

# What can we learn about temperature extremes from millennial-scale equilibrium climate simulations?

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Joint work Michael Stein<sup>2</sup>, Elisabeth Moyer<sup>2</sup>, Shanshan Sun<sup>2</sup>,  
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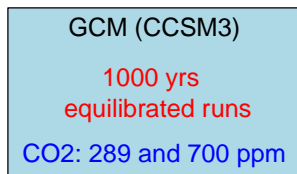
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Oct 24, 2016, State College, PA

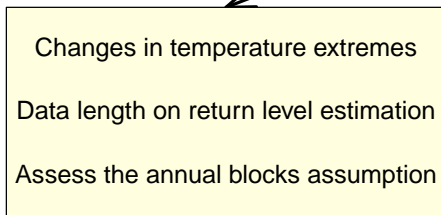
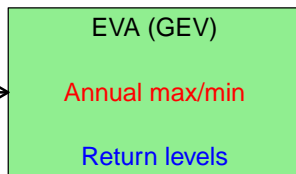


# Overview

## Climate Model

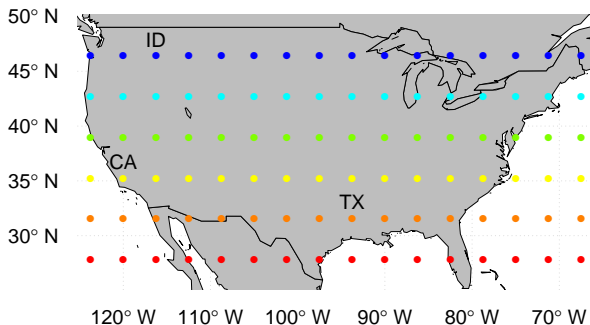


## Statistical Model



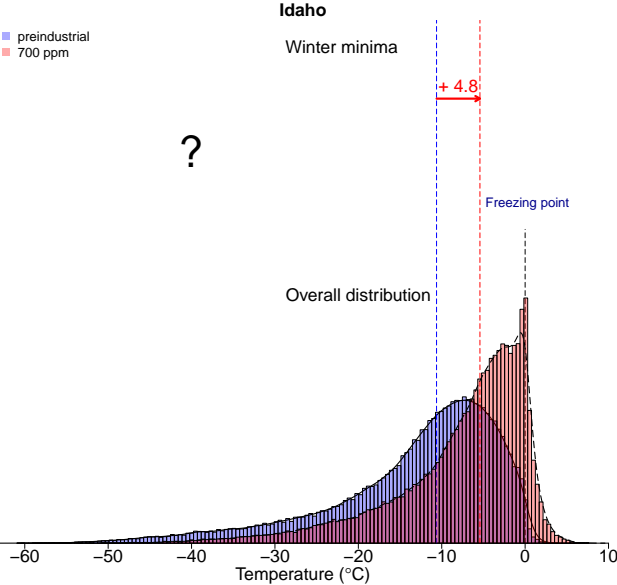
## Our millennial-scale CCSM3 runs

- ▶ Fully **equilibrated** pre-industrial and future (700 ppm CO<sub>2</sub>) climate simulations
- ▶ **1000-year** daily output ( $T_{\max}$ ,  $T_{\min}$ ) 😊
- ▶ T31 spatial resolution ( $3.75^\circ \times 3.75^\circ$  grid) 😞

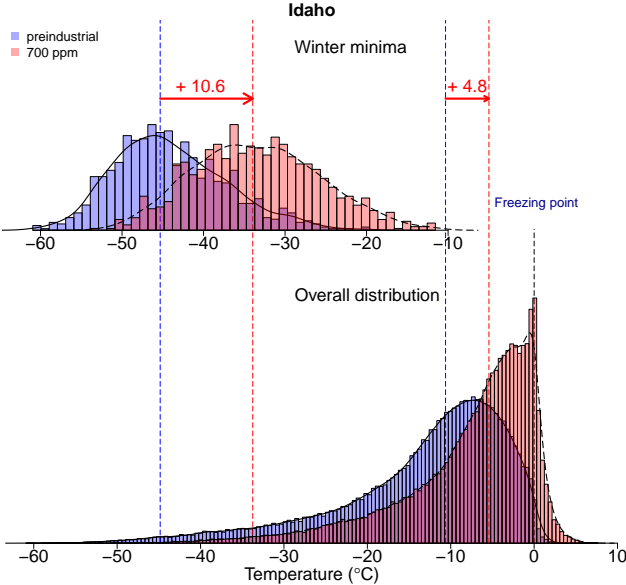


# Part 1: Changes in temperature extremes

# Extremes may shift differently than means



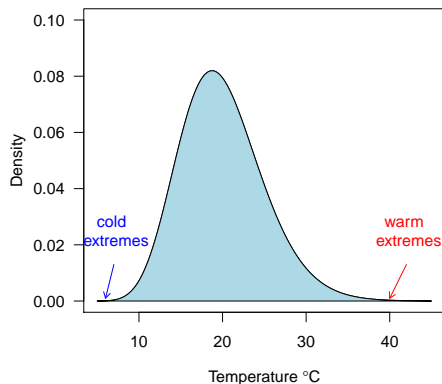
# Extremes may shift differently than means



# Generalized extreme value (GEV) distribution for sample maxima/minima

If  $Y_1, Y_2, \dots, Y_n$  is a random sample, then

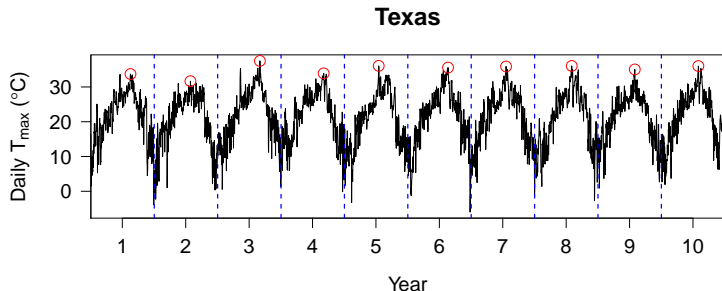
$$\max_{1 \leq i \leq n} Y_i \approx \text{GEV}(\mu_{(n)}, \sigma_{(n)}, \xi)$$



- ▶ Location ( $\mu_{(n)}$ ): the “center” of extremes
- ▶ Scale ( $\sigma_{(n)}$ ): the “spread” of extremes
- ▶ Shape ( $\xi$ ): the tail “heaviness” of extremes

# Model annual extremes as GEV distributions

- ▶ Example: annual maxima at Texas grid cell

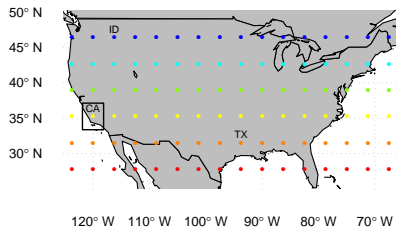
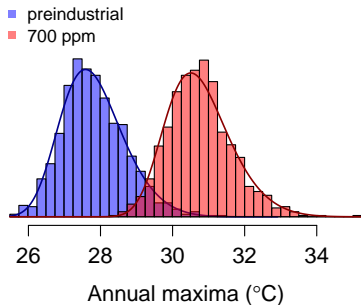


- ▶ Fit a  $\text{GEV}(\mu, \sigma, \xi)$  distribution to annual max/min



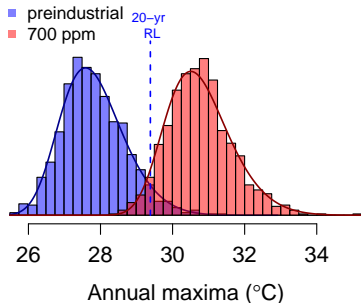
# Fit GEV to annual max

## California



# Compute the r-year return level for 289 ppm climate

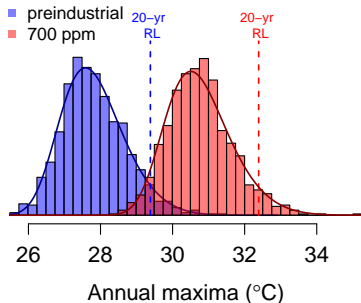
## California



$$\hat{R}L_{20} = \hat{\mu}_0 - \frac{\hat{\sigma}_0}{\hat{\xi}_0} \left\{ 1 - (-\log(1 - 0.05))^{-\hat{\xi}_0} \right\}$$

# Compute the r-year return level for 700 ppm climate

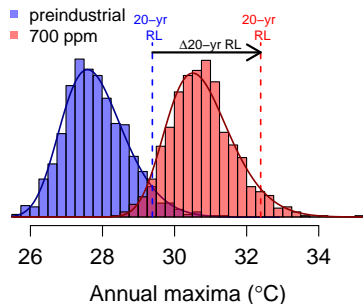
## California



$$\hat{R}L_{20} = \hat{\mu}_1 - \frac{\hat{\sigma}_1}{\hat{\xi}_1} \left\{ 1 - (-\log(1 - 0.05))^{-\hat{\xi}_1} \right\}$$

# Compute the difference in r-year return level

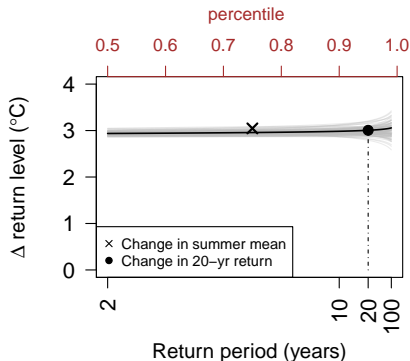
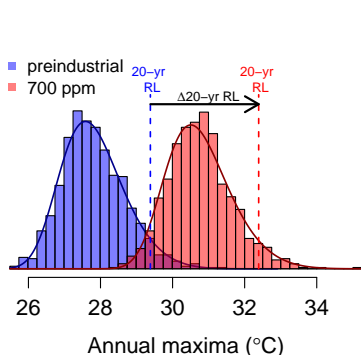
## California



$$\Delta 20\text{-yr RL} = \hat{R}L_{20} - \hat{R}L_{20}$$

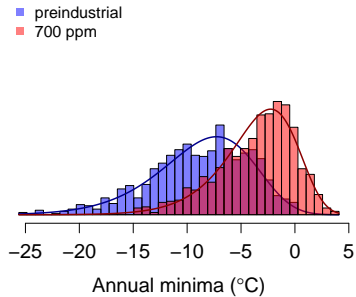
# Warm extremes shift with summer means

## California

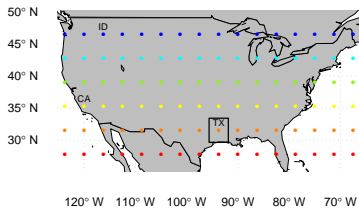


$$(\Delta\hat{\mu}, \Delta\hat{\sigma}, \Delta\hat{\xi}) = (2.9 **, 0.02, 0.01)$$

# Fit GEV to annual min

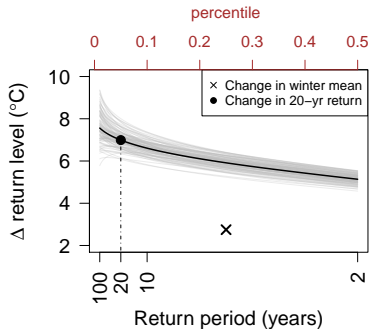
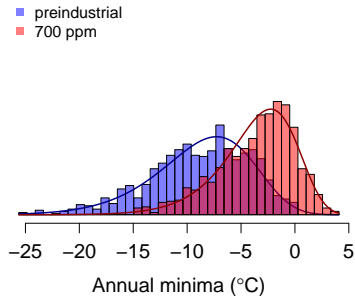


## Texas



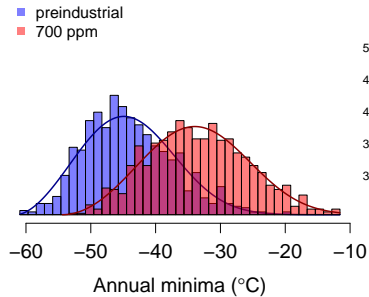
# Cold extremes shift more than winter means

## Texas

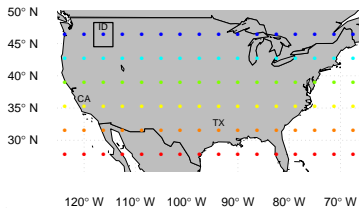


$$(\Delta\hat{\mu}, \Delta\hat{\sigma}, \Delta\hat{\xi}) = (4.8 **, -1.1 **, 0.05)$$

# Fit GEV to annual min



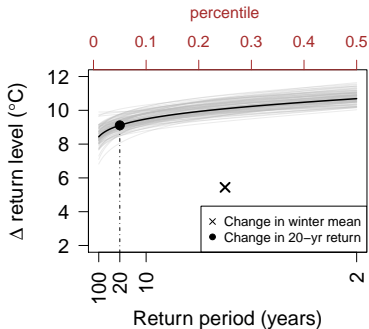
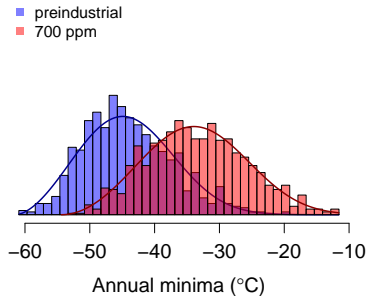
## Idaho





# Cold extremes shift more than winter means

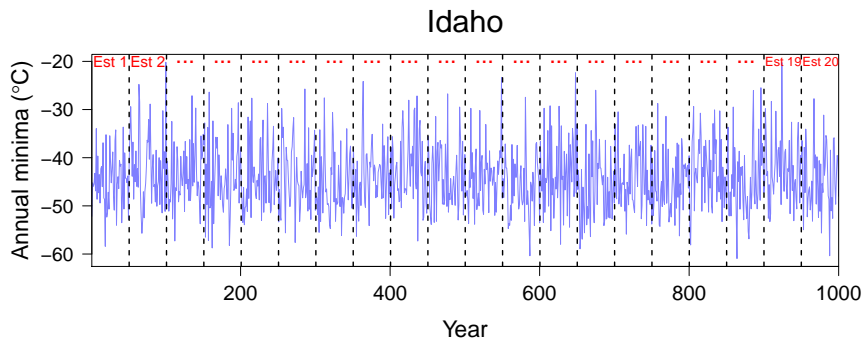
Idaho



$$(\Delta\hat{\mu}, \Delta\hat{\sigma}, \Delta\hat{\xi}) = (11.0 **, 0.7 **, 0.03)$$

## Part II: Data length on return level estimation

# What if we have shorter runs or data?

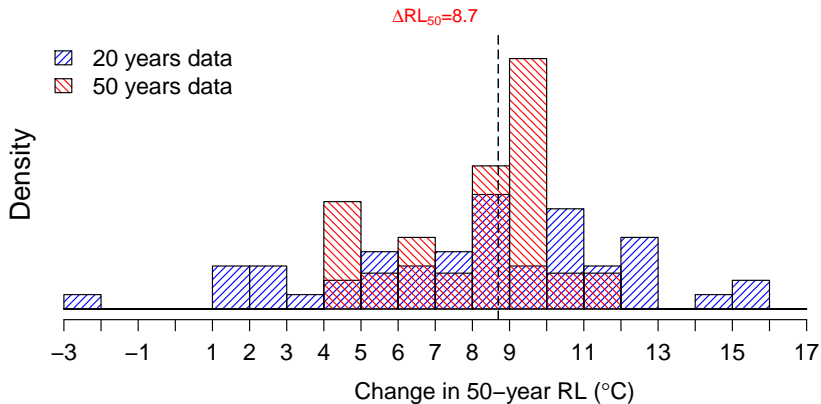


To assess the sampling error due to short runs

- ▶ divide the time series into segments
- ▶ refit GEV to each segment, compute the changes in return levels

# Sampling error is large for short runs

Estimates of change in 50-year RL



Sampling error:  $\mu$

Sampling error:  $\log(\sigma)$

Sampling error:  $\xi$

## Part III: Is annual block size long enough?



# GEV approximation to annual extremes

- ▶ **Extremal Types Theorem:**  $X_1, \dots, X_n \stackrel{\text{iid}}{\sim} F$ , then

$$\frac{\max_{i=1}^n X_i - b_n}{a_n} \xrightarrow{n \rightarrow \infty} G$$

- ▶ **In practice:** we assume annual extremes follow a GEV i.e.  
 $\max_{t=1}^{365} Y_t \sim \text{GEV}(\mu_1, \sigma_1, \xi_1)$
- ▶ **Question:** Is annual block size long enough?

# GEV approximation to annual extremes

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# Max-stability of GEV

- ▶ GEV is max-stable i.e.

$$G^k(a_k x + b_k) = G(x), \quad \forall k \in \mathbb{N}$$

for some  $a_k > 0$  and  $b_k$

- ▶ Fit GEV to annual extremes and decadal extremes and compare their  $\hat{\xi}$ 's

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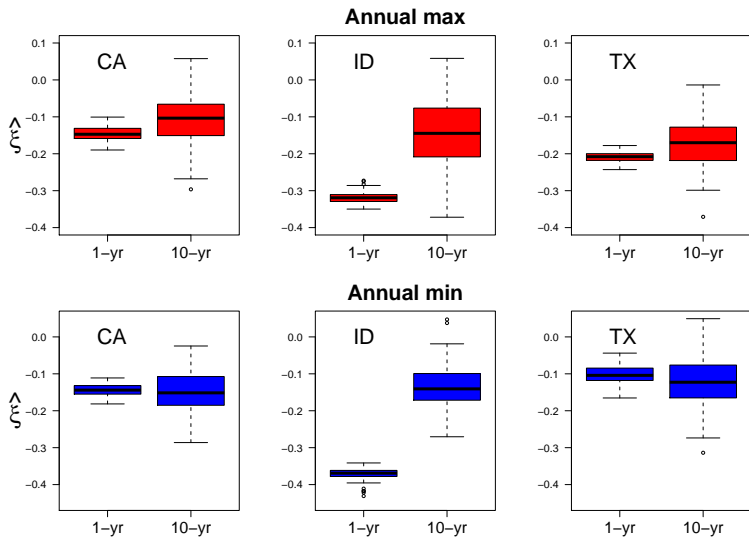
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- ▶ Fit GEV to annual extremes and decadal extremes and compare their  $\hat{\xi}$ 's

# Idaho, we have a problem





# Summary

## Changes in temperature extremes (in our model runs)

- ▶ **Warm extremes:** mainly due to the summer mean shifts
- ▶ **Cold extremes:** shifts larger than the winter mean shifts, with some changes in spread/skewness

## Data length on return level estimation

- ▶ **Sampling error is large for studying extremes in short datasets**

# Acknowledgments

- ▶ **RDCEP**: Center for Robust Decision Making on Climate and Energy Policy



- ▶ **STATMOS**: Research Network for Statistical Methods for Atmospheric and Oceanic Sciences



- ▶ Published in **Advances in Statistical Climatology, Meteorology and Oceanography (ASCMO)**