An approach to compensate for attenuation effects in reverse-time migration
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SUMMARY

Strong attenuation anomalies can significantly lower the resolution of the seismic image in reduced amplitude, shifted phase, and frequency content loss. To overcome these attenuation effects, I report a method to compensate for attenuation effects in reverse-time migration. Because attenuation effects include amplitude loss and velocity dispersion, to compensate for attenuation effects needs to take care of both. Unfortunately, conventional viscoacoustic/elastic modeling is difficult to use for such the attenuation compensation. In contrast, I show that it is easy to implement attenuation compensation in a novel viscoacoustic wave equation by reversing the sign of the absorption operator and leaving the sign of the dispersion operator unchanged. By testing this method in several synthetic examples, I believe that the proposed attenuation compensated imaging approach is promisingly helpful to improve the resolution of seismic images.

INTRODUCTION

Recently there are lots of attempts to perform attenuation compensation for RTM imaging using existing time domain two-way wave equations, such as the damping viscoscalar wave equation (Deng and McMechan, 2007) and the standard linear solid model for (Deng and McMechan, 2008). Zhang et al. (2010) derived a constant-Q wave equation using pseudo-differential operator and then modified it for extrapolating receiver wavefield with a normalized operator. But it is more or less ad hoc to build the normalized operators for modeling time reverse propagation in attenuating media.

In an earlier paper (Zhu and Harris, 2014), the time domain constant-Q wave equation is derived for modeling wave propagation in attenuating media. This viscoacoustic equation can accurately model seismic wave attenuation and dispersion behavior, say, constant Q in seismic frequency band. More importantly, the proposed wave equation has the decoupled attenuation and dispersion operators that have approved to be advantageous for attenuation compensation during time reverse modeling (Zhu, 2014). This proposed wave equation serves as the basis for the development of reverse time migration in attenuating media.

Based on this wave equation, I developed an RTM approach of compensating for attenuation effects (include amplitude loss and dispersion effects) via changing the sign of amplitude loss operator in the forward and backward propagation. The key here is reversing the sign of amplitude loss term but leaving the sign of dispersion term unchanged. Therefore, the proposed wave equation for backward propagation is physically time-invariant. Following that, I use the Marmousi model to illustrate effectiveness of compensating attenuation and dispersion effects during RTM.

METHODOLOGY

In this section, I will explain principally why we need to implement different treatment to amplitude loss and phase dispersion when we compensate for attenuation effects. Then, I briefly review the viscoacoustic wave equation with decoupled attenuation operators. Further, I introduce the methodology for implementing backward propagation via compensating for attenuation effects using the adjoint constant-Q wave equation.

Decoupled dispersion and loss for attenuation compensation during back-propagation

As shown in the top of Figure 1, an explosive source (red star) sends a signal into an attenuating medium. The waveform experiences attenuation during the forward propagation. For the principle of velocity dispersion in attenuating media, we knew that higher frequencies (labeled by ‘H’ in Figure 1) travel faster than lower frequencies (labeled by ‘L’ in Figure 1). Therefore, at receivers high frequencies advance to lower frequencies in the recorded waveform data. To back propagate the recorded data, I first flip the recorded data in time. Now lower frequencies are in the front of higher frequencies. Then, the reversed data is injected as boundary condition at receivers. If we can use the same velocity dispersion, i.e., higher frequencies will again travel faster than the lower frequencies, therefore they reconstruct the wavefields at the propagation time and finally arrive simultaneously at the original source. If not, high frequencies never catch up with low frequencies. We cannot reconstruct the wavefields. At the same time, the amplitude needs to be amplified. It means that we have to apply the ‘opposite’ process to boost the amplitude of the signal. Therefore, backward propagation with attenuation compensation applies the amplitude recovery rather than amplitude loss but the same dispersion effect. In the mathematical treatment, we have to treat the amplitude loss and velocity dispersion differently.
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Finally, the receiver wavefields can be reconstructed as the corresponding one in the mirrored forward propagation time. For example, the wavefield at forward propagation time \( t \) is equal to that at the backward propagation time \( T - t \).

\[
\eta = -c_0^{2γ} α_0^{2γ} \cos πγ \quad \text{and} \quad \tau = -c_0^{2γ-1} α_0^{2γ} \sin πγ,
\]

where the variable \( γ \) is defined as \( γ = 1/π \tan^{-1}(1/Q) \). The range of \( γ \) is \( 0 < γ < 1/2 \) with any positive \( Q \). The first term in the right side of equation 1 is related to dispersion effects, and the second is related to amplitude loss effects (Zhu and Harris, 2014). This is in contrast to other wave equations in which the attenuation and dispersion are encapsulated by a single term (Deng and McMechan, 2007, 2008; Carcione, 2010). In the following section, I show the viscoacoustic wave equation (1) to be advantageous for compensating for both amplitude loss and velocity dispersion effects in RTM.

Compensate for attenuation effects in RTM

Mathematically, back-propagation (time reverse) involves replacing time \( t \) by \(-t\) in equation 1. Accordingly, the viscoacoustic wave equation for back-propagation is given (Zhu, 2014)

\[
\frac{1}{c_0^{2}} \frac{∂^2 p_{b}}{∂t^2} = ω\mathbf{L} p_{b} - ω\mathbf{H} \frac{∂}{∂t} p_{b},
\]

which \( \tilde{t} = -t \) is the reversed time. \( p_{b}(x,−t) \) is a solution of equation 4 with the sign of the attenuation term reversed. The negative sign functions to compensate for attenuation effects during the propagation of the reversed wavefields. I emphasize that the first, dispersion-related term on the right hand side of equation 4 is time-independent and does not reverse sign (i.e. the frequency-dependent phase velocity remains unchanged in time). This explanation also appears to justify why we treat dispersion and amplitude loss operators differently.

To prevent high-frequency noise from growing exponentially, I apply a low-pass filter to the attenuation and dispersion operators in the spatial frequency domain when calculating the time-reversed wavefields. The implementation detail of attenuation compensation in RTM can be found in Zhu et al. (2014).

APPLICATIONS

I consider a realistic Marmousi model. Figure 2 shows velocity and the \( Q \) models. The top high attenuation zones are typically caused by the presence of a gas chimney. The size of the model is a \( 281 \times 1001 \) grid, with a grid spacing of \( dx = dz = 12.5 \) m. I used 20 grids PML absorbing boundary in the four sides to eliminate edge reflections. I had 97 sources, with each source being a Ricker wavelet with a center frequency of 20 Hz. The time step is 1 ms. The shot interval is 125 m, whereas the receiver spacing is...
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12.5 m with a total of 959 receivers. Sources and receivers are located at a depth of 62.5 m.

First, I generated two synthetic datasets - acoustic ($Q = \infty$) and viscoacoustic. They were calculated using a forward modeling scheme (equation 1). The reflections are attenuated in the viscoacoustic data, in particular reflections from the deep structures underneath the strong attenuation anomalies. I performed two types of RTM – acoustic RTM and the proposed attenuation compensated RTM.

For comparison, I produced the reference image by migrating acoustic data using acoustic RTM shown in Figure 3a. In the non-compensated case in Figure 3b, I used acoustic RTM to migrate viscoacoustic data. We found that the structure beneath the gas chimney zone lacks illumination. The anticline structure at the depth of 2 km disappears. The attenuation broadens the signals and absorbs the energy. Although I applied AGC to the non-compensated image and the amplitude of the image is boosted, the anticline structures at the depth of 2 km appear to the wrong location (not shown). Also, the shifted phase of the image at the bottom is compared to the reference one in Figure 3a.

Instead, I applied the proposed attenuation compensated RTM to the viscoacoustic data. I chose a Tukey filter with a cutoff frequency of 100 Hz and a taper ratio of 0.2 to suppress the noise growth during attenuation compensation.

Figure 2. Marmousi models: P-wave velocity (a) and $Q$ (b). The blue zones indicate low-$Q$ (high attenuation) gas cloudy areas.

Figure 3. (a) Acoustic RTM of acoustic data. (b) Acoustic RTM of viscoacoustic data. (c) $Q$-compensated RTM with viscoacoustic data regularized by a filter with cutoff frequency of 100 Hz.
Figure 3c shows the compensated image. Overall the amplitude is recovered. The top of anticline below the gas chimney zone appears to be better illuminated. The image resolution of the complex thin layer inside the gas chimney zone is improved, better interfaces continuity.

To show the dispersion effect in the image, I deliberately used the nondispersive operator $\eta L = V^2$ rather than the dispersive operator in equation 4. The velocity is nondispersive at the computational frequency range. I still reversed the sign of the absorption operator and re-ran RTM as above. The results are shown in Figure 4 (zoomed in the target area). Clearly, the amplitude after only absorption compensation is boosted, but the imaged anticline structure is still hard to track (see green arrows). This is because the amplitude of image may not be necessarily enhanced when there still exist phase shifts between source and receivers wavefields.

To look at the details of these images, I selected a depth cross-section at the lateral distance of 8 km. These comparisons are shown in Figure 5. Apparently, the middle panel (Figure 5b) gives the best match. In the compensated image (Figure 5b), I compensated for both amplitude loss and velocity dispersion. Therefore, it is remarkable that the phase and amplitude after compensation in Figure 5b are similar to those of the reference image in Figure 5a. Figure 5c shows the details how the only amplitude compensated image is off from the truth.

CONCLUSIONS

This study focuses on understanding the impact of attenuation effects on seismic images. Attenuation deteriorates seismic images by absorbing the amplitude and distorting the phase. The resulting images are dimmed for seismic interpretation to track structure interfaces. To improve the image resolution by reducing uncertainties, we proposed a strategy of compensating for attenuation effects by treating absorption and phase dispersion operators differently during propagation. Numerical tests clearly indicate that the reflectors in the conventional RTM might be dimmed due to high-attenuation geological environments but can be better imaged by this attenuation compensated RTM. It is worth noting that the phase dispersion ignored in practice as well as the absorption contribute to the recovery of the amplitude as well as the resolution of the seismic image.

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