On-line Appendices
for

Firm Dynamics, Job Turnover, and Wage Distributions in an Open Economy

by

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1 The Revenue Function

Given the value added net of exporting costs,

$$R(z,l) = \max_m \left\{ G(zl^{\alpha}m^{1-\alpha}) - Pm \right\}, \tag{1}$$

the first order condition for m is given by

$$Pm = (1-\alpha)\frac{(\sigma-1)}{\sigma}\exp\left[d_H + \mathcal{I}^x d_F(\eta^0)\right]\left(zl^{\alpha}m^{(1-\alpha)}\right)^{\frac{\sigma-1}{\sigma}},$$

which determines the optimal choice for m as

$$m = \left(\frac{(1-\alpha)}{P}\frac{\sigma-1}{\sigma}\exp\left[d_H + \mathcal{I}^x d_F(\eta^0)\right]\right)^{\frac{\sigma}{\sigma-1}\Lambda}(zl^{\alpha})^{\Lambda}, \tag{2}$$

where $\Lambda = \frac{\sigma-1}{\sigma-(1-\alpha)(\sigma-1)} > 0$. Using this expression to eliminate m from (1), and noting that

$$\frac{\sigma}{\sigma-1}\Lambda = 1 + (1-\alpha)\Lambda,$$

and

$$\frac{\sigma-1}{\sigma} + \Lambda\frac{(1-\alpha)(\sigma-1)}{\sigma} = \frac{(\sigma-1)}{\sigma}\left[1+(1-\alpha)\Lambda\right] = \Lambda,$$

yields gross revenue at state (z,l):

$$\begin{aligned} G(z,l) &= \exp\left[d_H + \mathcal{I}^x d_F(\eta^0)\right](zl^\alpha)^{\frac{\sigma-1}{\sigma}}\left\{\left(\frac{(1-\alpha)}{P}\frac{\sigma-1}{\sigma}\exp\left[d_H+\mathcal{I}^x d_F(\eta^0)\right]\right)^{\frac{\sigma}{\sigma-1}\Lambda}(zl^\alpha)^\Lambda\right\}^{\frac{(1-\alpha)(\sigma-1)}{\sigma}}, \\ &= \exp\left[d_H + \mathcal{I}^x d_F(\eta^0)\right](zl^\alpha)^{\frac{\sigma-1}{\sigma}}\left[\left(\frac{1-\alpha}{P}\right)\left(\frac{\sigma-1}{\sigma}\right)\exp\left(d_H+\mathcal{I}^x d_F(\eta^0)\right)\right]^{(1-\alpha)\Lambda}(zl^\alpha)^{\Lambda\frac{(1-\alpha)(\sigma-1)}{\sigma}}, \\ &= P^{-(1-\alpha)\Lambda}\left[(1-\alpha)\left(\frac{\sigma-1}{\sigma}\right)\right]^{(1-\alpha)\Lambda}\left(\exp\left[d_H+\mathcal{I}^x d_F(\eta^0)\right]\right)^{\frac{\sigma}{\sigma-1}\Lambda}(zl^\alpha)^\Lambda. \end{aligned}$$

We can now derive a parameterized version of the net revenue function

$$R(z,l) = \Delta(z,l)\,(zl^\alpha)^\Lambda - c_x\mathcal{I}^x(z,l). \tag{3}$$

From (2), optimal expenditures on intermediate inputs are:

$$Pm = P^{-(1-\alpha)\Lambda}\left[\left((1-\alpha)\frac{\sigma-1}{\sigma}\right)\exp\left[d_H+\mathcal{I}^x d_F(\eta^0)\right]\right]^{\frac{\sigma}{\sigma-1}\Lambda}(zl^\alpha)^\Lambda.$$

Subtracting this expression and fixed exporting costs from gross revenues yields:

$$\begin{aligned} R(z,l) &= G(z,l) - Pm - c_x\mathcal{I}^x \\ &= \left[1-(1-\alpha)\frac{\sigma-1}{\sigma}\right]P^{-(1-\alpha)\Lambda}\left((1-\alpha)\frac{\sigma-1}{\sigma}\right)^{(1-\alpha)\Lambda}\left(\exp\left[d_H+\mathcal{I}^x d_F(\eta^0)\right]\right)^{\frac{\sigma}{\sigma-1}\Lambda}(zl^\alpha)^\Lambda - c_x\mathcal{I}^x \\ &= \left[\frac{\sigma-(1-\alpha)(\sigma-1)}{\sigma}\right]P^{-(1-\alpha)\Lambda}\left((1-\alpha)\frac{\sigma-1}{\sigma}\right)^{(1-\alpha)\Lambda}\left(\exp\left[d_H+\mathcal{I}^x d_F(\eta^0)\right]\right)^{\frac{\sigma}{\sigma-1}\Lambda}(zl^\alpha)^\Lambda - c_x\mathcal{I}^x \\ &= P^{-(1-\alpha)\Lambda}\left(\frac{\sigma-1}{\sigma\Lambda}\right)\left((1-\alpha)\frac{\sigma-1}{\sigma}\right)^{(1-\alpha)\Lambda}\left(\exp\left[d_H+\mathcal{I}^x d_F(\eta^0)\right]\right)^{\frac{\sigma}{\sigma-1}\Lambda}(zl^\alpha)^\Lambda - c_x\mathcal{I}^x \\ &= \Theta P^{-(1-\alpha)\Lambda}\exp\left[d_H+\mathcal{I}^x d_F(\eta^0)\right]^{\frac{\sigma}{\sigma-1}\Lambda}(zl^\alpha)^\Lambda - c_x\mathcal{I}^x, \end{aligned}$$

where $\Theta = \left(\frac{1}{(1-\alpha)\Lambda}\right)\left[\frac{(1-\alpha)(\sigma-1)}{\sigma}\right]^{\frac{\sigma}{\sigma-1}\Lambda}$.

2 The Wage Functions

2.1 Hiring Wages

Note that depending on whether the firm is hiring or not, profits are given by

$$\pi(z',l,l') = \begin{cases} R(z',l') - w_h(z',l')l' - C(l,l') - c_p & \text{if } l' > l \\ R(z',l') - w_f(z',l')l' - C(l,l') - c_p & \text{otherwise,} \end{cases} \tag{4}$$

where c_p, the per-period fixed cost of operation, is common to all firms.

In order to characterize wages in hiring firms, we first determine the total surplus for a firm and a worker that are matched in the end-of-period state (z', l'). At the time of bargaining, the surplus that the marginal worker generates for the firm is given by

$$\Pi^{firm}(z', l, l') = \frac{1}{1+r}\left[\frac{\partial \pi(z', l, l')}{\partial l'} + \frac{\partial \mathcal{V}(z', l')}{\partial l'}\right].$$

Note that at the time of bargaining, the vacancy posting and matching process are over and the costs of vacancy postings are sunk. As a result, if bargaining fails, the firm is simply left with fewer workers. Thus we only use the relevant part of the profit function for hiring firms, i.e., when $l' > l$ in (4), denoted by $\pi(z', l, l')$. The surplus that a marginal worker generates consists of two parts: the current increase in the firm's profits, i.e., marginal revenue product net of wages, and the increment to the value of being in state (z', l') at the start of the next period. If the firm does not exit next period, i.e., if $\mathcal{V}(z', l') > 0$, the marginal worker will have a positive value only if the firm expands. Otherwise, the firm will incur the dismissal cost, c_f. If the firm exits, its expected marginal value from the current marginal hire will be zero.

Similarly, the surplus for the marginal worker who is matched by a hiring firm in the end-of-period state (z', l') is

$$\Pi^{work}(z', l') = \frac{1}{1+r}\left[w_h(z', l') + J^e(z', l') - (b + J^o)\right],$$

where the worker enjoys $w_h(z', l')$ in the current period, and starts the next period in a firm with the beginning-of-period state (z', l'). If bargaining fails, the worker remains unemployed this period, engages in home production of b, and starts the next period in state o.

The worker and firm split the total surplus by Nash bargaining where the bargaining power of the firm is given by β:

$$\beta \Pi^{firm}(z', l, l') = (1-\beta)\Pi^{wor\ker}(z', l').$$

Wages are thus determined as a solution to the following equation:

$$\beta\left[\frac{\partial \pi(z', l, l')}{\partial l'} + \frac{\partial \mathcal{V}(z', l')}{\partial l'}\right] = (1-\beta)\left[w_h(z', l') + J^e(z', l') - (b + J^o)\right]. \tag{5}$$

Note that theoretically we cannot rule out the case in which a firm hires in the current period and exits at the beginning of the next period. The bargaining outcome depends on the decision to exit or continue which is made by the time of bargaining. We analyze these two cases separately.

1. Exiting firms: If the firm is going to exit next period, i.e., $\mathcal{I}^c(z', l') = 0$, we have $\partial\mathcal{V}(z', l')/\partial l' = 0$ and $J^e(z', l') = J^u$ from the definition of J^e. In this case, $\partial\mathcal{V}(z', l')/\partial l'$ cancels with $J^e - J^o$ in (5) since $J^o = J^u$ in equilibrium. We are left with

$$\beta\frac{\partial\pi(z', l, l')}{\partial l'} = (1-\beta)[w_h(z', l') - b]. \tag{6}$$

Using the definition of $\pi(z', l')$ from (4), and rearranging terms, equation (6) becomes

$$\frac{\partial w_h(z', l')}{\partial l'}\beta l' + w_h(z', l') - \beta\frac{\partial R(z', l')}{\partial l'} - (1-\beta)b = 0,$$

which is the same as equation (10) in Bertola and Garibaldi (2001). From (3) we have:

$$\frac{\partial R(z', l')}{\partial l'} = \Delta\alpha\Lambda\left(z'\right)^{\Lambda}\left(l'\right)^{\alpha\Lambda-1}.$$

Here, we suppressed the dependence of $\Delta(\cdot)$ on l' since $\partial\Delta/\partial l' = 0$ if the firm's exporting decision does not depend on the marginal worker. Since workers bargain individually and simultaneously with the firm, no single worker will be taken as the marginal worker for the export decision. Accordingly, retracing Bertola and Garibaldi's (2001) derivation we obtain:

$$\begin{aligned}
w_h(z', l') &= (1-\beta)b + l^{-\frac{1}{\beta}}\int_0^l u^{\frac{1-\beta}{\beta}}\Delta\alpha\Lambda\left(z'\right)^{\Lambda}\left(l'\right)^{\alpha\Lambda-1}du \\
&= (1-\beta)b + \Delta\alpha\Lambda\left(z'\right)^{\Lambda}\left(l'\right)^{-\frac{1}{\beta}}\int_0^{l'} u^{\frac{1}{\beta}+\alpha\Lambda-2}du \\
&= (1-\beta)b + \frac{1}{\frac{1}{\beta}+\alpha\Lambda-1}\Delta\alpha\Lambda\left(z'\right)^{\Lambda}\left(l'\right)^{\alpha\Lambda-1} \\
&= (1-\beta)b + \frac{\beta}{1-\beta+\alpha\beta\Lambda}\underbrace{\Delta\alpha\Lambda\left(z'\right)^{\Lambda}\left(l'\right)^{\alpha\Lambda-1}}_{=\partial R(z', l')/\partial l'}.
\end{aligned}$$

In this case, the worker is paid a fraction of her marginal revenue plus her share of the outside option b.

2. Continuing Firms: In this case, we have $\mathcal{V}(z', l') > 0$. There is an expected gain from keeping the marginal worker because of the possibility of further hiring next period. The worker's expected gain in the beginning of the next period (when she still has a chance to leave the firm and search) is $J^e(z', l') - J^u$. In line with Bertola and Caballero (1994) and Bertola and Garibaldi (2001), we assume that the firm-worker pair shares the expected match surplus in the same manner they split current rents, i.e., $J^e(z', l') - J^u$ cancels with the expected gain of the firm in (5). In the event of

a contraction, however, the firm cannot enforce contracts that require laid-off workers to pay their share of firing costs. As a result, expected firing costs, $P_f(z', l')c_f$, are subtracted from firm surplus in the current period:

$$\beta \left[\frac{\partial \pi(z', l, l')}{\partial l'} - P_f(z', l')c_f \right] = (1-\beta)[w_h(z', l') - b],$$

Conditional on the firm not hiring, the possibility of losing one's job, $p_f(z', l)$, is

$$p_f(z', l) = \frac{l - L(z', l)}{l},$$

and the probability of being fired next period is then given by

$$P_f(z', l') = E_{z''/z'} \{ \left[1 - \mathcal{I}^h(z'', l') \right] p_f(z'', l') \}.$$

The wage schedule for expanding firms that will stay in the market next period is then given by

$$w_h(z', l') = (1-\beta)b + \frac{\beta}{1-\beta+\alpha\beta\Lambda} \Delta\alpha\Lambda \left(z'\right)^{\Lambda} \left(l'\right)^{\alpha\Lambda-1} - \beta P_f(z', l')c_f.$$

2.2 Firing Wages

To derive the firing wage schedule, we begin by writing the value of employment at a firing firm in the interim stage as

$$J_f^e(z', l) = \frac{1}{1+r} \left[p_f(z', l)(1+r)J^u + \left(1 - p_f(z', l)\right) \left(w_f(z', l') + J^e(z', l')\right) \right],$$

where $l' = L(z', l)$. This expression reflects the fact that workers who are not fired are paid just enough to retain them. Since workers are indifferent between staying and leaving, the two outcomes inside the bracket have equal value, i.e.,

$$w_f(z', l') + J^e(z', l') = (1+r)J^u,$$

which yields the wage schedule according to which workers in firing firms are paid:

$$w_f(z', l') = rJ^u - [J^e(z', l') - J^u].$$

3 Steady State Equilibrium

Let the transition density of the Markov process on z be denoted by $h(z'|z)$. Given a measure of aggregate expenditure abroad denominated in foreign currency, D_F^*, a steady state equilibrium for a small open economy consists of: a measure of domestic differentiated goods N_H; an exact price index for the composite good P; an aggregate domestic demand index for industrial goods D_H; aggregate income I; a measure of workforce in services L_s; a measure workers in differentiated goods sector L_q; a measure of workers searching for jobs in the industrial sector U; a measure of unemployed workers L_u; the job finding rate $\widetilde{\phi}$; the vacancy filling rate ϕ; the exit rate μ_{exit}; the fraction of firms exporting μ_x; the measure of entrants M; the value and associated policy functions $\mathcal{V}(z,l)$, $L(z,l)$, $\mathcal{I}^h(z,l)$, $\mathcal{I}^c(z,l)$, $\mathcal{I}^x(z,l)$, J^o, J^u, J^s, and J^e; the wage schedules $w_h(z,l)$ and $w_f(z,l)$; the exchange rate k; and end-of period and interim distributions $\psi(z,l)$ and $\widetilde{\psi}(z,l)$ such that:

1. **Steady state distributions:** In equilibrium, $\psi(z,l)$ and $\widetilde{\psi}(z',l)$ reproduce themselves through the Markov processes on z, the policy functions, and the productivity draws upon entry. In order to define the interim distribution, $\widetilde{\psi}(z,l)$, let $\widetilde{\widetilde{\psi}}(z',l)$ be the interim frequency measure of firms defined as

$$\widetilde{\widetilde{\psi}}(z',l) = \begin{cases} \int_z h(z'|z)\psi(z,l)\mathcal{I}^c(z,l)dz \text{ if } l \neq l_e \\ \psi_e(z') + \int_z h(z'|z)\psi(z,l)\mathcal{I}^c(z,l)dz \text{ if } l = l_e \end{cases}.$$

   Then, $\widetilde{\psi}(z',l)$ is given by

$$\widetilde{\psi}(z',l) = \frac{\widetilde{\widetilde{\psi}}(z',l)}{\int_{z'}\int_l \widetilde{\widetilde{\psi}}(z',l)dz'dl},$$

   while the end-of period distribution is

$$\psi(z',l') = \frac{\int_l \widetilde{\psi}(z',l)\mathcal{I}_{(L(z',l),l')}dl}{\int_{z'}\int_l \widetilde{\psi}(z',l)\mathcal{I}_{(L(z',l),l')}dz'dl},$$

   where $\mathcal{I}_{(L(z',l),l')}$ is an indicator function with $\mathcal{I}_{(L(z',l),l')} = 1$ if $L(z',l) = l'$.

2. **Market clearance in the service sector:** Demand for services comes from two sources: consumers spend a $(1-\gamma)$ fraction of aggregate income I on it, and firms demand it to pay their fixed operation and exporting costs, as well as labor adjustment and market entry costs. Aggregate income I itself is the sum of wage income earned by service and industrial sector workers, market services supplied by unemployed workers,

tariff revenues rebated to worker-consumers, and aggregate profits in the industrial sector distributed to worker-consumers who own the firms.

The average labor adjustment cost is given by

$$\overline{c} = \int_z \int_l C(l, L(z,l))\widetilde{\psi}(z,l)dldz.$$

The market clearance condition is then given by

$$L_s + bL_u = (1-\gamma)I + N_H(\overline{c} + c_p + \mu_x c_x) + Mc_e.$$

3. **Labor market clearing:** Total production employment in the industrial sector is given by

$$L_q = N_H \overline{l} = N_H \int_z \int_l l\psi(z,l)dldz,$$

where

$$\overline{l} = \int_z \int_l l\psi(z,l)dldz \tag{7}$$

is the sector's average employment. Every period a fraction μ_l of workers in that sector is laid off due to exits and downsizing:

$$\mu_l = \frac{\int_z \int_l [1 - \mathcal{I}^c(z,l)]l\psi(z,l)dldz + \int_z \int_l \mathcal{I}^c(z,l)\mathcal{I}^f(z,l)[l - L(z,l)]\psi(z,l)dldz}{\int_z \int_l l\psi(z,l)dldz}$$

Then, the equilibrium flow condition is

$$U\widetilde{\phi} = L_q \mu_l.$$

In equilibrium, a measure of $L_u = (1-\widetilde{\phi})U$ of workers who search do not find a job, and labor market clearing condition is given by

$$1 = L_s + L_q + L_u.$$

On the vacancies side, the aggregate number of vacancies in this economy is given by

$$V = N_H \int_z \int_l v(z,l)\mathcal{I}^h(z,l)\frac{\widetilde{\psi}(z,l)}{\mu_h}dldz = N_H \overline{v},$$

where

$$\overline{v} = N_H \int_z \int_l v(z,l)\mathcal{I}^h(z,l)\frac{\widetilde{\psi}(z,l)}{\mu_h}dldz, \tag{8}$$

is the average level of vacancies, and μ_h is the fraction of hiring firms:

$$\mu_h = \int_z \int_l \mathcal{I}^h(z,l)\widetilde{\psi}(z,l)dldz.$$

The total number of vacancies, V, together with U, determines matching probabilities $\phi(V,U)$ and $\widetilde{\phi}(V,U)$ that firms and workers take as given.

4. **Firm turnover:** In equilibrium, there is a positive mass of entry M every period so that the free entry condition

$$\mathcal{V}_e = \int_z \mathcal{V}(z,l_e)\psi_e(z)dz \leq c_e, \tag{9}$$

holds with equality. The fraction of firms exiting is implied by the steady state distribution and the exit policy function,

$$\mu_{exit} = \int_z \int_l [1 - \mathcal{I}^c(z,l)]\psi(z,l)dldz + \delta,$$

and measure of exits equals that of entrants,

$$M = \mu_{exit} N_H.$$

5. **Trade balance:** Adding up final and intermediate demand, total domestic expenditures on imported varieties equals $D_H\left(\tau_a\tau_c k\right)^{1-\sigma}$. Taking the import tariff into account, domestic demand for foreign currency (expressed in domestic currency) is thus $\frac{D_H(\tau_a\tau_c k)^{1-\sigma}}{\tau_a} = D_H\tau_a^{-\sigma}(\tau_c k)^{1-\sigma}$. Tariff revenue is given by $D_H\tau_a^{-\sigma}(\tau_c k)^{1-\sigma}(\tau_a - 1)$, and is returned to worker-consumers in the form of lump-sum transfers. Total export revenues are $\frac{kD_F^* P_X^{*1-\sigma}}{\tau_c}$ with the foreign market price index for exported goods P_X^* as defined in Section I.C in the paper. Trade is balance given by

$$\underbrace{\frac{D_H\left(\tau_a\tau_c k\right)^{1-\sigma}}{\tau_a}}_{\text{domestic demand for foreign currency}} = \underbrace{\frac{kD_F^* P_X^{*1-\sigma}}{\tau_c}}_{\text{export revenue}}.$$

The exchange rate k moves to ensure that this condition holds. Balanced trade ensures that national income matches national expenditure.

6. Workers are indifferent between taking a certain job in the undifferentiated sector and searching for a job in the industrial sector: $J^o = J^s = J^u$.

4 Numerical Solution Algorithm

To compute the value functions, we discretize the state space on a log scale using 550 grid points for employment and 60 grid points for productivity. We set the maximum firm size as 2000 workers and numerically check that this is not restrictive. In the steady state, a negligible fraction of firms reaches this size, which is also the case in the data. The algorithm works as follows:

1. Formulate guesses for $D_H, w_f(z,l), w_h(z,l), d_F$ and ϕ. Given ϕ, calculate $\widetilde{\phi} = (1-\phi^{\theta})^{1/\theta}$.

2. Given D_H, $w_f(z,l), d_F$, ϕ and $w_h(z,l)$, calculate the value function for the firm, $\mathcal{V}(z,l)$ as
$$\mathcal{V}(z,l) = \max\left\{0, \quad \frac{1-\delta}{1+r} E_{z'|z} \max_{l'} \left[\pi(z',l,l') + \mathcal{V}(z',l')\right]\right\}, \tag{10}$$
and find the associated decision rules for exiting, hiring, and exporting. Calculate the expected value of entry, $\mathcal{V}_e$, using equation (9). Compare $\mathcal{V}_e$ with c_e. If $\mathcal{V}_e > c_e$, decrease D_H (to make entry less valuable) and if $\mathcal{V}_e < c_e$, increase D_H (to make entry more valuable). Go back to Step 1 with the updated value of D_H and repeat until D_H converges.

3. Given $w_f(z,l), d_F$, ϕ and the converged value of D_H from Step 2, update $w_f(z,l)$. To do this, first calculate $J^e(z',l')$ using
$$J^e_h(z',l) = \frac{1}{1+r}\big[w_h(z',l') + J^e(z',l')\big], \tag{11}$$
and
$$\begin{aligned} J^e(z,l) = &\big[\delta + (1-\delta)\big(1-\mathcal{I}^c(z,l)\big)\big] J^u \\ &+ (1-\delta)\mathcal{I}^c(z,l) \max\big\{J^u, E_{z'|z}\big[\mathcal{I}^h(z',l)J^e_h(z',l) + (1-\mathcal{I}^h(z',l))J^u\big]\big\}, \end{aligned} \tag{12}$$
and imposing the equilibrium condition $J^u = J^o$. Given $J^e(z,l)$, update firing wage schedule using
$$w_f(z',l') = rJ^u - [J^e(z',l') - J^u]. \tag{13}$$
Compare the updated firing wage schedule with the initial guess. If they are not close enough go back to Step 1 with the new firing wage schedule and repeat Steps 1 to 3 until w_f converges. Note that if firing wages are too high, then $J^e(z,l)$—the value of being in a firm at the start of a period— is high, since the firm is less likely to fire workers. A high value of $J^e(z,l)$, however, lowers firing wages. Similarly, if firing wages are too low, then J^e is low, which pushes firing wages up.

4. Given d_F and ϕ, the converged value of D_H from step 2, and the converged value of $w_f(z,l)$ from Step 3, update $w_h(z,l)$ using

$$w_h(z',l') = (1-\beta)b + \frac{\beta}{1-\beta+\alpha\beta\Lambda} \underbrace{\Delta(z',l')\alpha\Lambda\left(z'\right)^{\Lambda}\left(l'\right)^{\alpha\Lambda-1}}_{=\partial R(z',l')/\partial l'} - \beta P_f(z',l')c_f. \qquad (14)$$

5. Given ϕ, the converged value of D_H from Step 2, the converged value of $w_f(z,l)$ from Step 3, and the converged value of $w_h(z,l)$ from step 4, calculate the trade balance. To do this:

   (a) Given firms' decisions, calculate $\psi(z,l)$ and $\widetilde{\psi}(z,l)$, the stationary probability distributions over (z,l) at the end and interim states, respectively.

   (b) Given $\widetilde{\psi}(z,l)$, calculate the average number of vacancies and the average employment in the industrial sector using equations (7) and (8).

   (c) Take a guess for N_H. Given N_H and $\overline{v}$, calculate the mass of unemployed U in the industrial sector from

   $$\phi(V,U) = \frac{M(V,U)}{V} = \frac{U}{((vN_H)^{\theta}+U^{\theta})^{1/\theta}},$$

   which is one equation in one unknown. Given U, calculate $L_u = (1-\widetilde{\phi})U$. Then, given $\overline{l}$, the size employment in the service sector is given by $L_s = 1 - L_u - N_H\overline{l}$. Given N_H, L_s, L_u, M (mass of entrants), and I (aggregate income), check if supply and demand are equal in the service sector:

   $$\underbrace{L_s + bL_u}_{\text{supply}} = \underbrace{(1-\gamma)\,I + N_H(\overline{c}+c_p+\mu_x c_x) + Mc_e}_{\text{demand}}.$$

   Update N_H until supply equals demand.

   (d) Given the value of N_H from Step 4c, calculate exports and imports. If exports are larger than imports, lower d_F; if exports are less than imports, increase d_F. Go back to Step 1 with the updated value of d_F, and repeat until convergence.

6. Given the converged value of D_H from Step 2, the converged value of $w_f(z,l)$ from Step 3, the converged value of $w_h(z,l)$ from Step 4, and the converged value of d_F from Step 5, update ϕ. In order to do that, first calculate EJ_h^e using (11). Given EJ_h^e and $\widetilde{\phi}$, calculate J^u using

$$J^u = \left[\widetilde{\phi}EJ_h^e + \frac{(1-\widetilde{\phi})}{1+r}\left(b+J^o\right)\right]. \qquad (15)$$

If $J^o > J^u$, increase ϕ (to attract workers to the differentiated goods sector) and if $J^o < J^u$, we lower ϕ (to make the differentiated goods sector less attractive). Go back to Step 2, and repeat until ϕ converges.

4.1 Estimation Procedure

In our policy experiments, we use the complete algorithm above to compute equilibrium outcomes for given a set of parameters, including the cost of entry c_e. In these experiments, both d_F and D_H are equilibrium objects that respond to changes in τ_a, τ_c and c_f. While estimating the model, however, we use the Olley-Pakes intercept $\tilde{d}_H$ estimated from

$$\ln G_{it} = \widetilde{d}_H + \mathcal{I}_{it}^x d_F(\eta_0) + \left[\frac{\sigma-1}{\sigma}(1-\alpha)\right]\ln(Pm_{it}) + \varphi(\ln l_{it-1}, \ln l_{it}) + \xi_{it}. \qquad (16)$$

to calculate firms' net revenue schedule $R(\cdot)$. Similarly, we treat d_F as a moment to be matched: given $\tilde{d}_H$ and the simulated value of the foreign market size parameter D_F^*, we calculate η using

$$\eta^o = \arg\max_{0\leq\eta\leq1} d_F(\eta) = \left(1 + \frac{\tau_c^{\sigma-1} D_H}{k^\sigma D_F^*}\right)^{-1}, \qquad (17)$$

which allows to use the implied d_F directly in our solution algorithm. The equilibrium price level P and exchange rate k can easily be solved in equilibrium so that trade balance holds and $\tilde{d}_H$ is consistent with D_H. Also, assuming that the economy is in a steady state with positive entry, we back out c_e by setting it equal to the equilibrium value of entry $\mathcal{V}_e$. This approach to discipline the cost of entry c_e is in line with the quantitative literature (Hopenhayn and Rogerson 1993). These shortcuts allow us to skip Steps 2 and 5d in the estimation and considerably reduce the computation time.

5 Further Results and Data Sources

5.1 Labor Units

Since workers are all identical in the model economy, we measure the labor input l in terms of "effective worker" units in our estimation. This allows us to control for the effects of worker heterogeneity on output. In the plant-level data we use to estimate our model for the pre-reform period, we observe five categories of workers: managerial, technical, skilled, unskilled, and apprentice. For a given plant-year, effective labor l is the sum of all workers in the plant, each weighted by the average wage (including fringe benefits) for workers in its category. For each category of worker, the average wage is based on the mean real wage in the entire 10-year panel and expressed as a ratio to the average real wage for unskilled workers during the same period. Thus wage weights are constant across plants and time, and the only source of variation in l is variation in the employment level of at least one category of worker.

After fitting the model to the pre-reform data, we simulate Colombian reforms and decreased trade costs in Section II of the paper. To evaluate the success of the model in explaining post-reform outcomes, we wish to compare the firm size distribution as predicted by the model to its empirical counterpart. Since we do not have access to the plant-level data from the post-reform period, we don't observe the above-described variables used to construct effective labor. While The Colombian Statistical Agency DANE publishes summary statistics on the size distribution of plants for the 2000-2006 period (`http://www.dane.gov.co/index.php/industria/encuesta-anual-manufacturera-eam`), these are based on the number of total employees.

The following procedure facilitates the comparison of model based and empirical size distributions in both periods (see Figure 4 in the paper). Using the pre-reform plant level, we first fit total number of workers to a polynomial of effective labor l. We then use the coefficients from this regression to convert model-generated effective labor l units to worker count for both the estimated pre-reform and simulated post-reform periods. The blue and red bars in Figure 4 in the paper—representing the model-based size distributions—are generated using this transformation. The black and white bars representing the empirical size distributions are generated directly from the data using total number of employees.

5.2 Sectoral Labor Flows in Colombia

The Colombian Statistical Agency DANE publishes monthly labor market indicators. We accessed the following link on September 26, 2013:

http://www.dane.gov.co/files/investigaciones/empleo/ech/totalNacional/Mensual/IML_MensualTnacional_01_08.xls.

The file is in Spanish but variable names can be easily translated using online translators. In this file, the worksheets titled "*ocup ramas trim tnal*" indicates monthly sectoral urban employment levels (*Población ocupada según posición ocupacional, CABECERAS*). The worksheet titled "*cesantes ramas trim tnal*" reports last sector of employment for the unemployed (*Población desocupada censate según ramas de actividad anterior, CABECERAS*). We exclude agriculture and mining, and aggregate service industries. The ratio of outflows from employment to unemployment gives sectoral transition rates. For the 2000-2006, average transition rates are 0.137 for manufacturing and 0.148 for services.

5.3 Size-Wage Relation

Table A1 shows the effect of size (measured by the number of workers) and productivity on wages in the data and the model.

Table A1: **Wage-Size Relationship Controlling for Productivity**

$\ln w = \alpha + \beta_l \ln l + \beta_z \ln z + \varepsilon$	**Non-expanding firms**		**Expanding firms**	
	Data	**Model**	**Data**	**Model**
β_l	0.075	-1.89	0.064	-1.3
	(0.002)	(0.048)	(0.002)	(0.018)
β_z	0.586	0.964	0.502	0.465
	(0.005)	(0.117)	(0.006)	(0.071)
R^2	0.46	0.65	0.51	0.58

Notes: l is effective workers defined in online Appendix 5.1, w is average wage per effective worker, and z is firm-level productivity implied by equation (30) *in the paper given the data and estimated parameter values.*

5.4 Isolated Effects of Changes in Tariffs, Firing Costs and Iceberg Trade Costs

Table A2 shows the results when tariffs, firing costs and iceberg trade costs are changed one at a time.

Table A2: **Isolated Effects**

	Baseline	(I)	(II)	(III)
τ_a (ad valorem tariff rate)	1.21	1.11	1.21	1.21
c_f (firing cost)	0.6	0.6	0.3	0.6
τ_c (iceberg trade cost)	2.5	2.5	2.5	2.1
Size Distribution				
20th percentile	16	17	16	17
40th percentile	25	26	24	27
60th percentile	39	41	38	46
80th percentile	78	84	78	104
Average firm size	46	49	46	57
Firm Growth Rates				
<20th percentile	1.15	1.14	1.12	1.17
20th-40th percentile	0.26	0.28	0.27	0.29
40th-60th percentile	0.18	0.19	0.18	0.22
60th-80th percentile	0.15	0.16	0.16	0.19
Aggregates				
% of firms exporting	1	1.339	0.989	2.191
Revenue share of exports	1	1.339	0.999	2.060
Exit rate	1	0.949	0.832	1.025
Job turnover	1	1.032	1.006	1.096
Mass of firms	1	0.918	1.001	0.764
Unemployment rate in the industrial sector	1	1.076	1.001	1.213
Industrial share of employment	1	0.985	1.002	0.949
Standard deviation of log wages (firms)	1	1.002	0.979	1.035
Standard deviation of log wages (workers)	1	1.002	0.978	0.989
Log 90-10 wage ratio (firms)	1	1.010	0.978	1.045
Log 90-10 wage ratio (workers)	1	1.020	0.981	1.009
Standard deviation of workers' value (J)	1	1.057	1	1.084
Log 90-10 ratio of workers' value (J)	1	1.036	0.973	1.066
Exchange rate (k)	1	0.97	1.05	0.727
Real income	1	1.042	0.993	1.180

Notes: Each column presents the outcomes from an isolated counterfactual scenario. Columns (I): reducing tariffs, Columns (II): reducing firing costs, Columns (III): reducing iceberg trade costs.

6 Robustness to the Choice of Model Period

To isolate the role of periodicity in driving our results, we hold the estimation strategy fixed by using our estimated revenue function and productivity process to approximate their quarterly counterparts.[1] Then we re-estimated remaining parameters using the same moment vector as in the annual baseline, aggregating simulated quarterly outcomes on flow variables to their annual equivalents, and taking simulated fourth quarter realizations on stock variables to be representative of their annual counterparts (as is done in the annual

[1] We emphasize "approximate" here because there is no analytical relationship linking the parameters of the annual objects to their quarterly counterparts. The reason is that our revenue function characterizes logs of flows, and thus annual variables are not linear combinations of quarterly variables.

manufacturing surveys).

Specifically, we kept our estimate of the elasticity of value added with respect to labor ($\alpha\Lambda$) based on annual data, and we chose the root of the quarterly productivity process to replicate our estimate of persistence in the annual process: $\rho_q = \rho_a^{1/4}$. Likewise, we adjusted the discount rate to $r_q = (1 + r_a)^{1/4} - 1$, and we shifted the log revenue function intercept $\widetilde{d}_H$ to put revenue flows on a quarterly basis. Finally, since we saw no good way to approximate the relationship between the variance of the innovations in the annual data ($\sigma^2_{z,a}$) and the variance of the innovations in the quarterly data ($\sigma^2_{z,q}$), we included $\sigma^2_{z,q}$ in the set of parameters to be estimated.

Tables A3 and A4 present the resulting parameter estimates and the fit of the model. The quarterly version doesn't fit as well as the annual baseline, perhaps because of the way we have constrained our revenue function estimates. Nonetheless, the quarterly results do give us some insight into the effects of periodicity choice on parameter estimates and model performance.

The major differences in parameter estimates are in the elasticity of substitution σ, the elasticity of the matching function θ, and the value of home production b. The change in b can be explained by the effect of model frequency on wage inequality. Allowing workers to search more frequently increases their reservation wages, which in turn affects the entire wage schedule. Other things equal, this would lower wage dispersion in the model. So, in order to still match the dispersion of *log* wages, the quarterly calibration lowers the constant term $(1 - \beta)b$ in the hiring wage schedule (14). It does so by reducing b from 0.403 to 0.28.[2] The other major change in parameter values is the decrease in matching function elasticity θ from 1.875 to 0.839. This compensates for the fact that, other things equal, switching to a quarterly frequency would have increased labor market tightness as workers enjoyed more opportunities to match with firms. In turn, this would have made it more difficult for firms hire, and thus shifted the simulated firm size distribution leftward. Dropping θ improves the ability of firms to meet workers over the relevant range of (U, V) values, and thus prevents this from occurring. Other parameter values such as exogenous exit rate δ and the initial firm size l_e drop in proportion to the change in model frequency.

Table A5 addresses the main question of interest: how robust are the policy experiments in the paper to the unit of time used in the model? That is, it redoes Table 4 using the quarterly version of our model. Note that here, as in the paper, the "reforms and globalization exercise" (3rd column) is based on a level of iceberg costs τ_c that induces the observed post-reform fraction of firms that export. However, for the quarterly version, a

[2]Note that the unit of account is the service sector wage per period, so $b = 0.28$ from the quarterly estimation is directly comparable to the b from the annual baseline.

Table A3: **Parameters Estimated with SMM - annual vs quarterly**

Parameter	Description	Annual	Quarterly
σ	Elasticity of substitution	6.831	7.954
α	Elasticity of output with respect to labor	0.195	0.218
β	Bargaining power of workers	0.457	0.463
θ	Elasticity of the matching function	1.875	0.839
δ	Exogenous exit hazard	0.046	0.018
c_h	Scalar, vacancy cost function	0.696	0.466
λ_1	Convexity, vacancy cost function	2.085	2.384
λ_2	Scale effect, vacancy cost function	0.302	0.307
b	Value of home production	0.403	0.280
l_e	Initial size of entering firms	6.581	2.560
c_p	Fixed cost of operating	10.006	9.882
c_x	Fixed exporting cost	100.23	65.674
c_e	Entry cost for new firms	25.646	23.665
σ_z	Standard deviation of the z process	0.135	0.0797

smaller reduction in τ_c is needed to hit this target.

Comparing these simulation outcomes with the annual baseline in the manuscript reveals several patterns. First, qualitative responses to tariffs, iceberg costs and firing costs (τ_a, τ_c, and c_f) are robust to the model's periodicity. Job turnover, firm-level wage dispersion and industrial sector unemployment all increase in response to reductions in the "reforms and globalization" experiment (3rd column). Also, as in the annual version of the model, the size distribution shifts rightward while there is a sizeable drop in the mass of firms. However, the quantitative responses of job turnover and wage dispersion are somewhat different. In our annual model, the "reforms and globalization" experiment increased job turnover by about 12 percent, leaving worker-level log wage dispersion relatively stable. In the quarterly model, the same experiment increased job turnover by only 4 percent, but increased the standard deviation of log wages by 9 percent. These contrasts reflect the shifts in parameter estimates described above. With a smaller home production payoff, b, wages are more sensitive (percentagewise) to firm characteristics (z, l). On the other hand, a lower matching function elasticity makes job finding and fill rates less responsive to changes in aggregate labor market conditions, muting the increase in job turnover.

Table A4: **Data-based versus Simulated Statistics - annual vs quarterly**

Moment	**Data**	**Annual**	**Quarterly**	**Size Distribution**	**Data**	**Annual**	**Quarterly**
$E(\ln G_t)$	5.442	5.253	5.199	20th percentile cutoff	14.617	15.585	16.787
$E(\ln l_t)$	3.622	3.636	3.587	40th percentile cutoff	24.010	25.773	26.024
$E(\mathcal{I}_t^x)$	0.117	0.108	0.126	60th percentile cutoff	41.502	41.432	44.707
$var(\ln G_t)$	2.807	3.329	5.348	80th percentile cutoff	90.108	79.109	75.48
$cov(\ln G_t, \ln l_t)$	1.573	1.788	2.681	**Firm Growth Rates**			
$var(\ln l_t)$	1.271	1.219	1.679	<20th percentile	1.421	1.234	1.477
$cov(\ln G_t, \mathcal{I}_t^x)$	0.230	0.251	0.320	20th-40th percentile	0.255	0.271	0.427
$cov(\ln l_t, \mathcal{I}_t^x)$	0.152	0.160	0.194	40th-60th percentile	0.209	0.183	0.259
$cov(\ln G_t, \ln G_{t+1})$	2.702	2.196	-0.254	60th-80th percentile	0.184	0.151	0.167
$cov(\ln G_t, \ln l_{t+1})$	1.538	1.556	3.524	**Aggregate Turnover/**			
$cov(\ln G_t, \mathcal{I}_{t+1}^x)$	0.225	0.278	0.621	**Wage Dispersion**			
$cov(\ln l_t, \ln G_{t+1})$	1.543	1.394	1.653	Firm exit rate	0.108	0.120	0.092
$cov(\ln l_t, \ln l_{t+1})$	1.214	1.161	1.276	Job turnover	0.198	0.240	0.315
$cov(\ln l_t, \mathcal{I}_{t+1}^x)$	0.152	0.185	0.209	Std. dev. of log wages	0.461	0.426	0.471
$cov(\mathcal{I}_t^x, \ln G_{t+1})$	0.220	0.279	0.285	**Olley-Pakes Statistics**			
$cov(\mathcal{I}_t^x, \ln l_{t+1})$	0.149	0.201	0.203	$(1-\alpha)\left(\frac{\sigma-1}{\sigma}\right)$	0.685	0.687	0.684
$cov(\mathcal{I}_t^x, \mathcal{I}_{t+1}^x)$	0.089	0.073	0.089	d_F	0.090	0.094	0.094

Table A5: **Effects of Reforms and Globalization under Quarterly Model Period**

	Baseline	Reforms	Reforms & Globalization
τ_a (ad valorem tariff rate)	1.21	1.11	1.11
c_f (firing cost)	0.6	0.3	0.3
τ_c (iceberg trade cost)	2.5	2.5	2.2
Size Distribution			
20th percentile	17	17	19
40th percentile	26	27	32
60th percentile	45	48	61
80th percentile	75	81	116
Average firm size	31	34	46
Firm Growth Rates			
<20th percentile	1.48	1.47	1.51
20th-40th percentile	0.43	0.43	0.52
40th-60th percentile	0.26	0.27	0.34
60th-80th percentile	0.17	0.16	0.21
Aggregates			
% of firms exporting	1	1.44	2.73
Revenue share of exports	1	1.45	2.73
Exit rate	1	0.98	1.04
Job turnover	1	0.99	1.04
Mass of firms	1	0.91	0.67
Unemployment rate in the industrial sector	1	1	1.13
Industrial share of employment	1	0.98	0.98
Standard deviation of log wages (firms)	1	1.05	1.12
Standard deviation of log wages (workers)	1	1.06	1.09
Log 90-10 wage ratio (firms)	1	1.05	1.14
Log 90-10 wage ratio (workers)	1	1.07	1.12
Standard deviation of workers' value (J)	1	1.14	1.09
Log 90-10 ratio of workers' value (J)	1	1.16	1.12
Real income	1	1.02	1.21

Note: Aggregate statistics in the bottom panel are normalized by their baseline levels.