Hybrid multiplicative time-reversal imaging reveals the evolution of microseismic events: Theory and field-data tests

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ABSTRACT

The generation of microseismic events is often associated with induced fractures/faults during the extraction/injection of fluids. A full characterization of the spatiotemporal distribution of microseismic events provides constraints on fluid migration paths in the formations. We have developed a high-resolution source imaging method — a hybrid multiplicative time-reversal imaging (HyM-TRI) algorithm, for automatically tracking the spatiotemporal distribution of microseismic events. HyM-TRI back propagates the data traces from groups of receivers (in space and time) as receiver wavefields, multiplies receiver wavefields between all groups, and applies a causal integration over time to obtain a source evolution image. Using synthetic and field-data examples, we revealed the capability of the HyM-TRI technique to image the spatiotemporal sequence of asynchronous microseismic events, which poses a challenge to standard TRI methods. Moreover, the HyM-TRI technique is robust enough to produce a high-resolution image of the source in the presence of noise. The aperture of the 2D receiver array (azimuth coverage in 3D) with respect to the microseismic source area plays an important role on the horizontal and vertical resolution of the source image. The HyM-TRI results of the field data with 3D azimuthal coverage further verify our argument by producing a superior resolution of the source than TRI.

INTRODUCTION

Induced seismicity related to underground extraction/injection of liquids has been widely reported (Suckale, 2009; Shapiro, 2015). Wherever the injection pressure exceeds a certain level, microcracks and fractures may be created and activated, the process being accompanied by the emission of P- and S-waves. The extent of microseismic activity is often taken to represent the extent of fracture propagation from the injection point (e.g., Maxwell et al., 2002). The distribution of microseismic sources is therefore needed to infer the spatial extent of induced fractures.

Tracking microseismic event propagation is analogous to locating earthquake sources in global seismology. The arrival-time inversion methods to locate earthquakes (for a review, see Thurber and Rabinowitz, 2000) are simple, but they often need traveltime picking. The microseismic data likely have hundreds of thousands of events recorded by hundreds or more receivers, and they may contain unidentifiable P- or S-wave signals emerging from strong background noise, e.g., surface microseismic data (Duncan and Eisner, 2010). These make traveltime picking very challenging. Several waveform-based location methods without picking have been developed recently. A common approach is the back-projection imaging (BPI) method that back-projects the seismic P-wave seismograms recorded at an array or at a seismic network to a grid of possible source locations (Kao and Shan, 2004). The BPI technique has been demonstrated to provide detailed images of earthquake rupture propagation (Ishii et al., 2007). Folesky et al. (2015) adopt a modified BPI technique to obtain the spatiotemporal evolution (rupture) of microseismic events, based solely on the phase and coherency of seismic array signals.

Alternatively, the time-reversal imaging (TRI) method that relies on fully simulating wave propagation in the earth is a promising...
source location technique. The principle of TRI of a seismic event is based on the back propagation of full seismic data recorded by all receivers, so that all the propagated energy focuses on its initial source position (e.g., McMechan, 1982; Gajewski and Tessmer, 2005). There are many strategies to improve TRI, e.g., attenuation compensation (Zhu, 2014, 2015), deconvolution imaging condition (Douma and Snieder, 2015), interferometric imaging conditions (Sava, 2011), and using multicomponents and pressure data (Li and van der Baan, 2016). Several authors extend the TRI method to process P- and S-waves simultaneously for the source location (Artman et al., 2010; Haldorsen et al., 2013; Xue et al., 2016; Li and van der Baan, 2017; Yang and Zhu, 2019). Most of these approaches are very well-demonstrated to image a single source or multiple synchronous sources (e.g., Larmat et al., 2006; Steiner et al., 2008), but they are challenging for tracking the evolution of multiple asynchronous sources clustered along the time axis, i.e., the rupture process. The reason is that the TRI of continuous data with multiple asynchronous sources likely mixes the outgoing wavefield from the focused source and the incoming wavefield from next source, i.e., lacking the source sink (Fink, 2006). Kremers et al. (2011) provide a good explanation of this challenge of TRI for imaging finite rupture processes.

This challenge could potentially be solved by a new hybrid multiplicative TRI (HyM-TRI) technique. A multiplicative TRI (or M-TRI) method back-propagates each receiver wavefield individually and replaces the summation operator by a multiplication operator. Then, it applies a causal integration over time (Claerbout, 2010) to obtain a source evolution image, where the dimension of time can be interpreted as a relative time between source events. The multiplicative operator ensures better resolution but at a greater cost (because it introduces many back-propagations). The HyM-TRI method (Sun et al., 2015) ought to mitigate this cost by applying the multiplicative operator to back-propagated wavefields computed from groups of receivers. In terms of the location, this method requires no origin time of the event and leads to its absolute location. In our previous work (Sun et al., 2015), we demonstrated this approach in acoustic examples with a simple time sequence of a few perfect point sources. It remains unclear whether HyM-TRI can deal with long recording data with multiple asynchronous sources, how robust HyM-TRI performs in the presence of noise, and if it is feasible to be applied to field data. This is what we will address in this paper.

The goals of the present paper are, first, to detail the theory of the HyM-TRI technique, and second, to assess whether or not this method can provide rupture parameters by accurately reconstructing the spatiotemporal evolution of source events. We therefore test the technique on two data sets: first, a fully synthetic (yet realistic) 2D microseismic data set (giving a lot of flexibility to our testing) and, second, a 3D field data set (provided by an industry third party). The spatial, temporal, and magnitude distributions of synthetic microseismic events are defined by a 2D statistical rupture propagation model. Then, synthetic microseismic data are simulated by solving the viscoelastic wave equation to generate continuous waveforms. Next, we test the imaging technique on synthetic data using multiple asynchronous events at a short period, sparse and limited aperture surface arrays, and downhole arrays. Finally, we present an application to a field microseismic monitoring data set from a Marcellus shale hydraulic fracturing site (Pennsylvania, USA).

**METHODOLOGY**

In this section, we briefly review the methodology of the time-reversal source imaging method and detail the HyM-TRI method proposed by Sun et al. (2015).

**Time-reversal imaging**

Considering a point source, we have an acoustic Green’s function $G(x_r, x_t, t)$, which represents an impulse response observed at a receiver $x_r$, due to a source at $x_t$. The data $d(x_r, x_t, t)$ are recorded at $x_r$, where $t$ is in the range $[0, T]$:

$$d(x_r, x_t, t) = G(x_r, x_t, t) * S(t),$$

where $S(t)$ is a source function and the symbol “*$” represents the time convolution.

The TRI principle states that all the wavefields back propagating in time from receivers coincide in the correct source location. It consists of three steps: (1) reversing the recorded data in time, (2) back-propagating the time-reversed data as sources from receiver locations through an appropriate earth model, and (3) applying the focusing imaging condition. The back propagation of recorded seismic data $d(x_r, t)$ can be written mathematically as

$$W_d(x, t) = G(x, x_r, t) * d(x_r, T - t),$$

where $x$ is the space coordinate. Thus, the TRI is

$$I_{TR}(x) = ||W_d(x, t)||_{IC},$$

where $|| ||_{IC}$ denotes the focusing imaging condition, e.g., the maximum amplitude. Various source imaging conditions can be used to obtain the source image $I_{TR}(x)$ (e.g., Larmat et al., 2009; Sava, 2011; Douma and Snieder, 2015). The advantage of TRI is the avoidance of picking arrival times, which is usually the major factor for introducing uncertainties. TRI has been used for low signal-to-noise data, such as microseismic records or earthquake data, where we cannot easily pick the arrival times of the events (Steiner et al., 2008).

Figure 1a shows the forward propagation from a source. Figure 1b schematically illustrates the procedure of the TRI method, which back propagates the recorded data at three receivers simultaneously into the subsurface and searches for a focusing point with the maximum amplitude through the whole time axis. With a single source, the final image would be a focusing of the waves back-propagating until the final backward time step. However, in the case of multiple asynchronous sources with different onset times in continuous data, the focusing (e.g., peak amplitude) at intermediate times is difficult or even impossible to pick when it interferes with other back-propagated wavefields (corresponding to multiple sources). The reasons for this are, first, stacking (summing) the back-propagated wavefields from all the receivers at once results in an image that contains nonzeros across all wave propagation paths. Second, owing to the lack of the sink that absorbs elastic energy so that the time-reversed wavefield is canceled after focusing, a final focus will act as an initial source and continue to propagate in the computational domain (Fink, 2006). This makes these TRI methods not well-suited to image the migration of multiple asynchronous sources.
Hybrid multiplicative time-reversal imaging

The idea of M-TRI was inspired by the realization of distributed sensor networks for volcano earthquake monitoring, where the distributed data processing is performed on a single sensor for in situ and real-time needs (Song et al., 2009). Rather than back propagating the recorded data from all the receivers simultaneously, we treated the wavefield back propagated from each single receiver independently; i.e., each receiver treated as a single virtual source is broadcast into the medium. Then, we define microseismic hypocenters as the locations where all the back-propagated wavefields coincide in space and time. For a single source event, the receiver wavefields are multiplied and stacked over time to obtain a high-resolution source image (Sun et al., 2015).

In the case of multiple asynchronous sources, stacking over time only provides the image of all possible sources in space but it does not provide the spatiotemporal evolution (sequence) of sources. Here, we perform the causal integration over time (Claerbout, 2010) to highlight the importance of having the time evolution of accumulated seismicity source events in space as suggested by Sun et al. (2015). Specifically, we compute a new image-movie:

\[
I_{\text{M-TRI}}(x, t) = \int_0^t \prod_{i=1}^N W_d^{(i)}(x, \tau) d\tau,
\]

where \(N\) denotes the number of receivers. The image movie \(I_{\text{M-TRI}}(x, t)\) is an evolving map of microseismicity in time \(t\) that can be used to track rupture propagation. The last snapshot of \(I_{\text{M-TRI}}(x, t)\) corresponds to a stacked image \(I_{\text{M-TRI}}(x)\) of all source images. Replacing the causal integration by crosscorrelation in equation 4, M-TRI will be identical to the crosscorrelation imaging condition proposed by Nakata and Beroza (2016), which leads to \(I_{\text{M-TRI}}(x)\).

Figure 1c shows that we back-propagate the recorded trace from each receiver as a receiver wavefield. Applying the imaging condition in equation 4 to back-propagated receiver wavefields leads to nonzero values corresponding only to the focused source. The peak amplitude of \(I_{\text{M-TRI}}(x, t)\) is considered to be a focused source. Considering continuous data with multiple events, the peak amplitude evolution in time and space potentially provides estimates for source characters (e.g., migration velocity, direction, and extent).

Contrary to TRI, in which the entire data volume is back-propagated at once, equation 4 has to carry out back propagation from each receiver (say, \(N\) receivers), leading to an \(N\) times increase in the computational cost. To improve the efficiency in our implementation, we group the data and back-propagate each group (Figure 1d) before applying the imaging condition (Sun et al., 2015). We call this technique HyM-TRI. The imaging step is thus replaced with

\[
I_{\text{HyM-TRI}}(x, t) = \int_0^t \prod_{i=1}^g \prod_{j=1}^{n_i} W_d^{(i-1)\times n_i + j}(x, \tau) d\tau,
\]

where \(g\) is the total number of groups and \(n_i\) is the number of receivers in each group \(i\) (\(\sum_{i=1}^g n_i = N\)). Using such data groups leads then to a “\(g\) times” increase in the computational cost (instead of an \(N\) times increase). Empirically, \(g \geq 3\) will be effective enough to help to minimize crosstalk. If we use the entire data as one group, the HyM-TRI reduces the autocorrelation TRI (AC-TRI) (Artman et al., 2010).

To illustrate the resolution of the four above-mentioned imaging techniques (TRI, AC-TRI, M-TRI, and HyM-TRI), we set up an ideal acquisition with a circular receiver array (20 receivers) and one point source (a Ricker wavelet with 40 Hz dominant frequency) located at the origin as shown in Figure 2a. We use a homogeneous acoustic model with a P-wave velocity of 2500 m/s, a density of 2.2 g/cm³, and the wavelength is 62.5 m. The receiver spacing (distance) is 156 m. Figure 2b shows four source images, and Figure 2c compares their vertical cross sections passing through the theoretical source location. The bottom panel in Figure 2c shows that the TRI method gives the lowest image resolution (i.e., it has the highest uncertainty on the source location) with a resolution of approximately \(\lambda/2.5\). Because AC-TRI is almost equal to \(I_{\text{TRI}}^2\), the AC-TRI method (the dashed black line) gives slightly better image resolution and suppresses low-frequency noise (two-side tails). Remarkably, the new \(I_{\text{HyM-TRI}}\) method produces a higher resolution (approximately \(\lambda/12\) at half of the maximum amplitude) than the previous two. HyM-TRI correlates four groups of data to reach a compromise between the low computing cost of TRI and the high image resolution of M-TRI (then allowing a resolution of approximately \(\lambda/5\)). Increasing the number of groups will improve the resolution but at the price of increasing the computation time (see the details in Table 1). The M-TRI and HyM-TRI methods attenuate low-frequency artifacts significantly. Note that we show the stacked images because in this specific case, we are considering a single source.

We also test the robustness of the above four methods in the presence of noise. We add strong noise (signal-to-noise ratio — S/N = 1 of peak amplitude) to a sample trace in Figure 3a, where the signal...
(black) is completely hidden. Figure 3b shows the source images by the four methods with normalized amplitude. It is not surprising that the TRI image is strongly contaminated by the noise. The AC-TRI, M-TRI, and HyM-TRI methods all produce images of the source, whereas HyM-TRI (the red line) is best in terms of the source location (the red line in Figure 3c) and the resolution. But, M-TRI (the blue line) seems to give a shifted location. These tests illustrate the robustness of the HyM-TRI in the presence of noise.

In addition, the peak amplitudes of multiple asynchronous sources after focusing may be very different, which could hide small events. To balance peak amplitudes from different event magnitudes, we design a normalization operator with a local sliding-window

$$\hat{I}(x, t) = \frac{I(x, t)}{\max \{\max_{\nu \in [t-\tau/2, t+\tau/2]} I(x, \nu), \epsilon\}},$$  \hspace{1cm} (6)

where \(\tau\) is the window size and \(\epsilon\) is a small threshold number to avoid dividing by zero. This operator normalizes a time slice (centered at time \(t\)) of the image cube by dividing by its maximum value, so that small-amplitude events are enhanced in the given time window. Similarly, this normalization can be done in space.

**Numerical parameterization of HyM-TRI**

To numerically back-propagate seismic data from receivers, we use the pseudospectral method to solve the second-order constant-density acoustic wave equation. To implement the 2D HyM-TRI method, we group each subset data into four groups divided by its index order (e.g., we divide 21 receivers into four groups as \([1, 2, 3, 4, 5]\) [6, 7, 8, 9, 10] [11, 12, 13, 14, 15] [16, 17, 18, 19, 20, 21]) and we inject the traces of one group at the corresponding receiver locations into the same velocity model as used before.

In the first demo example, we use a velocity gradient model with four assumed microseismic sources (the red dots in Figure 4a). The model is discretized on a 140 x 140 grid with 15 m spacing in the vertical and horizontal directions. The seismic sources are assumed to be a Ricker wavelet with a peak frequency of 20 Hz, with a time step of 1 ms. The recorded surface data are shown in Figure 4b.

We test TRI and HyM-TRI. In HyM-TRI, we divide the data into four groups and each group has 35 receivers. Figure 5 shows snapshots of (a) TRI and (b) HyM-TRI corresponding to four focused sources. The TRI image (Figure 5a) suffers from identifying the focused sources from interfered wavefields from other events, whereas the HyM-TRI image presents four well-focused sources with almost no artifacts (Figure 5b).

**Table 1. Computer time and resolution versus the number of receiver groups in Figure 2.**

<table>
<thead>
<tr>
<th>Group</th>
<th>(g = 1)</th>
<th>(g = 4)</th>
<th>(g = 8)</th>
<th>(g = 20)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Computer time (s)</td>
<td>50</td>
<td>200</td>
<td>400</td>
<td>1000</td>
</tr>
<tr>
<td>Resolution ((\lambda = 62.5) m)</td>
<td>(\lambda/3)</td>
<td>(\lambda/5)</td>
<td>(\lambda/8)</td>
<td>(\lambda/12)</td>
</tr>
</tbody>
</table>

Figure 2. (a) Experiment setup. The red star denotes a point source, and downward triangles \(\nabla\) are receivers. (b) Comparison of the image resolution from four imaging techniques, TRI, AC-TRI, M-TRI, and HyM-TRI. (c) Their cross sections through the center point.

Figure 3. (a) A sample trace with noise (S/N = 1 of peak amplitude) in color red and noise-free trace in black. (b) Comparisons of the image resolution from four imaging techniques, TRI, AC-TRI, M-TRI, and HyM-TRI of noisy data. (c) Their cross sections through the center point.
To process continuous data, due to limited computer memory, we may need to split continuous data into small segments of continuous data. What happens if one event is split? We purposely split the last event by cutting a 1.8 s-length segment of full synthetic data (Figure 4b), where the last event waveform is incomplete. By dividing data into four groups, the HyM-TRI image in Figure 6a presents a slightly weak focused point corresponding to the rightmost event location. Using six groups, because group 1 contains zeros of the last event, the rightmost event focus disappears (see Figure 6b). Therefore, if the careful grouping strategy would be implemented to avoid the situation that one group contain all zeros of the incomplete event, the incomplete event can still be located. Ideally, we should avoid splitting by writing the intermediate $I_{HyM-TRI}(x,t)$ into the disk and then read them for later use.

MODELING OF MICROSEISMICITY

To fully test the HyM-TRI method, we use a synthetic 2D data set. We numerically model a microseismicity scenario (fluid-injection-induced microseismicity) using a realistic distribution of seismic events. This section aims to describe the design of such a synthetic data set.

Rupture propagation modeling

We use a statistical rupture propagation model to define the spatial, temporal, and magnitude distributions of a cloud of microseismic events. Figure 7 shows a horizontally layered geologic model that we use in this example and the associated physical properties are given in Table 2. This represents a relatively simple and realistic model not referring to any specific field site. The injection point is located at $x = 1000$ m and $z = 1025$ m. The 2D distribution of events is defined by a bivariate normal distribution (BND) with horizontal and vertical means $\mu_x = 0$, $\mu_z = 0$, respectively; standard deviations $\sigma_x = 87.5$ m, $\sigma_z = 6$ m, respectively; and a correlation coefficient of one (e.g., Jagdish and Campbell, 1996). These values of $\sigma_x$ and $\sigma_z$ cause the hypocenters’ cloud to develop predominantly in the horizontal direction (e.g., Fischer et al., 2008). The BND defines that the spatial location of the events in 2D space and microseismic sources can be infinitely close but not spatially coincident.

To compute the synthetic seismic data, we use a pseudospectral seismic modeling algorithm that requires the geologic model to be discretized on a regular grid (see “Seismic modeling” section). Consequently, the location of each source is rounded to the closest grid point of the numerical mesh. Figure 8 shows the 2D distribution of 496 microseismic sources. The coordinate system of Figure 8 is centered on the injection point. All of the events are confined within layer 5 (see Figure 7).

The cracks propagate with different velocities along the vertical and horizontal directions (e.g., Fischer et al., 2008). We assume an elliptical distribution of the average rupture velocity with velocities in the horizontal and vertical directions of $v_{rx} = 1$ m/s and $v_{rz} = 0.5$ m/s, respectively. The average velocity of each event is then perturbed by a normal distribution with standard deviation $\sigma_v = 0.1$ m/s. The origin time of the $j$th microearthquake is computed as $t_j^0 = d_j^x/v_{rx} + d_j^z/v_{rz}$, where $d_j^x$ and $d_j^z$ are the horizontal and vertical distances between the injection point and the $j$th event, respectively. Figure 8a shows the origin times of the gridded

![Figure 4](image-url)  
(a) Microseismic source locations overlaid on a P-wave velocity model. (b) A data gather of four events.

![Figure 5](image-url)  
(a) Individual source location by (a) TRI and (b) HyM-TRI at 0.1, 0.6, and 1.1 s. No time integration is applied in equation 5 in this case.

![Figure 6](image-url)  
The HyM-TRI of four events where the last event is incomplete. Comparison of (a) four groups of data and its HyM-TRI image in the right panel and (b) six groups of data and its HyM-TRI image.
microseismic cloud; when a source point is characterized by multiple events, the highest origin time is shown. The seismicity starts from the injection point, which is located at $x = z = 0 \text{ m}$ on Figure 8a.

We compute the event magnitudes with the probability density function (Palacios et al., 2006)

$$f(M_w) = \begin{cases} \beta \exp\left[-\frac{1}{\beta}(M_w - M_{0w})\right], & M_w \geq M_{0w}, \\ 0, & M_w < M_{0w} \end{cases}, \quad (7)$$

where $M_w$ is the moment magnitude, $M_{0w}$ is the minimum magnitude considered in the data set and $\beta = b / \ln(10)$, and $b$ being the $b$ value of the Gutenberg-Richter law. In this study, $M_{0w} = -1$. The computed magnitudes are randomly distributed within the cloud and are in the range $(-1, 0.78)$ with a $b$ value of the Gutenberg-Richter law of 1.42. The $b$ values higher than 1 are typical of microseismic activity related to fluid injection, where the events are caused by rock fracturing due to the increased pore pressure. Figure 8b shows the moment magnitude distribution of the microseismic events. The injection point is again located at coordinates (0, 0).

Figure 9a and 9b shows space-time plots illustrating the temporal evolution of the microseismic cloud in the horizontal and vertical directions, respectively. The magnitude of each event is also indicated. The injection point is located at spatial coordinate 0 m, for the $x$- and $z$-axes, and time 0 s. The seismicity spreads mainly horizontally and symmetrically with respect to the injection point.

Seismic forward modeling

The synthetic seismograms are computed with a 2D modeling code based on the isotropic viscoelastic stress-strain relation (e.g., Carcione, 2014). The algorithm uses a staggered Fourier pseudospectral method for computing the spatial derivatives and a fourth-order Runge-Kutta time scheme for calculating the wavefield recursively in time, which is used to minimize the numerical dispersion in long-time simulations. The anelasticity is described by standard linear solid (Zener) model, with one relaxation mechanism. The numerical grid discretizing the geologic model (Figure 7), has 1024 gridpoints in the horizontal direction ($x$) and 512 gridpoints in the vertical direction ($z$), with a constant grid spacing of $dx = dz = 2.5 \text{ m}$. The time-sampling interval used in the computation is $dt = 0.1 \text{ ms}$. Absorbing boundary conditions are set around the physical domain using perfectly matched layers (Martin et al., 2010); we observe no edge reflections on the records.

Each point source is modeled as a pure compressional/dilatational stress of the rock (explosion). For each point source, the time...
History is then a Ricker time function with a dominant frequency of 50 Hz. The amplitude of each explosion is set independently through the seismic moment $M_0$, which is computed as (Hanks and Kanamori, 1979)

$$M_0 = 10^{(2(M_s + 10.7))},$$

using the moment magnitude $M_s$ values shown in Figure 8b, and where $M_0$ is given in erg ($1\text{erg} = 10^{-7} \text{kg} \times \text{m}^2/\text{s}^2$).

We record the pressure field using surface and borehole seismic receiver arrays. The surface array contains 1024 traces (one for each gridpoint), and the two borehole arrays (located at $x = 500$ and 1700 m, respectively) contain 512 traces each. The full synthetic microseismic data are 260 s long with a time sampling of 2.5 ms. Figure 10a shows the synthetic seismograms recorded at the surface receiver array for a limited time window, which shows that some events are very close in time. The noisy data (see Figure 10b) are produced from the noise-free synthetic data by globally adding a Gaussian random noise (with a variance of 0.0001) using $\text{sfnoise}$ of the open-source platform Madagascar (Fomel et al., 2013). Using noise-free and noisy synthetic data, we devise a few numerical experiments to show the features of the HyM-TRI method in the “Synthetic examples” section.

Because of limited computer memory, to deal with this 260 s synthetic data to produce the HyM-TRI image (equation 5), we equally split our 260 s synthetic data into 10 subsets each 26 s in length.

SYNTHETIC EXAMPLES AND RESULTS

To show the advantages and limitations of the HyM-TRI method, we devised six numerical experiments: two synthetic tests imaged with a complete receiver array data, and noisy data and then four synthetic tests imaged with sparse receiver arrays, limited-aperture array, single downhole array, and dual downhole arrays. We detail these experiments and their results in the four following subsections.

Ability of source separation in space and time: TRI versus HyM-TRI

First, we compare the cumulative source images obtained by TRI and HyM-TRI from the noise-free and noisy synthetics to test their performance in the case of closely occurring sources. Figure 11a and 11c displays snapshots of the TRI results, and Figure 11b and 11d presents their corresponding HyM-TRI reconstructions. First of all, random noise apparently pollutes the TRI images but does not influence the HyM-TRI results due to multiplication and causal integration (Figure 11c and 11d). Second, the HyM-TRI results give high-resolution spike-like source images, whereas the TRI results show focused energy with sidelobes and a large width (Figure 12a and 12b). Moreover, the TRI images contain more artifacts (patterns of intersecting rings). Based on the maximum amplitude, we pick the source locations, and seven of them from HyM-TRI and TRI exhibit small deviations ($<30 \text{m} \approx \lambda/3$) from the true event locations. And, due to interfering waves from other events, the fourth picked location (the fourth panel in Figure 11a) is far from the true location (error: $\approx 270 \text{m} \approx 3\lambda$). In reality, this is often the case when the later arrivals from different sources interfere with the previous focused source when two events are close in time. Figure 13 shows the stacked TRI and HyM-TRI results of accumulating the images in the interval (60.2, 61.9) s in Figure 11 to collapse the time axis. The TRI method provides a low-resolution source image with artifacts, whereas the HyM-TRI method still performs better and cleaner, even with strong noise. We conclude that the HyM-TRI method shows higher resolution in space and time than the standard TRI method (Figure 11).

Full 260 s synthetic data

We now apply the proposed method to all the events. Figure 14a shows six snapshots of the cumulative source images using the HyM-TRI method (equation 5). The area defined by the red dashed line shows the actual “source-propagating” (fracture front, which is reasonably constrained inside the imaged source area. The color scale indicates the image amplitude. Rather than picking the maximum amplitude as the source point, we prefer to have the imaged source area. We argue that the colored area may better present the resolution of the imaging method. From top to bottom, we can estimate the rupture direction, propagation velocity, and extent. We find that the rupture is bilateral and propagates horizontally up to 250 m (see the bottom panel in Figure 14a). We show the imaging of the noisy data in Figure 14b. Compared with Figure 14a, the imaged source areas are not very different because the noise does not add up coherently over all the receivers when applying the multiplication. At late times (final 50 s), the sparsity of the events (six events) might cause the stack of noise (the bottom panel in Figure 14b) appearing as strong artifacts outside the source region. Again, the evolution of sources in space and time is tracked temporally and the rupture is bilateral and propagates horizontally up to 250 m (see the bottom panel in Figure 14a).

In this case, the estimated horizontal propagation velocity can be confidently estimated, at approximately $250 \pm 40 \text{ m/s}$/250 s $= 1 \pm 0.16 \text{ m/s}$, which reasonably approximates the true rupture propagation velocity $1 \pm 0.10 \text{ m/s}$. The rupture propagation in the vertical direction is not linear and reaches its maximum extent in approximately 30 s (Figure 9b). We show the imaged sources between 5
and 30 s in Figure 15. The rupture propagates along the horizontal and vertical directions (see arrows). Because of the lower resolution (elongated focus) in the vertical direction, the estimation of the vertical propagation velocity is more uncertain.

Sparse and limited-aperture acquisition

So far, we have considered a very ideal data-availability scenario: First, we used a dense array (1024 receivers); second, this array had a perfect regular distribution (constant receiver spacing of 2.5 m); and third, this array was fully covering the cloud of events (the surface array distribution was centered on the injection point). Such a favorable situation may actually not be achievable in field experiments. To simulate a more realistic scenario, we first consider using random subsets, reducing the dense array to 21 and four receivers, as shown in Figure 16a and 16b, respectively. Then, we apply HyM-TRI as described above. Figure 17a and 17b shows the source imaging sections for the 21-receiver experiment and for the four-receiver experiment, respectively. With 21 receivers distributed from 0 to 2500 m, the imaged sources are not visually different from the imaging with all receivers (Figure 14). When reducing the data to four receivers located at 250, 875, 1500, and 2250 m, the imaged sources show artifacts. We find that a too-sparse receiver array significantly degrades the vertical resolution but still has little impact on the horizontal resolution.

We also test a unilateral coverage of the cloud events by using only the left side of the surface array with all 512 receivers distributed evenly from 0 to 1250 m (Figure 17c). The imaged source area in Figure 17c obviously exhibits a directional effect on the imaging due to the sole contribution of the left-side receiver wavefield. Again, the vertical resolution gets a little worse but the horizontal resolution changes only subtly.

Single and dual downhole arrays

In the previous subsection, we show that surface receiver arrays provide good horizontal resolution of the source imaging. Here, we show that downhole receiver arrays can complement surface arrays in the vertical direction. Downhole receiver array logging is a very common acquisition in practical microseismic surveys. We first consider a single vertical downhole array at a distance of 1700 m, as shown in Figure 7. The depth of the array is from 2.5 to 875 m (with

![Figure 11. Comparison of (a and c) TRI and (b and d) HyM-TRI at eight successive times using noise-free and noisy data shown in Figure 10a and 10b, respectively. At 60.81 s, interfered artifacts in the fourth panel of (a) have larger amplitude and the maximum amplitude deviates from the true location.](http://library.seg.org/)
350 receivers), i.e., stopping just above the injection layer. Figure 18a shows the source imaging sections. The maximum amplitude is trapped around the interface. This is possibly because multiplication of different groups of borehole receiver array data spreads wavepath energy in the area between the true source locations and the well.

One solution is to use multiple wells for monitoring because recent case studies (Warpski et al., 2007; Douglas et al., 2007; Murer et al., 2012) show that this improves the accuracy of microseismic locations. We simulate a second downhole array at a horizontal distance of 500 m. We use the same array length as the previous one (positioned at 1700 m). In the implementation of 350 receivers for each array, we group them into six contiguous groups. We find that the resolution of the source image with two wells that have a wide aperture has been improved significantly, as shown in Figure 18b.

**APPLICATION TO FIELD MICROSEISMIC DATA**

In this section, we apply our HyM-TRI method to 3D field microseismic data that were collected during Marcellus shale hydraulic fracturing in Pennsylvania, USA (Tan and Engelder, 2016). The surface monitoring arrays are designed to be a star-shape distribution with 1082 single-component geophones in Figure 19. The 3D layered velocity model used for imaging is constructed from a 1D sonic log. The raw data (single z-component) were processed with DC removal and a band-pass filter (5–8–30–50 Hz) in ProMAX software, and the P- and S-wave phases of one event are clearly identified in Figure 20. For HyM-TRI, we group the data into...
one group, five groups, and 10 groups as the 10 star-shape arrays in
Figure 19 and we integrate the full record length (2.7 s).

The computational 3D model is discretized into a $201 \times 232 \times 161$
regular grid. The grid spacings are $x = 24.4$ m, $y = 24.4$ m, and
$z = 15.2$ m. The time step is 1.5 s. We use a 3D finite-difference
acoustic wave modeling scheme to back propagate data for 3D
HyM-TRI. The finite-difference scheme is sixth-order accurate in
space and second-order accurate in time.

Figure 17. Cumulative source images at times 0.25, 20, 80, 140, 200, and 250 s using
HyM-TRI method with (a) only 21 traces shown in Figure 16a, (b) only four traces
shown in Figure 16b, and (c) left-side of seismogram shown in Figure 16c. The dashed
red line overlain on the image shows the true locations of the corresponding sources.

Figure 18. Cumulative source images at times 0.25, 20, 80, 140, 200, and 250 s using
HyM-TRI method with (a) single (right-side) downhole array at 1700 m, (b) two downhole arrays (left-side array
at 500 m). The dashed red line shows true locations of correspond-

Figure 21 shows the HyM-TRI image of the waveforms with one
group, five groups, and 10 groups, respectively. The HyM-TRI with
one group in Figure 21a is equivalent to the AC-TRI (close to TRI).
With more groups, the HyM-TRI image exhibits a much cleaner
focus of the event that is close to the location provided by the third
party. But, a few artifacts appear around the sharper focus, which
may be caused by the complex P waveforms. We also examine the
HyM-TRI result with 20 groups, and it is very close to the one with
10 groups. Other observations are that (1) the noise does not seem to affect the quality of the
focus and (2) the S-wave focus is not visible in the
HyM-TRI image. Partially, because the S
wave energy is relatively small and the imaging
velocity is the P-wave velocity, it is not expected
that the S-wave focus coincides with the P-wave
focus. However, the low-resolution AC-TRI
image exhibits strong artifacts above the true
location.

Next, we examine the other nine events and
estimate the location by searching for the maxi-
mum value. All locations are compiled in Table 3.
Because we use surface arrays, the error in depth
location is relatively large (maximum errors
153 m). By projecting to the top view, all esti-
imated locations are shown in Figure 22 as stars
versus the reference locations (crosses). Consid-
ering that different locating methods and differ-
ent velocity model are used in the commercial
processing, we believe that the locations are sat-
sfactorily determined by HyM-TRI.

**DISCUSSION**

In this study, we present the HyM-TRI method
based on multiplication between receiver wavefields rather than
stacking in TRI. This multiplication can enhance the source focus,
and it also suppresses nonsource artifacts. With surface monitoring
arrays, the single source image by HyM-TRI is well-focused in
space and time with almost no artifacts. In general, the horizontal
resolution is higher than the vertical resolution in source images

Figure 19. The 3D view of the wells (in red) and the surface geo-
phone arrays (black). The P-wave velocity model is constructed
from the sonic log.
(e.g., Figures 9b, 9d, and 11). The vertical resolution is dependent on the azimuth of the receiver geometry with respect to the microseismicity area. In practice, the wide azimuth coverage condition is not hard to satisfy by designing the surface microseismic survey with several star-shaped arrays (Duncan and Eisner, 2010), as is used in our field-data example. In addition, our synthetic example results (2D) encourage us to show that the horizontal and vertical evolutions of the microseismic rupture can be monitored, even with a sparse monitoring array.

Compared with the M-TRI method, the HyM-TRI has three merits: (1) It is using a hybrid strategy to group receivers to reduce the computational cost in practical 3D applications, (2) it can produce a time-dependent image that represents the (spatial and temporal) evolution of asynchronous sources, and (3) it is robust in the presence of noise.

There is no general rule for grouping, and how to group the data likely depends on the receiver distribution. Maximizing the multiplication of all grouped receiver wavefields from a larger aperture data array will enhance the focus. Based on our experience, and with a dense array coverage, we found that several groups (usually fewer than 10) retain enough imaging resolution from the pure M-TRI, while reducing the computational cost to a reasonable level.

The possible waveform polarity due to the source complexity (e.g., double-couple or nondouble-couple sources) likely leads to defocusing in the source image if we use raw waveforms. In this case, it is necessary to preprocess the waveforms before the application of the source imaging methods (e.g., TRI and HyM-TRI). For example, McMechan et al. (1985) preprocess the earthquake waveforms to construct a true amplitude section by filtering and extrapolating waveforms with a given velocity model. Beskardes et al. (2018) compare three methods (envelope, short-term averaging/long-term averaging, and kurtosis) to regularize the waveform as an input of waveform-based source imaging.

Because TRI and HyM-TRI are wave-equation-based methods, similar to active-source reverse time migration, they are sensitive to the errors of the a priori velocity model derived from sonic logs or seismic tomography. The velocity errors are likely propagated to the source location in space and time. One solution is to use the sophisticated full-waveform inversion of perforation shots to improve the velocity model. A more ambitious strategy is to simultaneously estimate the source location and velocity model using joint full-waveform inversion (Sun et al., 2016).

The 2D example (260 s length) with four groups of receivers, run in parallel with OpenMP, took approximately 1 h in a Linux workstation (Intel Xeon central processing unit [CPU] v4 3.00 GHz) with 40 threads. Because the proposed HyM-TRI method involves computing wave propagation from each group of receivers, the computational cost of HyM-TRI is proportional to the number of groups and is g times higher than the conventional TRI technique, where g is the number of groups in HyM-TRI. In the 3D field-data example, with 10 groups of waveform data (2.7 s length), the HyM-TRI took approximately 2 h on the same computing architecture. The current

Figure 20. Waveform data of an example microseismic event after preprocessing (DC removal and band-pass filtering).

Figure 21. Imaging results of one event by HyM-TRI with one group, five groups, and 10 groups, respectively. The red line is across the reference event location.
Table 3. Reference locations \((x_{\text{ref}}, y_{\text{ref}}, z_{\text{ref}})\) (by Schlumberger), estimated locations \((x_{\text{est}}, y_{\text{est}}, z_{\text{est}})\), and the absolute value of location errors (e.g., \(|x_{\text{est}} - x_{\text{ref}}|\)) of 10 events. Units are in meter

<table>
<thead>
<tr>
<th>Events</th>
<th>(x_{\text{ref}})</th>
<th>(y_{\text{ref}})</th>
<th>(z_{\text{ref}})</th>
<th>(x_{\text{est}}(\text{errors}))</th>
<th>(y_{\text{est}}(\text{errors}))</th>
<th>(z_{\text{est}}(\text{errors}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2170</td>
<td>2756</td>
<td>2012</td>
<td>2072 (98)</td>
<td>2877 (121)</td>
<td>1981 (31)</td>
</tr>
<tr>
<td>2</td>
<td>2853</td>
<td>3097</td>
<td>2012</td>
<td>2853 (2)</td>
<td>3097 (2)</td>
<td>2012 (6)</td>
</tr>
<tr>
<td>3</td>
<td>2682</td>
<td>3121</td>
<td>1981</td>
<td>2516 (174)</td>
<td>3170 (49)</td>
<td>2134 (153)</td>
</tr>
<tr>
<td>4</td>
<td>2341</td>
<td>2828</td>
<td>2027</td>
<td>2292 (49)</td>
<td>2706 (122)</td>
<td>2134 (107)</td>
</tr>
<tr>
<td>5</td>
<td>3072</td>
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<td>1981 (31)</td>
</tr>
<tr>
<td>6</td>
<td>3316</td>
<td>3536</td>
<td>1981</td>
<td>3048 (268)</td>
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<td>1981 (5)</td>
</tr>
<tr>
<td>7</td>
<td>2780</td>
<td>2950</td>
<td>1981</td>
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<td>2926 (24)</td>
<td>2134 (153)</td>
</tr>
<tr>
<td>8</td>
<td>3194</td>
<td>2658</td>
<td>1996</td>
<td>3414 (97)</td>
<td>2926 (24)</td>
<td>2134 (122)</td>
</tr>
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<td>9</td>
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<td>2012</td>
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<td>2660 (2)</td>
<td>2134 (138)</td>
</tr>
<tr>
<td>10</td>
<td>2292</td>
<td>3146</td>
<td>1936</td>
<td>2292 (2)</td>
<td>3194 (38)</td>
<td>1981 (45)</td>
</tr>
</tbody>
</table>

Figure 22. Comparison of the estimated locations (dots) by HyM-TRI and the reference locations (crosses) provided by Schlumberger. We use a different color for each event, with the “+” symbol being our locations and “o” being Schlumberger’s location. The same event is shown in the same color. The color bar denotes depth errors from the reference locations.

high-performance cluster (HPC) architecture would speed up its implementation. In a recent study, Xue et al. (2016) demonstrate that, with a realistic 3D microseismic monitoring geometry, a microseismic event location method based on TRI is typically 30 times faster using a graphics processing units implementation than with its CPU counterpart. This suggests the potential of the HyM-TRI technique toward real-time imaging with state-of-the-art HPC clusters.

CONCLUSION

We have presented an HyM-TRI algorithm for automatically tracking the spatiotemporal distribution of many microseismic events. HyM-TRI back propagates the data traces from a group of receivers (in space and time) as receiver wavefields, multiplies receiver wavefields between all groups, and applies a causal integration over time to obtain a source evolution image. We evaluated the HyM-TRI technique in the synthetic and field data sets. Our three main conclusions are

1) Although the standard TRI method — due to the source sink — does not allow us to map the migration of multiple asynchronous sources (rupture process), the HyM-TRI can present the spatiotemporal evolution image of sources, i.e., rupture parameters (e.g., the hypocenter point, rupture propagation velocity, direction and extent).
2) With ideal noise-free data, the M-TRI can achieve the highest resolution (approximately \(\lambda/12\)) of the image of a single point source with high computational costs but the M-TRI tends to introduce artifacts in the presence of strong noise in the data. With several groups, HyM-TRI preserves satisfactory resolution with a much lower computational cost and, more importantly, is more resilient to noisy data.
3) The aperture of the 2D receiver array (azimuth coverage in 3D) with respect to the microseismic source area plays an important role on the horizontal and vertical resolution of the source image. The HyM-TRI results of the field data with 3D azimuthal coverage further verify our argument by producing a superior resolution of the source than TRI.

We anticipate that HyM-TRI can be applied to a variety of passive seismic cases, e.g., microseismic monitoring of subsurface injected \(\text{CO}_2\) leaks into the caprock and the geothermal activity, and earthquake locations.

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DATA AND MATERIALS AVAILABILITY

Data associated with this research are confidential and cannot be released.

REFERENCES
