IMPACT OF PVC DYNAMICS ON SHEAR LAYER RESPONSE IN A SWIRLING JET

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ABSTRACT

Combustion instability, or the coupling between flame heat release rate oscillations and combustor acoustics, is a significant issue in the operation of gas turbine combustors. This coupling is often driven by oscillations in the flow field. Shear layer roll-up, in particular, has been shown to drive longitudinal combustion instability in a number of cases.

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systems, including both laboratory and industrial combustors. One method for suppressing combustion instability would be to suppress the receptivity of the shear layer to acoustic oscillations, severing the coupling mechanism between the acoustics and the flame. Previous work suggested that the existence of a precessing vortex core (PVC) may suppress the receptivity of the shear layer, and the goal of this study is to first, confirm that this suppression is occurring, and second, understand the mechanism by which the PVC suppresses the shear layer receptivity. In this paper, we couple experiment with linear stability analysis to determine whether a PVC can suppress shear layer receptivity to longitudinal acoustic modes in a non-reacting swirling flow at a range of swirl numbers. The shear layer response to the longitudinal acoustic forcing manifests as an m=0 mode since the acoustic field is axisymmetric. The PVC has been shown both in experiment and linear stability analysis to have m=1 and m=-1 modal content. By comparing the relative magnitude of the m=0 and m=-1,1 modes, we quantify the impact that the PVC has on the shear layer response. The mechanism for shear layer response is determined using companion forced response analysis, where the shear layer disturbance growth rates mirror the experimental results. Differences in shear layer thickness and azimuthal velocity profiles drive the suppression of the shear layer receptivity to acoustic forcing.

INTRODUCTION

Combustion instability, or the coupling between flame heat release rate oscillations and combustor acoustics, is a significant issue in the operation of gas turbine combustors. Combustion instability events have been linked to reduce engine operability, increased emissions, and, in the most severe cases, engine hardware failure [1]. These instabilities are particularly prevalent in lean-burn, low-emissions combustion systems, as the flames are less stable at these conditions.

Combustion instabilities in gas turbines are typically driven through a coupling mechanism, or an intermediary oscillation that is excited by combustor acoustic fluctuations and results in flame heat release rate oscillations [2]. Two coupling mechanisms are particularly prevalent: equivalence ratio coupling and velocity coupling. In this study, we consider the
velocity-coupling mechanism, where velocity fluctuations in the flow field drive heat release rate oscillations. These velocity fluctuations can stem from the acoustic field itself or from oscillations in the flow field that supports the flame. A number of studies have shown that the oscillations in the shear layer, driven by acoustic excitation, can drive the coupling between acoustics and the flame in a number of different combustor configurations. Fundamental experiments in backwards-facing step combustors have shown that coupling of shear layer oscillations with combustor acoustics can lead to violent oscillations in the flame [3, 4]. This same principle is at work in bluff-body stabilized flames [5, 6], as well as swirling flames [7, 8]. In all these cases, acoustic perturbations excite vortex roll-up of the shear layer, where these vortices cause significant wrinkles along the flame front. These flame area fluctuations drive fluctuations in heat release rate, which drives the thermoacoustic feedback cycle.

Most recently, work by Kirthy et al. [9] highlighted the importance of self-excited shear layer stability in exciting pressure oscillations in a lean premixed backward-facing step combustor. They showed, from a hydrodynamic stability analysis, that one of the unstable thermoacoustic states observed in the experiments of Hong et al. [10] is due to the fact that the shear layer mode in the combustor becomes self-excited with a frequency close to the fundamental acoustic mode of the combustor. Thus, it is conceivable that similar self-excited instabilities exist in a swirl stabilized gas turbine combustor and can excite pressure oscillations in the same way.

In swirling flows, two shear layers may be present as a result of combustor configurations and the geometry of the flow field. These flows typically contain a vortex breakdown (VB) bubble, which is a recirculation zone along the centerline of the flow [11]. As a result of the VB bubble,
there is a shear layer that develops along the edge of the VB bubble, referred to as the “inner shear layer,” and one that emanates from the separation point of the nozzle, referred to as the “outer shear layer.” Vortical disturbances in both shear layers can result in flame wrinkling, depending on the flame shape [12, 13].

Control methods for combustion instabilities have typically not directly manipulated velocity-coupling mechanisms to suppress the instabilities, but instead have either passively damped the acoustic mode [1] or counteracted the equivalence ratio fluctuations in the system [14]. In this work, we explore a new method for suppressing the shear layer velocity-coupling mechanism in the swirling flow through the activation of a precessing vortex core. A PVC is the manifestation of a global instability in the flow field, which causes large helical oscillations in the center region of the flow [15]. This instability results in high-amplitude, narrowband oscillations in the flow field at the PVC frequency, which is determined by the base flow rather than the acoustics of the combustor.

Previous results from Mathews et al. [16] suggested that the presence of a PVC could suppress the receptivity of the shear layer in a swirling flow to longitudinal acoustic forcing. In this experiment, we quantified the receptivity of the shear layer to acoustic oscillations through a velocity transfer function, where the input was the acoustic excitation and the output was the resulting shear layer response, quantified with the vorticity fluctuation at the forcing frequency. This transfer function was measured for a range of swirl numbers and two frequencies, showing that the response of the shear layer significantly decreased when a PVC was present for the operating conditions considered in this study.
Given these previous results, the goal of this work is two-fold. First, we aim to confirm that shear layer oscillations are suppressed by the presence of a PVC over a larger frequency range than was explored previously. We do this by better quantifying which dynamical features in the flow are the result of shear layer response versus PVC oscillations using an azimuthal mode decomposition. We recognize that while this experimental test matrix is more extensive than that of the previous study, it is not exhaustive enough to categorically show that the PVC can suppress shear layer receptivity. Instead, we use linear stability analysis to understand the mechanism by which the shear layer oscillations are suppressed in flows with a PVC; this analysis provides insight into the mechanism driving changes in the flow’s behavior that experimental data alone could not provide. This mechanism can be generalized, and hence applied to other operating conditions and experimental geometries.

The outcomes of the present work could offer a new methodology for suppressing combustion instability in gas turbine combustors. The stability analysis that complements the experimental results in this study shows the mechanism by which the PVC alters the base flow, resulting in reduced sensitivity to external perturbations. In future, this type of analysis could be used to guide swirler design for instability suppression. More generally, hydrodynamic instability analyses, such as the one presented in this paper, provide insight into the mechanisms controlling the coherent unsteady dynamics of flow structures in combustors, using time-averaged base flow data as input. This represents a computationally inexpensive, physics-based approach to reduced-order modelling that can potentially provide useful insight into the characteristics of coherent heat release unsteadiness in gas turbine combustor flows during early stages of the
combustor design process. This insight could then potentially improve the reliability of predictive combustion instability modelling tools currently used in industry.

The remainder of this paper is organized as follows. First, we describe the experimental configuration used, as well as the theoretical formulation used to predict the stability of the flow. Next, we discuss the base flow used for the stability analysis at a range of swirl numbers. Then, we use an azimuthal decomposition to quantify the flow field response to acoustic excitation to differentiate between the dynamics of the PVC and those of the shear layer. Finally, we use linear stability analysis to support our experimental observations and explain the mechanism by which shear layer response is suppressed in the presence of a PVC.

**EXPERIMENTAL CONFIGURATION**

The experimental facility used in this study is identical to the one used in Mathews et al. [16]. The main components of the setup, shown in Figure 1, relevant to the analysis performed in this work are: the two-microphone injector nozzle (upper right), the swirler chamber with a variable, radial-entry swirler (lower right), and the settling chamber with two speakers to provide longitudinal acoustic forcing.

The pressure transducers are located 2.54 cm and 7.62 cm downstream of the swirler, and 6.92 cm and 1.84 cm from the nozzle exit, respectively. In the remainder of this paper, the location of the pressure transducer (PT) located closer to the nozzle exit will be referred to “near nozzle”, and the location of other PT as “near swirler.” The pressure data is recorded at 20 kHz for a duration of 3 sec; the pressure spectra shown in this paper are the average of 30 ensembles. The swirler is actuated by a stepper motor in the base of the experiment and allows for the blades to be set between 65° and -65° with a 2.5° resolution. Results will be presented in terms of
geometric swirl number. The volumetric flow rate is set to 30 SCFM with a maximum variance of 0.5 SCFM; the bulk flow velocity of air is 28 m/s with a Reynolds number based on jet diameter of 35,000. More details of the experiment can be found in Mathews et al. [16]. All results in this study are non-reacting.

![Figure 1. Experimental facility](image)

**Diagnostics**

Velocity fields are captured in the $r$-$\theta$ plane using particle image velocimetry (PIV) with images from an SA5 Photon CMOS high speed camera, set in double frame mode. The camera is mounted with the lens in the $r$-$x$ plane, and the laser plane is imaged using a periscope configuration. The lens points directly at an aluminum coated, right angle, first surface mirror, mounted 25.4 cm away. The mirror is suspended 46.3 cm directly above the nozzle. The dimensions of the mirror face are 75 mm by 106.1 mm. The mirror redirects the field of view of the camera such that it is shifted 90° from the $r$-$x$ plane and captures the $r$-$\theta$ plane.
A Hawk/Darwin Duo Nd-YAG, 532 nm wavelength, 60 W laser is used for PIV. For all cases tested, the laser is directed through a -50 mm sheet optics to produce a laser sheet in the \( r-\theta \) plane. The laser sheet is set to a height of \( x/D=1.7 \), which is twice the convective wavelength of the vorticity fluctuations produced with 600 Hz forcing. This convective wavelength was determined in work performed by Mathews et al. [16], and choosing this laser plane allows for direct comparisons of the \( r-\theta \) data in this study and the velocity transfer functions at 600 Hz reported in Mathews et al..

The sampling rate of the PIV system is 5 kHz with an interframe time ranging from 17 - 23 \( \mu s \) depending on swirl number. Images are recorded for a 1 second duration, yielding 5000 frames per test case. The PIV system is triggered simultaneously with the PTs. Aluminum oxide particles with a nominal diameter of 1-2 \( \mu \)m are used as tracer particles and can accurately follow flow perturbations up to a frequency of 4000Hz. Velocity vectors are calculated in DaVis 8.3.1 without any pre-processing or masking. Cross-correlation with multi-pass iterations with decreasing window sizes is used. The first pass is a 32x32 pixel interrogation window with a 50% overlap followed by 2 passes with a 16x16 pixel interrogation with a 50% overlap. During vector post-processing there are two methods used to reject vectors. First, if the vector is more than 3 times the RMS of the surrounding vectors, the vector is removed and replaced. Additionally, universal outlier detection removes and replaces spurious vector results.

**Data Analysis**

Proper orthogonal decomposition is used to extract the most energetic motions in the flow field [17]. The results of this analysis produce a spatial mode shape and a temporal mode amplitude; here the power spectral density (PSD) of the temporal mode amplitude is presented
in order to analyze the spectral content of each mode’s oscillation. The time-averaged field is subtracted from the data before the POD analysis so as to only decompose the fluctuating component. An eigenvalue decomposition method is used for calculating the POD. At each instant in time, the transverse and radial component velocities are converted into both spatial and temporal modes, as well as their mode energies.

Additionally, an azimuthal mode decomposition is used to interpret the motion of the disturbances in the $r$-$\theta$ plane. This decomposition requires that the velocity field be mapped on a cylindrical coordinate system. Figure 2 shows an example time-averaged velocity magnitude field with the points that make up the cylindrical grid in the $r$-$\theta$ plane.

![Figure 2. Interpolated points in cylindrical coordinates (white dots) on velocity magnitude in $r$-$\theta$ plane](image)

The cylindrical vector field is calculated by performing a cubic spline interpolation on the Cartesian vector field at each of the white dots in Figure 2. A third-order fit was found to most accurately represent the data by comparing interpolated points to known values. In the case of
this study, the cylindrical grid is formed with 32 radial increments and 120 azimuthal increments. The number of increments was calculated to maintain the spatial resolution that was captured with the Cartesian grid from the PIV system at the nozzle radius. At radii smaller than that of the nozzle, the resolution is increased, and at radii larger than that of the nozzle, the resolution is decreased.

We decompose the velocity field at each radius of the data in cylindrical coordinates using the form shown in Eq. 1, where \( \hat{B}_{i,m} \) is the strength of azimuthal mode, \( m \), which is derived from the fluctuating velocity field, \( u' \), at each frequency, \( \omega \).

\[
\hat{B}_{i,m} = \frac{1}{2\pi} \int_{0}^{2\pi} \hat{u}'_{i}(r, x, \theta, \omega) e^{-im\theta} d\theta
\] (1)

The resulting modes represent motions of varying levels of axisymmetry in the flow field. For example, mode \( m=0 \) describes axisymmetric motion in the flow field; the acoustic forcing used in this study should result in \( m=0 \) motion as the forcing is longitudinal. Modes \( m=1 \) and \( m=-1 \) are representative of spinning modes in opposite directions with a “period” of one oscillation around the circumference of the flow [18].

THEORETICAL FORMULATION

We perform a linearized global forced response analysis in this paper to gain physical insight into the influence of the PVC on the forced flow response characteristics. The governing linearized Navier-Stokes equations for the coherent oscillating field around the time-averaged base flow can be derived as follows. The instantaneous quantities in the nonlinear Navier-Stokes equations, \( q = [u_1, u_2, u_3, p] \), are decomposed into contributions from their time-averaged, \( \bar{q} \), coherent fluctuations, \( q' \), and incoherent turbulence fluctuations, \( q'' \), as \( q = \bar{q} + q' + q'' \). Next, governing equations for \( \bar{q} \) are derived by time averaging the resulting equations. Further,
equations for $q'$ can be derived by phase averaging the nonlinear equations and subtracting the equations for $\bar{q}$ from these. Finally, linearized governing equations for $q'$ can be derived by neglecting terms nonlinear in $q'$. The system of equations resulting from this procedure, written using tensor index notation, are as follows,

$$\frac{\partial u'_i}{\partial x_i} = 0 \quad (2)$$

$$\bar{\rho} \left( \frac{\partial u'_i}{\partial t} + \bar{u}_j \frac{\partial u'_i}{\partial x_j} + u'_j \frac{\partial \bar{u}_i}{\partial x_j} \right) = -\frac{\partial p'}{\partial x_j} + \frac{\partial \tau'_{ij}}{\partial x_j} + \frac{\partial}{\partial x_j} \bar{\rho} \left( \bar{u}''_i u''_j - \langle u''_i u''_j \rangle \right) \quad (3)$$

where, $\tau'_{ij}$ is the coherent fluctuation stress and $p'$ is the fluctuating hydrodynamic pressure.

Equations 2 and 3 have been non-dimensionalized using the nozzle diameter ($D$) and bulk velocity ($U_o$) at the exit of the nozzle, as the length scale and the velocity scale respectively. The last term on the right in Eq. 3, represents the transport of coherent fluctuating momentum by turbulence fluctuations. Following prior studies, we model this term using a gradient transport assumption expressed using a turbulent viscosity, $\bar{\mu}_t$ [19-22]. The latter is determined from time resolved PIV measurements by assuming axisymmetric time-averaged base flows and using the procedure suggested by Oberleithner et al. [20].

Longitudinal acoustic forcing in the present setup can excite the shear layer at the nozzle exit, causing it to roll up into coherent structures further downstream. In the limit of spatially varying base flows for which the length scale over which the base flow changes is large compared to the length scale of the flow response, the characteristics of the latter to forcing to can be determined using WKBJ [23, 24] theory up to leading order as follows. Equations 2 and 3 are written in cylindrical coordinates with the $x$-axis aligned along the streamwise direction. Thus,
for a forcing angular frequency, $\omega_f$, and azimuthal wavenumber, $m$, the leading order response of the flow can be written as follows,

$$q'(x, r, \theta, t) = \hat{q}^+(r) \exp \left[ i \left\{ \int_0^x k^+(\omega_f, m, x') dx' + m \theta - \omega_f t \right\} \right]$$  \hspace{1cm} (4)$$

where we have assumed that the time-averaged base flow is axisymmetric and varies slowly along the streamwise direction, $x$. Thus, in Eq. 4, $\hat{q}^+$ is the local spatial eigenvector and $k^+$ is the local spatial eigenvalue corresponding to the local base flow profile at the streamwise location, $x'$, for an excitation frequency, $\omega_f$. The superscript ‘+’ denotes solutions corresponding to downstream propagating disturbances. Also, boundary conditions, $q' \to 0$ as $r \to \infty$, and kinematic compatibility conditions at $r = 0$ are imposed [25].

Base flow fields for the above analysis are determined by time-averaging stereo-PIV measurements. Smooth radial profiles at each streamwise location are then fit to both $\bar{U}_x$ and $\bar{U}_\theta$ data using a linear combination of piecewise cubic spline fits and a least squares fit to the base flow model proposed by Oberleithner et al. [20]. The governing equations (Eqs. 2 and 3) are discretized using the Chebyshev pseudospectral collocation method. The physical space, $r \in [0, \infty]$, is truncated to $r \in [0, 100 D]$ and mapped to the computational space, $\eta \in [-1,1]$ using a grid mapping procedure that modifies that suggested by Bayliss et al. [26]. The resulting equations are written as a generalized matrix eigenvalue problem using the companion matrix method [27] as $A \hat{q}_d = kB \hat{q}_d$, where $\hat{q}_d$ is the discretized spatial eigenvector. The matrices $A$ and $B$ are functions of $\omega_f$. The above discrete matrix eigenvalue problem is solved using the MATLAB eig function. The resulting global response solution is then computed using the discrete local eigenvectors and eigenvalues using Eq. 4.
RESULTS

Results from Mathews et al. [16] showed, through spectral analysis of the flow in the $r$-$x$ plane, that shear layers are less receptive to acoustic forcing at high swirl numbers where a PVC is present. In this study, we analyze the flow in the $r$-$\theta$ plane to better capture the PVC dynamics while still measuring shear layer response. The results are divided into four sections. First, we describe the base flow as a function of swirl number in both the $r$-$\theta$ and $r$-$x$ planes. Next, we consider the self-excited dynamics of the flow in the form of a PVC, which occur at three of the swirl numbers investigated, $S=0.79$, 1.05, and 1.43. Then, we quantify the flow response in the $r$-$\theta$ plane to 600 Hz forcing, showing the relative strength of the PVC and shear layer dynamics. We extend this analysis by also analyzing the flow’s response to other forcing frequencies, showing that the PVC suppresses flow response to a range of frequencies. Finally, we describe the mechanism by which this suppression occurs using forced response analysis of the base flows described at the beginning of the results.

Base flow variations with swirl number

Seven swirl numbers are studied in this work at various forcing conditions, as shown in Error! Reference source not found.. For all cases, the jet is non-reacting, the bulk flow velocity is 28 m/s, the jet is unconfined, and the incoming air is not preheated.
Table 1. Test matrix with flow state and PVC frequencies

<table>
<thead>
<tr>
<th>Swirl number</th>
<th>Flow state</th>
<th>PVC frequency</th>
<th>Forcing frequencies</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>No VB</td>
<td>-</td>
<td>600 Hz</td>
</tr>
<tr>
<td>0.18</td>
<td>No VB</td>
<td>-</td>
<td>600 Hz</td>
</tr>
<tr>
<td>0.38</td>
<td>No VB</td>
<td>-</td>
<td>400–700 Hz</td>
</tr>
<tr>
<td>0.56</td>
<td>Intermittent VB</td>
<td>-</td>
<td>600 Hz</td>
</tr>
<tr>
<td>0.79</td>
<td>Weak PVC</td>
<td>770-815 Hz</td>
<td>600 Hz</td>
</tr>
<tr>
<td>1.05</td>
<td>PVC</td>
<td>840 Hz</td>
<td>600 Hz</td>
</tr>
<tr>
<td>1.43</td>
<td>Strong PVC</td>
<td>1060 Hz</td>
<td>400-700 Hz</td>
</tr>
</tbody>
</table>

Figure 3. Variation of time-averaged flow field (base flow) with swirl number in r-x (top) and r-θ (bottom) planes

Error! Reference source not found. presents the time-averaged flow fields at each swirl number in the r-x plane and r-θ plane, shown in the top and bottom row, respectively. The black horizontal line in each r-x image is drawn at x/D = 1.7 and marks the downstream distance of the r-θ cuts. The time-averaged streamwise velocity component is shown in color for the data in the r-x plane; the colorbar for the data in the r-θ plane is the time-averaged azimuthal velocity. Streamlines are shown on each plot to visualize the direction of the flow.

As swirl number increases, each plane has defining features that illustrate the shift from uniform, non-swirling flow, to VB, and finally to the formation of a PVC. In the r-x plane, at S=0.00
and 0.18, the flow from the nozzle is uniform and the jet only weakly spreads. This spatially constant behavior is attributed to the lack of swirling motion. As swirl number increases, the streamwise velocity decreases everywhere in the flow. This reduction is due to the momentum of the flow being re-distributed from the streamwise direction to the azimuthal direction. At $S=1.43$, the streamwise velocity is approximately 10 m/s less than a case with low swirl. A pocket of negative velocity near the nozzle is indicative of VB and increases in strength as the swirl number increases from 0.79. At high swirl numbers, the streamlines trace a recirculation zone in the center of the jet, which is a defining feature of VB.

The plots of the $r-\theta$ plane also show changes in flow as swirl number increases. It is important to note that that reason that azimuthal velocity is negative in these plots is because the swirler is set to negative angles and thus produces a clockwise rotation of the flow. One of the most obvious changes in the $r-\theta$ plots is the radial extent of the swirling motion, illustrated by the streamlines. At $S=0.18$, a slight swirl is defined, and only extends as far as the diameter of the nozzle. Whereas at $S=1.43$, the swirl pattern clearly shows jet spreading. It is also important to note that only unforced cases are presented in Error! Reference source not found.. But, as shown by Mathews et al. [16], the time-averaged flow does not change with forcing, and so these plots are accurate for each swirl number, regardless of forcing.

**Self-excited dynamics of the flow**

POD is used to illustrate the self-excited dynamics of the flow field as a function of swirl number. Figure 4 shows the strength of modes 1-20 for all swirl numbers. At swirl numbers of 0.79 and above, there exist one or two initial modes with higher energies than the rest, indicating
the presence of self-excited dynamics. It is these high-energy modes that we consider in the analysis of the self-excited dynamics.

![Figure 4: Energies of POD modes 1-20 for all swirl numbers](image)

**Figure 4. Energies of POD modes 1-20 for all swirl numbers**

It is evident from the mode energies that swirl numbers of 0.79 and above contain a PVC. The frequencies of these PVCs were confirmed through acoustic measurements using pressure transducers in the swirler nozzle. Figure 5 shows the reconstructed azimuthal velocity for the POD modes that contain spectral content at the PVC frequency. A reconstruction is performed for three swirl numbers, as shown in the top row of the figure. The second row shows the frequency spectra of mode 1, the highest energy POD mode. The plots in the third row are the pressure spectra at each swirl number. These spectra were obtained using the near-nozzle pressure transducer.

The reconstructed mode shapes shown in Figure 5 are indicative of a PVC, as the velocity oscillations indicate a helical motion in the flow field. As the swirl number increases, the size of the PVC grows radially as a result of the radially-expanding flow. The spectra of the first POD
temporal modes show a defined peak that occurs at the same frequency as the peak obtained using pressure measurements for all three swirl numbers. These peaks occur at the PVC frequency. This is particularly evident for swirl numbers $S=1.05$ and 1.43, for which the frequency is 840 Hz and 1060 Hz, respectively. At a swirl number of $S=0.79$, the PVC is noticeably weaker and less coherent. This is evidenced by a broad peak around 800 Hz in POD mode 1 and a bump on the pressure spectra at 800 Hz, which is barely above the noise floor.

![Figure 5. POD reconstruction and frequency spectrum of swirl numbers that cause a PVC to be formed](image)

The azimuthal dynamics of the PVC can be quantified using an azimuthal decomposition, as shown by O’Connor and Lieuwen [28], which can be directly compared to linear stability analysis, as shown in Hansford et al. [18]. Figure 6 shows the strength of modes $m=1$ and $m=-1$, which have been shown in this work and others to be the dominant modes for precessing vortex cores in swirling flows. Here, the mode strength is given by the strength of the power spectral
density (PSD) of the radial velocity spectrum at each mode, and is plotted as a function of radius to show the spatial extent of these oscillations in the flow field.

The radial distributions of PVC strength in Figure 6 align well with the POD mode shapes shown in Figure 5. The PVC oscillations peak near \( r/D = 0.7 \). The mode strength for both modes increases as swirl number is increased from \( S = 0.79 \) to 1.05, which is reflected in the POD and pressure measurements as well. The mode strengths for the highest swirl number, \( S = 1.43 \), are lower than those at \( S = 1.05 \), which is a somewhat unexpected result given that the pressure fluctuations from the PVC are significantly stronger at the highest swirl number case. We believe that the reduction in mode strength is the result of a number of factors. Predominantly, as the PVC is extremely strong at this condition, the centrifugal force on the seeding particles is significant and seed is thrown from the center of the flow outwards radially, resulting in lower seeding densities than in the lower-amplitude PVC cases.

![Figure 6. Strength of PVC modes \( m=1 \) and \( m=-1 \) for three swirl numbers as a function of radius](image)

The distribution of energy at the PVC frequency in azimuthal mode space is shown in Figure 7. We see that for all three swirl numbers, the PVC manifests in both \( m=1 \) and \( m=-1 \)
motions, with only small contributions from other modes. Most notably, the mode strength at mode $m=0$ is low for all cases. This is the result of the highly non-axisymmetric motion of the PVC, which precesses around the flow. The lack of $m=0$ motion in the PVC will be used for identification of the forced response in the system in the next section.

![Figure 7. Distribution of modes at the PVC frequency for three swirl numbers and $r/D=0.7$](image)

**Flow response to acoustic forcing**

While the PVC motion is concentrated in the $m=1$ and $m=-1$ modes, the flow’s response to axisymmetric acoustic forcing will manifest as an $m=0$ response. The axisymmetric input disturbance, from the longitudinal forcing, will result in axisymmetric excitation of the flow at the base of the flow, causing axisymmetric disturbances that convect downstream. We use this difference in modal dynamics to investigate the relative strength of the acoustic response and self-excited dynamics of the flow in this section. Figure 8 shows the strength of the $m=0$ mode at 600 Hz for a range of swirl numbers, where the flow has been forced at constant-amplitude, 600 Hz longitudinal forcing. The forcing conditions match those presented in Mathews et al. [16]; for brevity, we do not repeat those results here, but discuss them at length in the previous work.
The mode strength is plotted as function of radius to illustrate the spatial extent of the flow response. Data for $S=0$ is not shown due to difficulty in measuring fluctuating radial and azimuthal velocity components in the $r-\theta$ plane for a non-swirling jet.

![Figure 8. Strength of the $m=0$ mode for a range of swirl numbers at the forcing frequency, 600Hz](image)

The radial distributions of the $m=0$ mode show the differences in the response of the flow as a function of swirl number. At the three low-swirl conditions, $S=0.18$, 0.38, and 0.56, where no vortex breakdown is present and hence no PVC exists, the flow responds strongly to the acoustic excitation. The peak response is near $r/D=0.5$, which is the location of the shear layer in these flows. These data are obtained at $x/D=1.7$, the same location with the velocity transfer function was obtained in Mathews et al. [16], which also shows strong flow response in the $r-x$ plane at these three swirl numbers. At $S=0.79$, the response to acoustic forcing is significantly less, although a weak response is still measured. Here, a weak PVC is present, which seems to dampen, but doesn’t completely suppress, the response of the flow to acoustic excitation. There is no
meaningful response of the flow field at the two highest swirl numbers, \( S = 1.05 \) and 1.43, where a PVC is present.

The modal distribution of the flow response at the forcing frequency is shown in Figure 9a, where mode strengths are calculated at the 600 Hz forcing frequency and \( r/D = 0.5 \). This figure shows that the three lowest swirl numbers have significant \( m = 0 \) dynamics, as was seen in the radial distributions as well. There is very little non-axisymmetric motion at these swirl numbers, which indicates that the flow field responds axisymmetrically to the axisymmetric forcing. Linear stability analysis of the flow field at \( S = 0.18, 0.38, \) and 0.56 has shown that these flows do not contain any pockets of absolute instability, and are convectively unstable throughout. This stability condition indicates that the flow field is receptive to external forcing, and that the flow will respond not only to the forcing frequency, but also the forcing symmetry. This symmetry-matching response has been shown in studies using transverse acoustic forcing of non-swirling jets, including Kusek et al. [29] and Leyva et al. [30].

The modal distribution of the PVC dynamics is not significantly affected by the presence of acoustic forcing in the cases where a PVC is present. Figure 9b shows the modal distribution of fluctuations at 600 Hz and the PVC frequency for \( S = 0.79, 1.05, \) and 1.43. The fluctuations at the PVC frequency are still situated at \( m = 1 \) and \( m = -1 \), which is the same distribution as in the unforced case. Again, the mode strength for the \( S = 1.43 \) case should not necessarily be interpreted quantitatively as a result of issues of seeding at this condition. However, the trend in modal distribution is representative of the dynamics of the flow field. The modal strength of the fluctuations at 600 Hz are negligible in all cases, which is likely a result of the suppressed receptivity of the flow field with a PVC.
The reduced response at the high swirl numbers, where a PVC is present in the flow field, is present at other forcing frequencies as well. To show this, we have considered two swirl numbers, $S=0.38$ and $S=1.43$, where the flow at $S=0.38$ does not have a PVC and has been shown to have a significant response to 600 Hz forcing, and the flow at $S=1.43$ contains a PVC and does not respond to acoustic forcing. Figure 10 shows the strength of mode $m=0$ as a function of radius for these two swirl numbers and a range of frequencies. These frequencies were chosen so that they do not overlap with the PVC frequency, which could result in non-linear coupling between the acoustics and the PVC.

**Figure 9. Distribution of modes at 600Hz for six swirl numbers and $r/D=0.5$ (a), and Distribution of modes at 600Hz and the PVC frequency for the three highest swirl numbers and $r/D=0.5$ (b)**
Figure 10. Strength of the $m=0$ mode for $S=0.38$ and $S=1.43$ at a range of forcing frequencies

The $m=0$ mode strengths in Figure 10 indicate that the $S=0.38$ flow field responds to longitudinal acoustic forcing at a range of frequencies. While the response is not the same for all frequencies, there is a coherent, spectrally-narrow response at the forcing frequency, which is mirrored in the POD spectra shown in Figure 11. Here, we show spectra from both the near-nozzle pressure transducer and the POD; the POD spectra have been scaled to illustrate their content rather than their absolute values. The pressure transducer responds to pressure fluctuations from both acoustic and hydrodynamic sources, and in the acoustically forced cases, the peaks at the forcing frequency are likely to be entirely acoustic content. At $S=0.38$, the POD, which is only a reflection of the flow behavior, shows strong, narrow-band response to the acoustic forcing at these frequencies.

At $S=1.43$, the response of the flow field at $m=0$ is equally non-existent at the forcing frequency, as is shown in the radial distribution of mode strengths in Figure 10. This fact is reflected in the pressure and POD spectra shown in Figure 12, where the POD
spectra have been scaled to fit on plots with the pressure spectra. In comparing the pressure spectra at $S=0.38$ in Figure 11 and $S=1.43$ in Figure 12, a few interesting points are noticeable. First, the overall sound pressure level is higher at the higher swirl number; this is likely a result of not just the presence of a PVC, but also an increase in flow turbulence, and hence flow noise, that results from the PVC motion. Second, the acoustic forcing frequencies can be seen in the pressure spectra for both cases, and in the $S=1.43$ case, the peak from the PVC is also visible at approximately 1060 Hz. This peak is matched by a peak in the POD spectrum, which indicates that the source of the pressure oscillations at 1060 Hz is the PVC. However, the POD spectra for $S=1.43$ do not contain significant peaks at the acoustic forcing frequencies, indicating a lack of flow response. We have inspected the spectrum of each POD mode, and no coherent peaks can be found at the forcing frequency in any of the modes for $S=1.43$. 

Figure 11. Response of flow at swirl number of $S=0.38$ to acoustic forcing as seen by pressure transducer and POD modes at various forcing frequencies. Forcing at 400Hz (a), 500Hz (b), 600Hz (c), and 700Hz (d).
Response suppression mechanism

The experimental results presented thus far have provided evidence for the suppression of the shear layer hydrodynamic response by action of the PVC. We next present results from the linear hydrodynamic forced response analysis to provide a possible explanation for the suppression of shear layer receptivity by the PVC. Figure 13 shows the variation of the spatial amplification envelope, i.e., the exponential term in Eq. 4, determined using the most locally spatially amplified axisymmetric ($m=0$) $k^+$ branch at various swirl numbers. Figure 13 shows the variation of the amplitude of the envelope of the flow response for an excitation frequency of 600 Hz at the various values of $S$ considered in the experimental study. Note that for $0 < S < 0.79$, the flow field response has a maximum downstream of the jet exit plane that exceeds unity. This result suggests that for this frequency, a large axisymmetric ($m=0$) response of the flow field may be expected as seen in the experiments (see 9a). However, for $S > 0.79$, the envelopes

Figure 12. Response of flow at swirl number of $S=1.43$ to acoustic forcing as seen by pressure transducer (PT) and POD modes at various forcing frequencies. Forcing at 400Hz (a), 500Hz (b), 600Hz (c), and 700Hz (d).
monotonically decay with downstream distance. This trend shows that the axisymmetric \((m=0)\) response decays with downstream distance. Again, this is consistent with the experimental results presented earlier, which show that there is no axisymmetric \((m=0)\) hydrodynamic response when the flow has a PVC.

![Figure 13. Spatial amplification envelope of flow response to forcing at 600Hz](image)

Next, fig. 14 shows the variation of the peak spatial amplification, \textit{i.e.}, the maxima of the curves shown in fig. 13, with Strouhal number \(St = \frac{f \delta_{OSL}}{U_o}\), where, \(f\) is the forcing frequency, \(\delta_{OSL}\) is the time averaged outer shear layer thickness for each value of \(S\), and \(U_o\) is the axial flow velocity. We determine \(\delta_{OSL}\) in this paper as the thickness parameter in a hyperbolic tangent fit to the spatial variation of the time-averaged axial velocity component in the outer shear layer region at the nozzle exit. This is because longitudinal forcing perturbs the shear layer formed due to flow separation from the nozzle exit in an axisymmetric fashion. Therefore, the relevant length scale that determines the receptivity of the shear layer to longitudinal forcing would be the outer shear layer thickness at the exit of the nozzle. Figure 14 shows that for \(0 < S < 0.79\), the peak spatial amplification greatly exceeds unity across all forcing frequencies considered in this study.
However, for $S > 0.79$, the peak spatial amplification remains near 1 across all forcing frequencies, which shows that velocity disturbances uniformly decay along the streamwise direction at these values of $S$.

![Figure 14. Variation of peak value of envelope as a function of non-dimensional excitation frequencies at various swirl numbers](image)

The reasons for this behavior can be seen in fig. 15, which shows the radial variation of $\bar{U}_x$ and $\bar{U}_\theta$ at various swirl numbers for a streamwise location $x/D=0.3$. This streamwise location corresponds to the location where the local spatial growth rate is maximum across $S$ and also the closest location from the nozzle exit where spatial stability analysis is performed. Figure 15a shows the radial variation of $\bar{U}_x$, from which it can be clearly seen that the outer shear layer thickness significantly increases for $S = 1.05$ and 1.43, where a PVC is present. Also, the peak axial velocity magnitude has also reduced significantly when compared with the $S < 1.05$ cases. Both these factors, in effect, would reduce the time-averaged azimuthal vorticity strength across the outer shear layer. Figure 15b shows the radial variation of $\bar{U}_\theta$ for various $S$. It is clear that for $S=1.05$ and 1.43, the outer shear layer thickness in the $\bar{U}_\theta$ profile becomes larger than that seen
in the $S < 1.05$ cases. Also, the peak $\bar{U}_\theta$ values along the radial direction for $S = 1.05$ and 1.43 are lower than those for $S < 1.05$. All of these results are typical and are seen at other axial locations as well.

![Figure 15: Radial profiles at various swirl numbers, $S$, of time averaged base flow velocity components (a) $\bar{U}_x$ (b) $\bar{U}_\theta$, at $x/D = 0.3$](image)

Recent local stability analyses [31, 32] have shown that the primary mechanisms causing the growth of fluctuating vorticity production in these types of base flows are the Kelvin-Helmholtz mechanism associated with the shear layers in the axial velocity profile and the centrifugal mechanism induced by swirl. Hence, the increase in the outer shear layer thicknesses in both $\bar{U}_x$ and $\bar{U}_\theta$ profiles for $S = 1.05$ and 1.43 result in a progressive weakening of the KH mechanism. Likewise, a large reduction in the radial maximum of $\bar{U}_\theta$ at $S = 1.05$ and 1.43, results in a weakening of the centrifugal instability mechanism. Hence, the imposed harmonic forcing at a large range of frequencies does not induce significant levels of coherent fluctuating vorticity generation in the flow when $S > 0.79$, i.e., when the base flow is altered by the PVC, causing the hydrodynamic response to the imposed forcing to decay spatially.
CONCLUSIONS AND FUTURE WORK

In this work, we have demonstrated how a precessing vortex core can suppress the receptivity of a swirling flow field to longitudinal acoustic forcing through both experiment and linear stability analysis. Data obtained at a range of swirl numbers is used to quantify the self-excited dynamics of the flow as a function of swirl number. Measurements in the $r$-$\theta$ plane are used to calculate POD modes to understand the structural characteristics of these flows, and azimuthal decompositions of the radial velocity fluctuations in this place are used to quantify the azimuthal behavior of the disturbances. We show that flows with a PVC display strong oscillations at modes $m=1$ and $m=-1$.

We measured flow response to acoustic forcing at a range of forcing frequencies, quantifying the response of the flow to the longitudinal forcing using the $m=0$ mode at the forcing frequency as a marker for flow response. Flow fields without a PVC respond to the acoustic forcing through narrow-band shear layer oscillations, while flows with the PVC do not respond at any of the forcing frequencies. Forced response analysis of the base flow shows that these differences in response are driven by the shear layer thicknesses and a weakening of the KH mechanisms in the flow field.

The results of this study could have significant implications for the suppression of combustion instability in lean-burn, low-NO$_x$ gas turbines. Suppression of combustion instability is an important design criterion, and designing a system that is naturally resistant to instability may be both operationally and economically favorable to designing passive and active control mechanisms. There are significant strides in understanding that must be made, though, before this method of instability suppression is robust enough for industrial use.
Follow-on work to the results presented in this study focuses in two directions. First, we plan to further investigate the dynamics of the PVC and how linear stability analysis can be interpreted in the case of very strong PVCs. As linear stability analysis has proved to be a particularly useful tool for understanding both the self-excited behavior and forced response of swirling flows, it is critical that this tool be extended to regimes where nonlinear oscillations are important. Second, it is important to note that a PVC will not universally suppress response of the flow to external excitation. In some cases, like those described by Steinberg et al. [33], the PVC can nonlinearly couple with the acoustically-driven motions. We plan to use a combination of linear stability analysis and experiment to understand how this PVC design may be achieved. In this way, PVC as a combustion dynamics suppression methodology may be realizable.

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NOMENCLATURE

\begin{align*}
D & \quad \text{Nozzle diameter} \\
KH & \quad \text{Kelvin-Helmholtz} \\
POD & \quad \text{Proper orthogonal decomposition} \\
PSD & \quad \text{Power spectral density} \\
PT & \quad \text{Pressure transducer} \\
PVC & \quad \text{Precessing vortex core} \\
S & \quad \text{Swirl number}
\end{align*}
SVD  Singular value decomposition

VB  Vortex breakdown

k  Axial wavenumber

m  Azimuthal wavenumber

P  Pressure

q  Disturbance vector

r  Radial coordinate

t  Time

u  Velocity

x  Axial coordinate

θ  Azimuthal coordinate

μt  Turbulent viscosity

τ  Shear stress

ω  Angular frequency

REFERENCES


