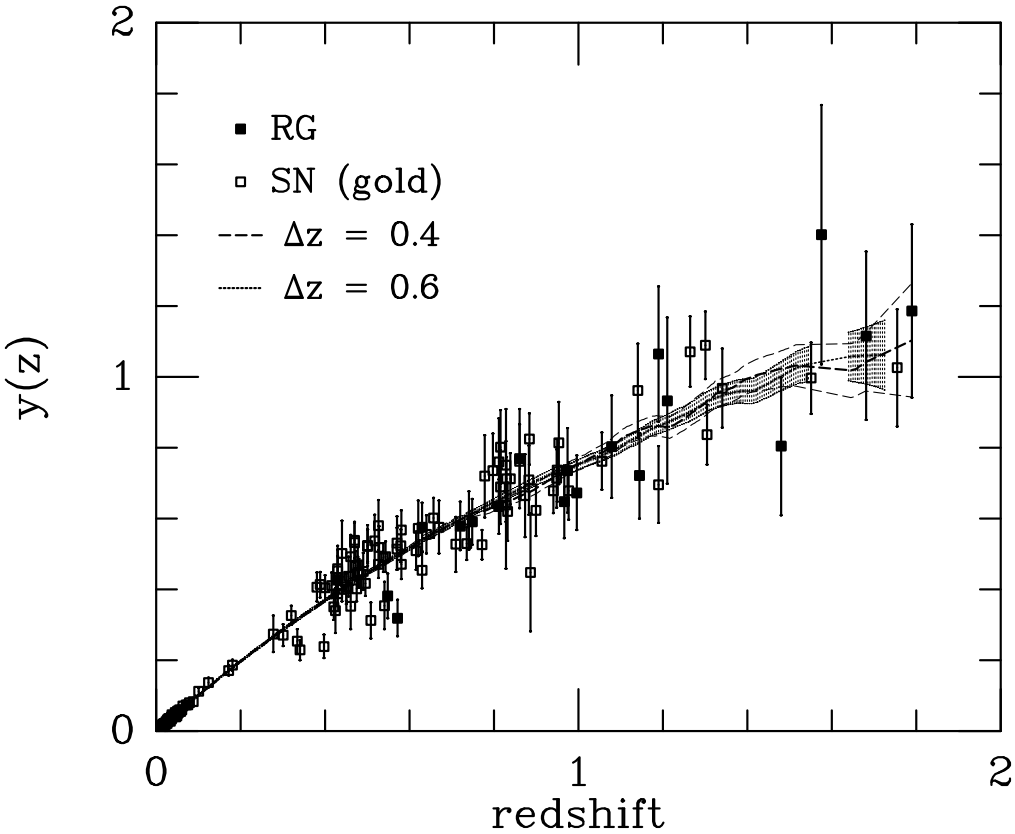

Figure 3. Dimensionless coordinate distances $y(z)$ to 20 radio galaxies and the "gold" sample SNe as a function of z. The smoothed values of y along with their 1 σ error bars obtained for window function widths $\Delta z = 0.4$ (dashed lines) and 0.6 (dotted line and hatched error range) are also shown. Note again that the new high-redshift SNe values agree quite well with those of the high-redshift RGs.

methodology can be used to determine the evolution of the dark energy properties and the observed kinematics of the universe with an increasing precision and confidence.

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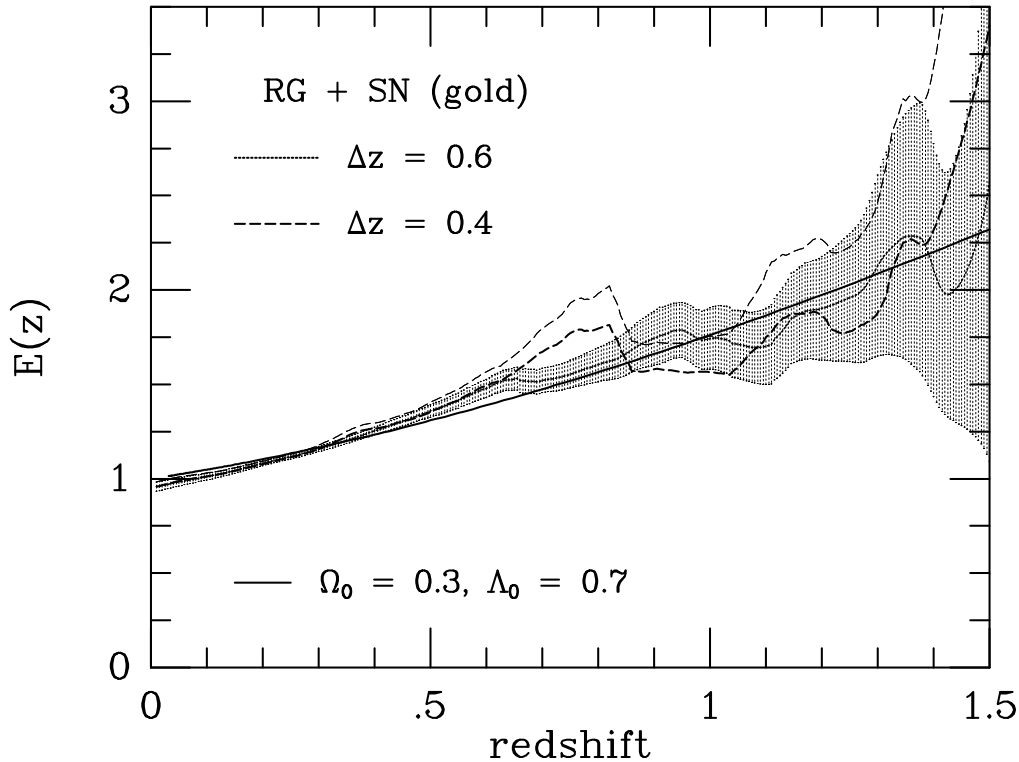

Figure 4. The derived values of the dimensionless expansion rate $E(z) \equiv (\dot{a}/a)H_0^{-1} = (dy/dz)^{-1}$ obtained with window functions of width $\Delta z = 0.4$ and their 1 σ error bars (dashed lines) and 0.6 (dotted line and hatched error range). At a redshift of zero, the value of E is $E_0 = 0.97 \pm 0.03$. The value of $E(z)$ predicted in a spatially flat universe with a cosmological constant and $\Omega_0 = 0.3$ is also shown, and provides a reasonable fit to the data.

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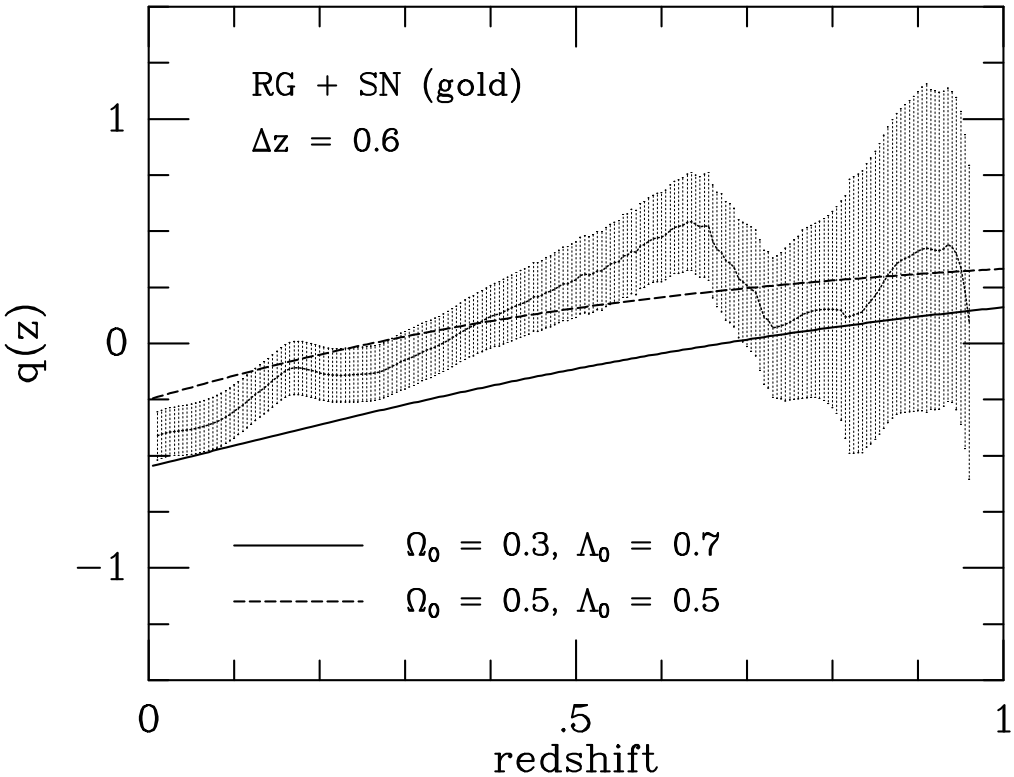

Figure 5. The derived values of deceleration parameter $q(z)$ (see equation 2) and their 1 σ error bars obtained with window function of width $\Delta z = 0.6$ applied to the RG plus gold SNe sample. The universe transitions from acceleration to deceleration at a redshift $z_T \approx 0.4$. The value of the deceleration parameter at zero redshift is $q_0 = -0.35 \pm 0.15$. Note that this determination of $q(z)$ only depends upon the assumptions that the universe is homogenous, isotropic, expanding, and spatially flat, and it does not depend on any assumptions about the nature of the dark energy, or the correct theory of gravity. Solid and dashed lines show the expected dependence in the standard Friedmann-Lemaitre models with zero curvature, for two pairs of values of Ω_0 and Λ_0.

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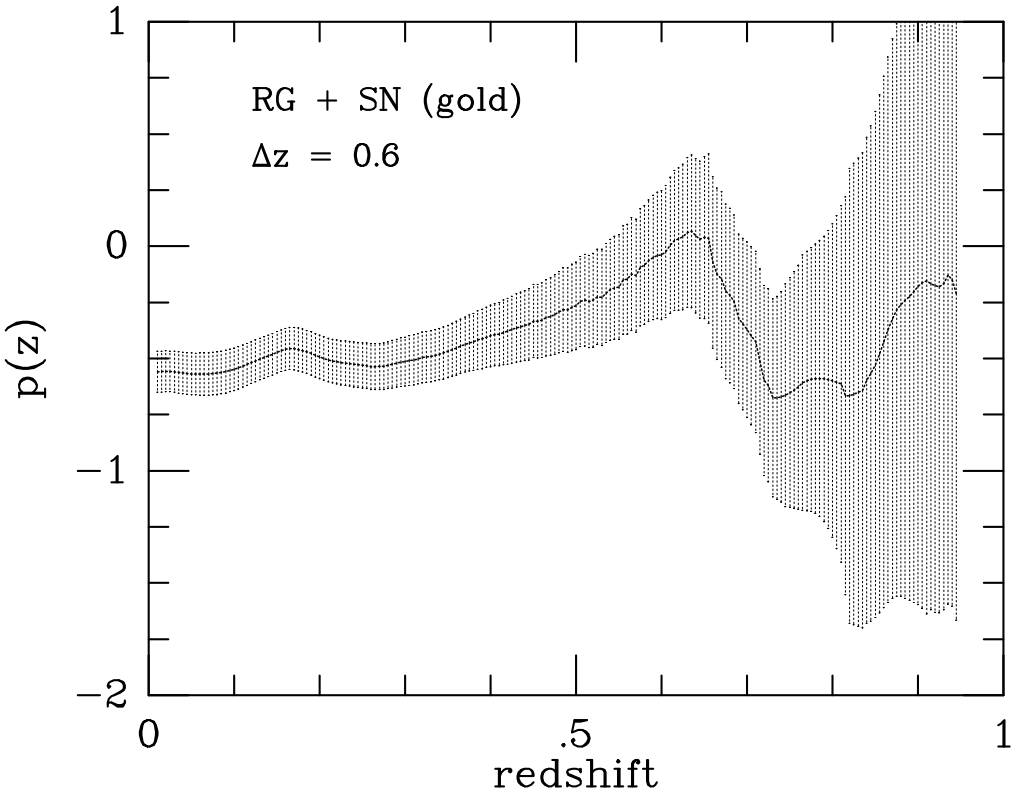

Figure 6. The derived values of dark energy pressure $p(z)$ (see equation 3), obtained with window function of width $\Delta z = 0.6$. This derivation of $p(z)$ requires a choice of theory of gravity, and General Relativity has been adopted here. The value at zero redshift is $p_0 = -0.6 \pm 0.15$.

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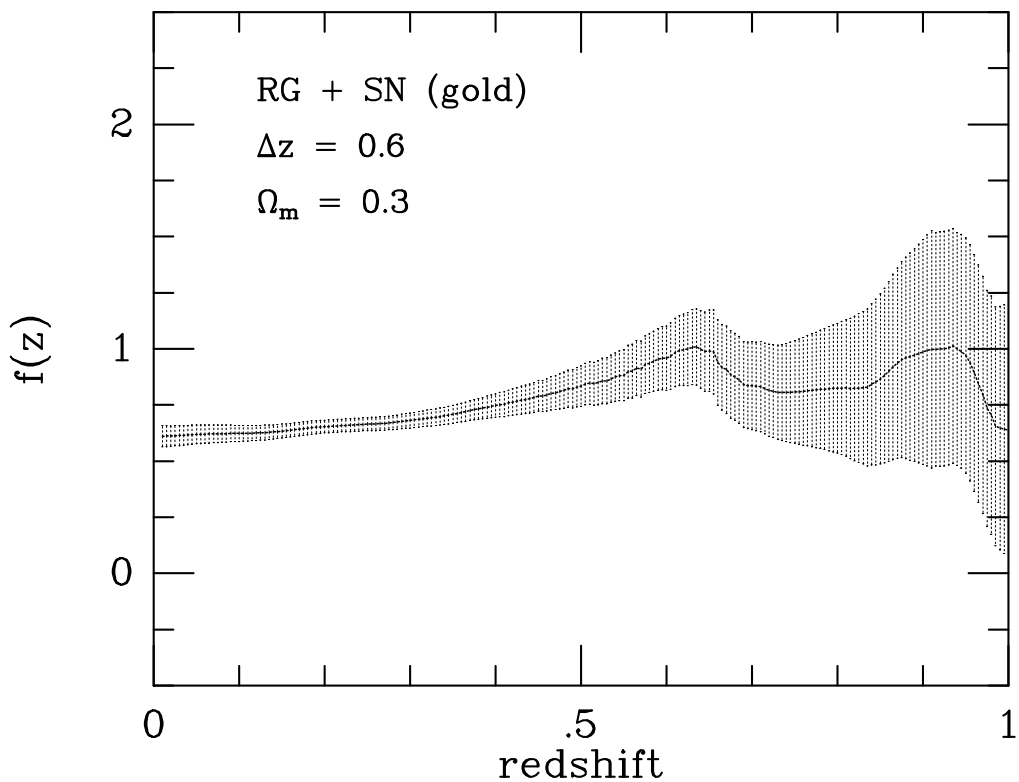

Figure 7. The derived values of the dark energy density fraction $f(z)$ (see equation 4), obtained with window function of width $\Delta z = 0.6$. This derivation of $f(z)$ requires of theory of gravity and the value of Ω_0 for the nonrelativistic matter; General Relativity has been adopted here, and $\Omega_0 = 0.3$ is assumed. The value at zero redshift is 0.62 ± 0.05.

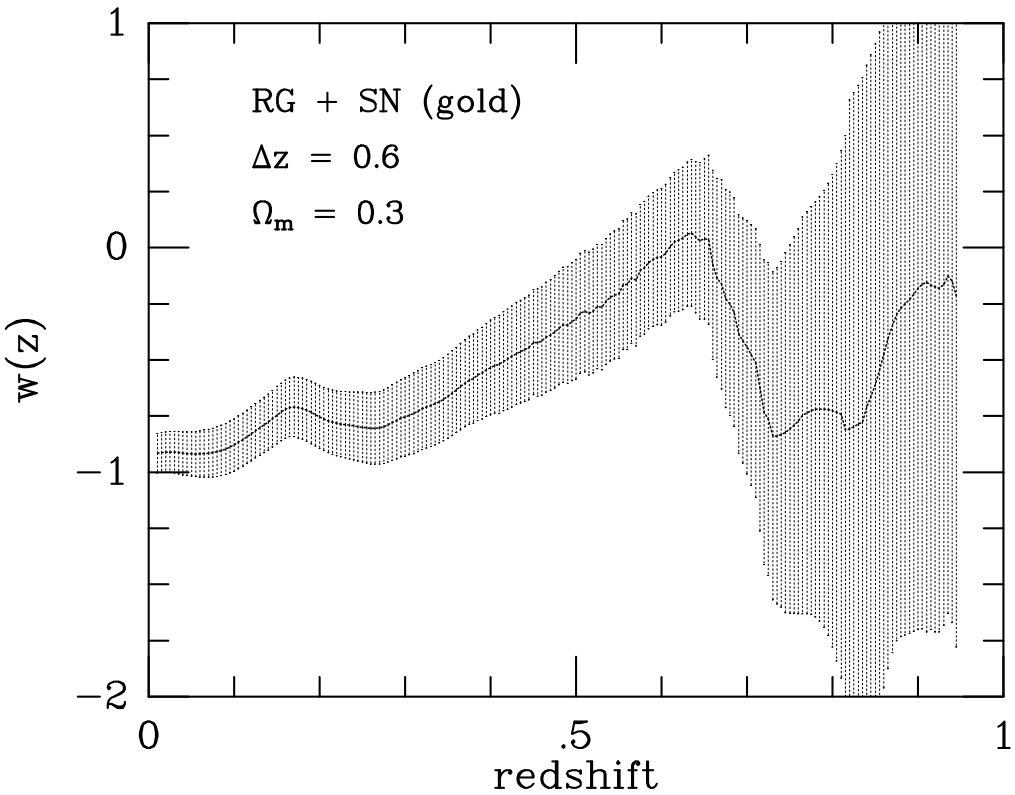

Figure 8. The derived values of the dark energy equation of state parameter $w(z)$ (see equation 5), obtained with window function of width $\Delta z = 0.6$. This derivation of $w(z)$ requires of theory of gravity and the value of Ω_0; General Relativity has been adopted here, and $\Omega_0 = 0.3$ is assumed. The value at zero redshift is $w_0 = -0.9 \pm 0.1$, consistent with the cosmological constant models.